

## Topic 7 (Day 1) - 6.1 Slope of a Line

**Slope** is a measure of steepness. Some real life examples of slope include:

- in building roads one must figure out how steep the road will be (See pg 343 “The World of Math”)
- skiers/snowboarders need to consider the slopes of hills in order to judge the dangers, speeds, etc
- when constructing wheelchair ramps, slope is a major consideration
- when building stairs, one must consider the slope of them so they are not too steep to walk on
- when building roller coasters slope is considered both for safety and entertainment
- Steepness of a roof is considered in the safety of walking on a roof. Steeper roofs are more expensive to shingle.

Formula

$$\text{SLOPE} = m = \frac{\text{Rise}}{\text{Run}}$$

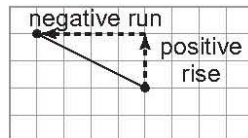
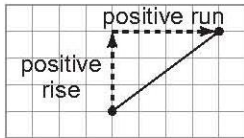
**RISE:**The change in the vertical distance.

Moving up is **positive**.....Moving down is **negative**.

**RUN:**The change in the horizontal distance.

Moving right is **positive**.....Moving left is **negative**.

**SLOPE:** Measures the **Steepness** of a line. We use the letter “m” to represent slope.

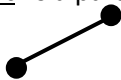


**Concept #28** - 6.1 Correctly determine the slope of a line or line segment using the graph or the formula when given two points, explain the meaning of zero or undefined slopes, draw a line given its slope and a point on the line (**NC**) (**Skill**)

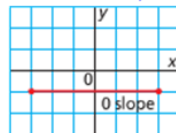
Definitions:

**Line:** a straight one dimensional figure having no thickness and extending to infinity in both directions.

**Line Segment:** is a part of a line that is bounded by two distinct end points and contains every point on the line between its endpoints.



**Zero Slope**



For a horizontal line segment, the change in y is 0 and x increases. The rise is 0 and the run is positive.

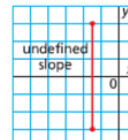
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \frac{0}{\text{run}}$$

$$\text{slope} = 0$$

So, any horizontal line segment has slope 0.

**Undefined Slope**



For a vertical line segment, y increases and the change in x is 0. The rise is positive and the run is 0. For a vertical line segment, y could decrease and the rise would be negative.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

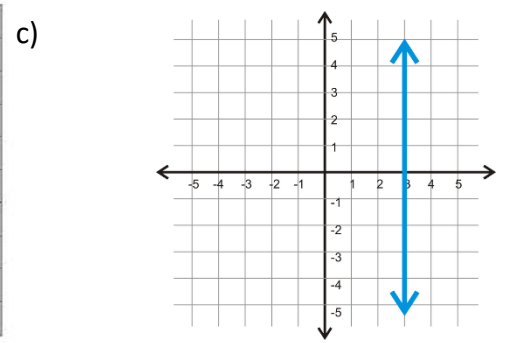
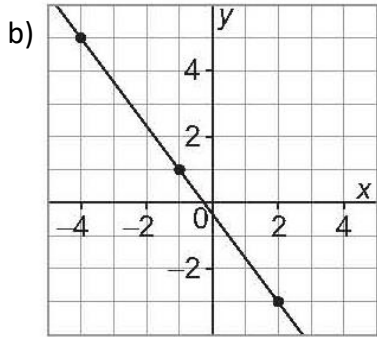
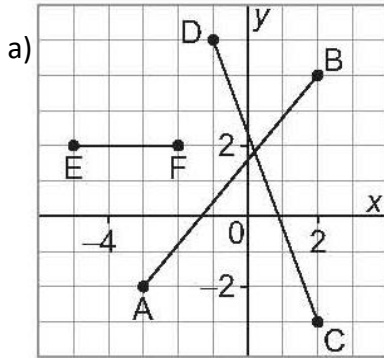
$$\text{Slope} = \frac{\text{rise}}{0}$$

A fraction with denominator 0 is not defined.

So, any vertical line segment has a slope that is undefined.

The slope of a line that is approaching a perfectly vertical line would have to be infinity, but infinity is not a real number, so we just have to say that the idea of slope is not defined or doesn't make sense for a vertical line. Due to different interpretations in different situation, we sometimes see the answer to a vertical slope written as **UNDEFINED** but sometimes the answer will also be given as **INFINITY**.

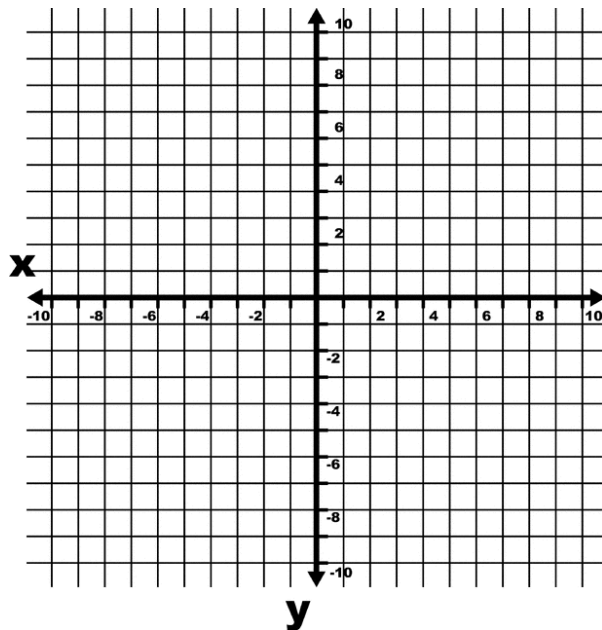
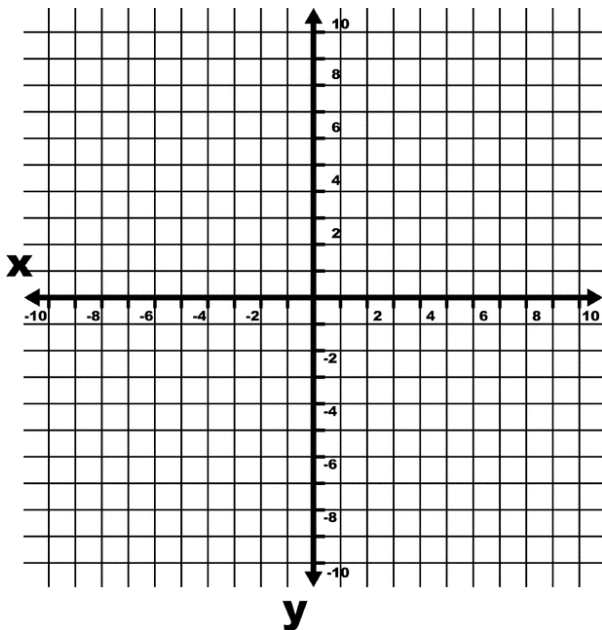
**Example #1:** Find the slope of each line or segments in the following diagrams.



**Example #2:** a) Draw line segments that have the following slopes. .

i)  $m = \frac{2}{5}$  (start at the origin)

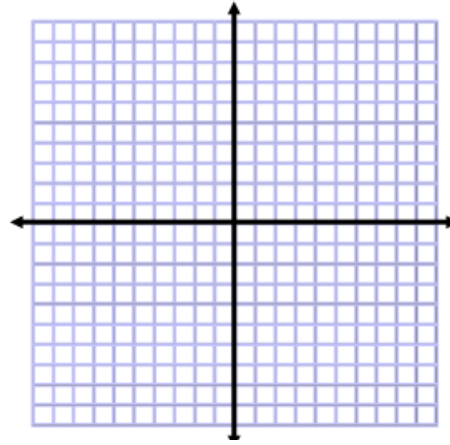
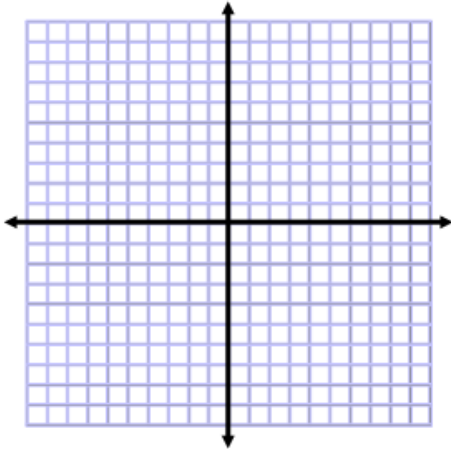
ii)  $m=4$  (Start at the point (3, -2))



b) Draw a line that has the following slopes.

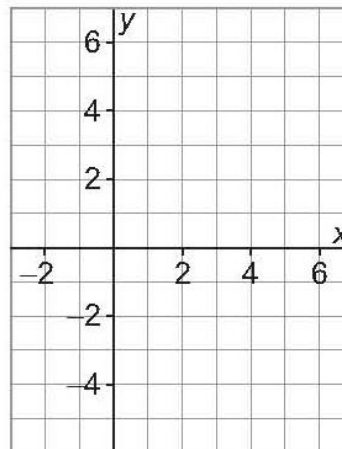
i)  $m = \frac{-7}{3}$  (Choose any starting point you wish)

ii)  $m = -2$  (Start at  $f(4) = 3$ )



**Example #3:** Find the slope between the points

A(-2, -4) and B(3, 5) by drawing the points

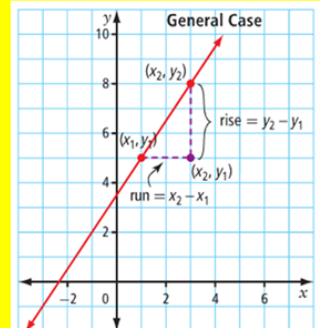


Instead of drawing a line when given two points that are on the line to find the slope we can use the “Two point slope formula”

### Slope of a Line

A line passes through A( $x_1, y_1$ ) and B( $x_2, y_2$ ).

$$\text{Slope of line AB} = \frac{y_2 - y_1}{x_2 - x_1}$$



Slope Formula:

Given two different ordered pairs of the form (x, y) we will call the first ordered pair ( $x_1, y_1$ ) and the second ordered pair ( $x_2, y_2$ ). To find the slope between these points we will use the following formula:

$$\text{Slope} = m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note: Later in Calculus we will use the following notation: Slope =  $\frac{\Delta y}{\Delta x}$

**Example #4** Find the slope using each piece of information from a linear relation.

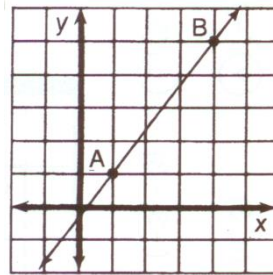
a) A(4, -7) B(-2, 6)

b) A linear function,  $y=f(x)$  has these conditions:

$f(-3)=9$  and  $f(1)=9$

d) A(6,-4), B(8, -4)

e)



**Example #5:** A ladder is leaning against a wall with a slope of 0.375. If the ladder has a rise of 3m what is the run? What does the run represent in this situation?

# REVIEW OF Linear Relations

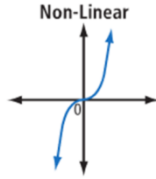
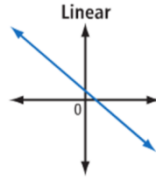
## REVIEW MATH 9

### How can we tell if a relation is linear?

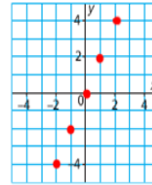
Linear vs Non-Linear Relations

➔ From a graph....

Linear relations have graphs that are straight lines



Linear



➔ From an equation....

When a linear relation is written as an equation, it will contain one or two variables and its degree will be 1.

#### Linear Relations

$$x = 7$$

$$3m + 2n = -12$$

$$y = -\frac{2}{3}x + 5$$

#### Non-Linear Relations

$$2x + y^2 = 6$$

$$h = k^3$$

$$xy = 3$$

➔ From a table of values...

In linear relations, values of y increase or decrease by a constant amount as values of x increase or decrease by a constant amount. Horizontal and vertical lines are exceptions.

#### Linear Relation

	x	y	
+1	2	8	+3
+1	3	11	+3
+1	4	14	+3
	5	17	

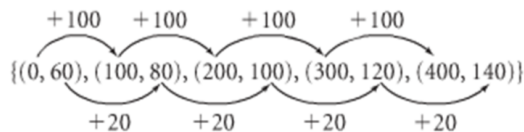
#### Non-Linear Relation

	x	y	
+1	2	8	+19
+1	3	27	+37
+1	4	64	+61
	5	125	

➔ From ordered pairs....

A linear relation will have ordered pairs where the first coordinates will increase or decrease at a constant rate as the second coordinate will increase or decrease at a constant rate. It is important that the first coordinates are listed in numerical order!

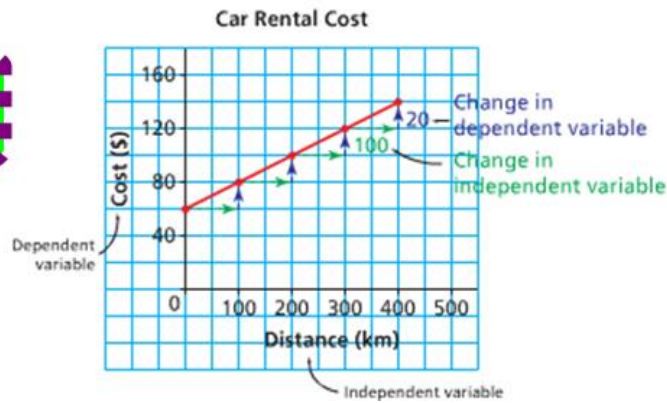
#### Linear Relation



## Topic 7 ( Day 2) - 5.7 Interpreting Graphs of Linear Functions

**Concept #29** -5.6/5.7 Understand and determine the rate of change of a linear relation (NC) (Skill & Problem Solving)

### Rate of Change



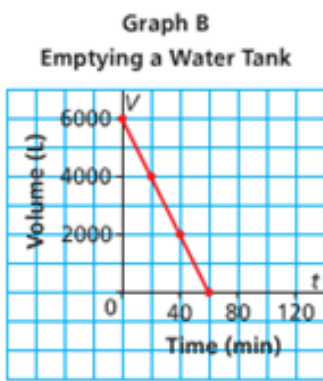
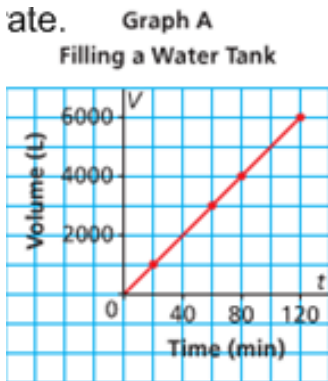
The rate of change can be expressed as a fraction. The rate of change is the slope of the line but given context to the situation. When a situation has context, the rate of change has units. These units should be included in the calculation. They help us to understand what the rate of change represents:

$$\text{Rate of change} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in dependent variable}}{\text{change in independent variable}} = \frac{\$20}{100 \text{ km}} = \$0.20/\text{km}$$

The rate of change is \$0.20/km, that is for each additional 1 km driven the rental cost increases by \$0.20

**The rate of change is constant for a linear relation.**

**Example #1:** A water tank on a farm near Swift Current holds 6000L. Graph A represents the tank being filled at a constant rate. Graph B represents the tank being emptied at a constant rate.



a) Determine the rate of change of each relation, then describe what it represents

b) On Graph B. How many litres are in the tank after 20mins?

c) On Graph A. How many litres are left in the tank after 40 mins?

**Example #2:** Since the speed of light is faster than the speed of sound, you see lightning before you hear the sound of the thunderclap. If a thunderstorm is 1100 m away, the sound of thunder is heard in 3.2 s. If the storm is 4950 m away, the sound reaches you in 14.5 s.

- a) Determine the average rate of change, to the nearest metre per second.
- b) What does this rate of change represent?
- c) If you hear thunder 30 s after you see lightning, approximately how far away is the storm?

**Topic 7 (Day 2)- Assignment Pg 308 #3,4,5, 6b, 14 Page 319 #7, 10, 13 AND the following questions:**

1. In 1800, the wood bison population in North America was estimated at 168 000. The population declined to only about 250 animals in 1893. That year, Wood Buffalo National Park was established on the Alberta/Northwest Territories border. In 2006, there were about 5600 bison in the park.
  - a) What was the average rate of change in the bison population from 1800 to 1893? Describe the meaning of this rate.
  - b) What was the average rate of change in the bison population from 1893 to 2006? Describe the meaning of this rate.
2. The mountain pine beetle is infesting many forests in British Columbia and Alberta. In 2004, about 1000 infested trees were counted in Alberta. In 2007, the number of infested trees in the province was about 2.8 million.
  - a) Determine the average rate of change per year.
  - b) What does this rate of change represent?
  - c) Predict the number of infested trees in Alberta in 2012.

**ANSWERS**

- 1)
  - a)-1804 bison/year. The wood bison population diminished at a rate of approximately 1804 wood bison per year.
  - b) 47 bison/year. The wood bison population increased at a rate of approximately 47 wood bison per year.
- 2) a) 933000 Trees /Year                      b) On average each year 933 000 trees were infested with pine beetles in Alberta
- c) approx. 74 650 000 infested trees in Alberta in 2012

## Topic 7 (Day 3) - 5.7 Interpreting Graphs of Linear Functions (X & Y Intercepts)

**Concept #30:** 5.7 Determine and interpret the intercepts of a linear function given the graph or the equation (NC)  
(Skill & Problem Solving)

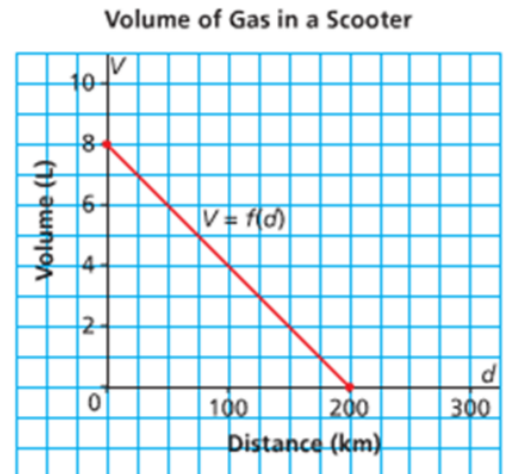
### Vertical and Horizontal intercepts (also Known as X and Y Intercepts)

- The place where the graph crosses the vertical axis is known as the VERTICAL INTERCEPT (or the Y INTERCEPT). The ordered pair of this point will always have a 0 in the first spot – in general we can say that the Horizontal Intercept is the ordered pair  $(0, y)$  although it is also known as  $(0, b)$
- The place where the graph crosses the Horizontal axis is known as the HORIZONTAL INTERCEPT (or the X INTERCEPT). The ordered pair of this point will always have a 0 in the second spot – in general we can say that the Horizontal Intercept is the ordered pair  $(x, 0)$

### Example #1:

The graph shows the fuel consumption of a scooter with a full tank of gas at the beginning of a journey.

- Write the coordinates of the points where the graph intersects the axes.
- Determine the vertical and horizontal intercepts.
- Describe what the points of intersection represent.
- What is the average rate of change?
- What are the domain and range of this function?



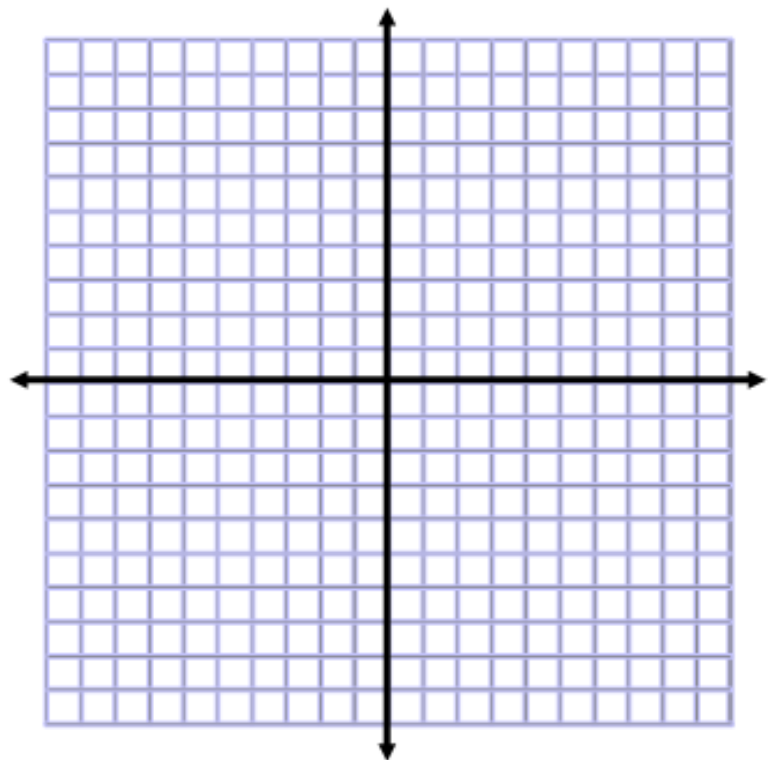


**Example #2**

1) Determine the x and y intercept of each linear function. Sketch a graph of the linear function using the intercepts. Label each function with its equation

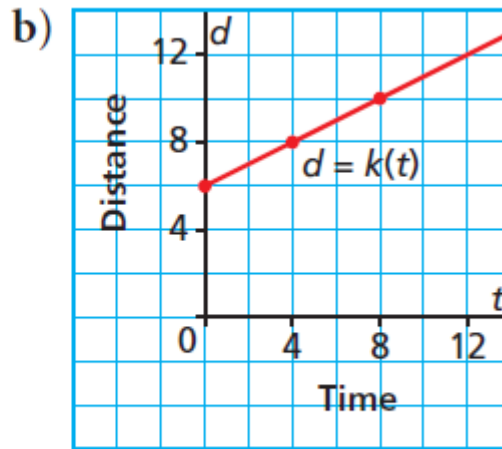
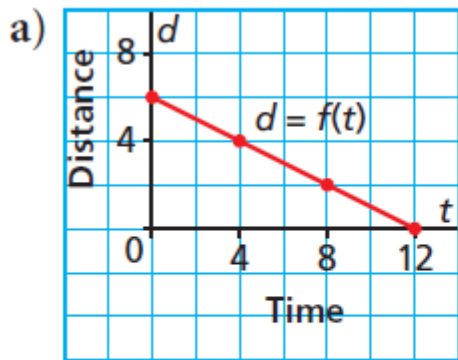
a)  $f(x) = -2x+7$

b)  $2x - 3y - 6 = 0$



**Example #3**

Which graph has a rate of change of  $\frac{1}{2}$  and a vertical intercept of 6? Justify your answer



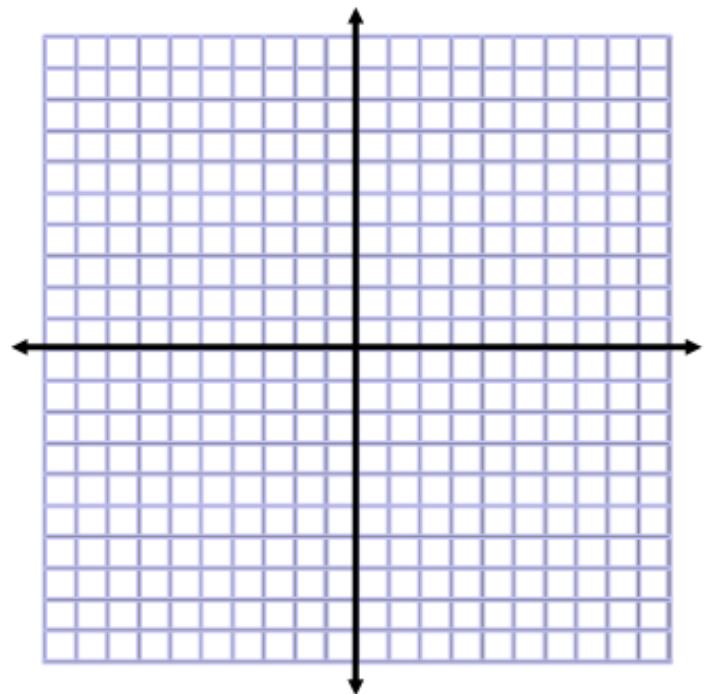
**Example #4:**

Sketch each linear relation and identify the intercepts

a)  $y - 3 = 0$

b)  $x + 4.5 = 0$

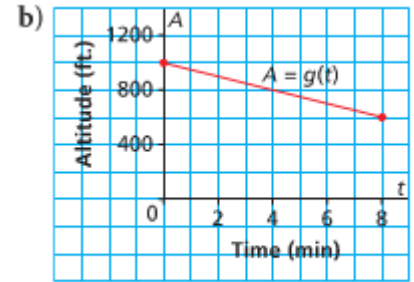
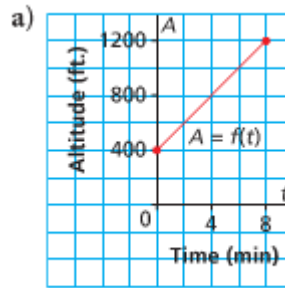
c)  $y = 0$



**Topic 7 (Day3) 5.7 ASSIGNMENT Pg 319 #6** (Find the X and Y intercepts and graph the linear function), **#8**, and extra questions below

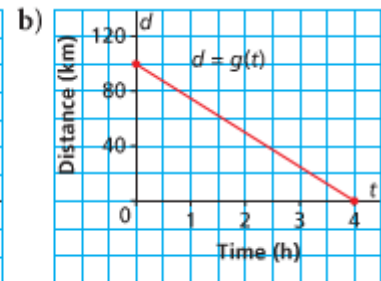
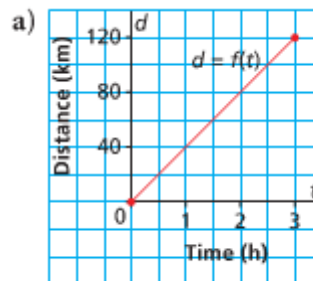
1. Each graph shows the altitude,  $A$ , feet, of a small plane as a function of time,  $t$  minutes. For each graph:

- Determine the vertical intercept. Write the coordinates of the point where the graph intersects the axes. What do these coordinates represent?
- Determine the domain and range
- Determine the dependent variable and the independent variable.
- Determine the rate of change



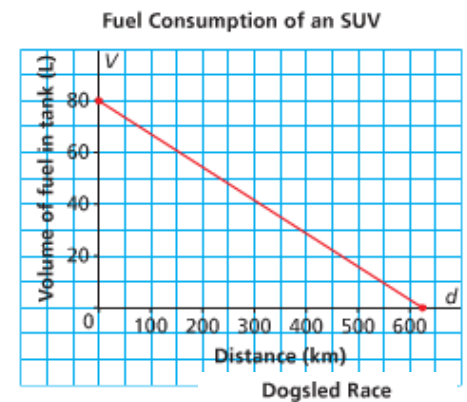
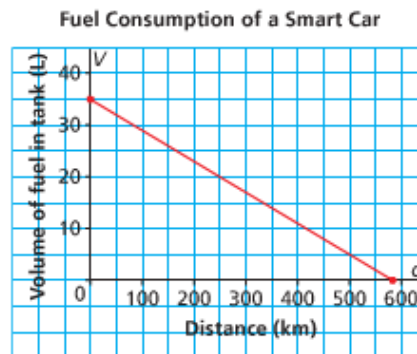
2. Each graph below shows distance,  $d$  kilometers, as a function of time,  $t$  hours. For each graph:

- Determine the vertical and horizontal intercepts. Write the coordinates of the points where the graph intersects the axes. What do these coordinates represent?
- Determine the domain and range
- Determine the dependent variable and the independent variable.



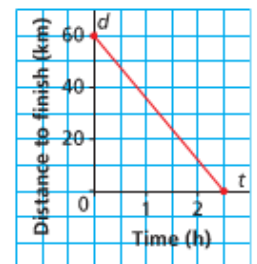
3. A Smart car and an SUV have full fuel tanks, and both cars are driven on city roads until their tanks are nearly empty. The graphs show the fuel consumption for each vehicle.

- Determine the vertical and horizontal intercepts. Write the coordinates of the points where the graph intersects the axes. What do these points represent?
- Determine the domain and range.
- Determine the dependent variable and independent variable.



4. This graph shows the distance to the finish line,  $d$  kilometers, as a function of time,  $t$  hours, for one dogsled in a race near Churchill, Manitoba.

- What was the length of time it took the dogsled to finish the race? What do we call this point on the graph?
- How long was the race in kilometers? What do we call this point on the graph?
- Determine the domain and range.
- Determine the dependent and independent variables.



**Answers**

- 1 a) Graph A: (0, 400) This is the altitude of the plane at 0 minutes  
Graph B: (0, 1000) This is the altitude of the plane at 0 minutes
- b) Graph A: D: [0, 8] or  $\{t | 0 \leq t \leq 8, t \in \mathbb{R}\}$  R: [400, 1200] or  $\{A | 400 \leq A \leq 1200, A \in \mathbb{R}\}$   
Graph B: D: [0, 8] or  $\{t | 0 \leq t \leq 8, t \in \mathbb{R}\}$  R: [600, 1000] or  $\{A | 600 \leq A \leq 1000, A \in \mathbb{R}\}$
- c) Both Graphs – time is independent; altitude is dependent  
d) 100ft/ min
- 2 a) Graph A: vertical – (0,0), horizontal (0,0) These show the distance at time 0.  
Graph B: vertical – (0, 100) at 0 minutes the distance is 100 km  
Horizontal – (4, 0) it takes 4 minutes to have a distance of 0 km
- b) Graph A: D: [0, 3] or  $\{t | 0 \leq t \leq 3, t \in \mathbb{R}\}$  R: [0, 120] or  $\{d | 0 \leq d \leq 120, d \in \mathbb{R}\}$   
Graph B: D: [0, 4] or  $\{t | 0 \leq t \leq 4, t \in \mathbb{R}\}$  R: [0, 100] or  $\{d | 0 \leq d \leq 100, d \in \mathbb{R}\}$
- c) Both graphs: time is independent; distance is dependent
- 3 a) Graph A: vertical (0, 35) before driving the tank has 35L of fuel  
Horizontal (575, 0) the tank runs out at 575 km  
Graph B: vertical (0, 80) before driving the tank has 80 L of fuel  
Horizontal (625, 0) the tank runs out at 625 km
- b) Graph A : D: [0,575] or  $\{d | 0 \leq d \leq 575, d \in \mathbb{R}\}$  R:[0,35] or  $\{V | 0 \leq V \leq 35, V \in \mathbb{R}\}$
- c) Both graphs – distance is independent; volume is dependent
- 4 a) 2.5 hours; the horizontal intercept  
b) 60 km; the vertical intercept
- c) D: [0, 2.5] or  $\{t | 0 \leq t \leq 2.5, t \in \mathbb{R}\}$  R: [0, 60] or  $\{d | 0 \leq d \leq 60, d \in \mathbb{R}\}$
- d) time is independent; distance is dependent

## Topic 7 ( Day4) - 6.2 Slopes of Parallel and Perpendicular Lines

**Concept #31** -6.2 Determine whether two lines are parallel or perpendicular (NC) (Skill & Problem Solving)

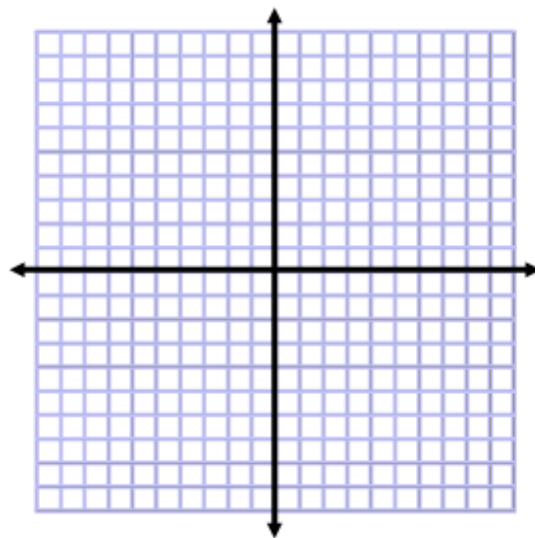
### Investigation A:

Draw 2 different line segments with a slope of  $\frac{7}{5}$

**How are the line segments the same?**

**How are they different?**

**What kind of lines are they?**



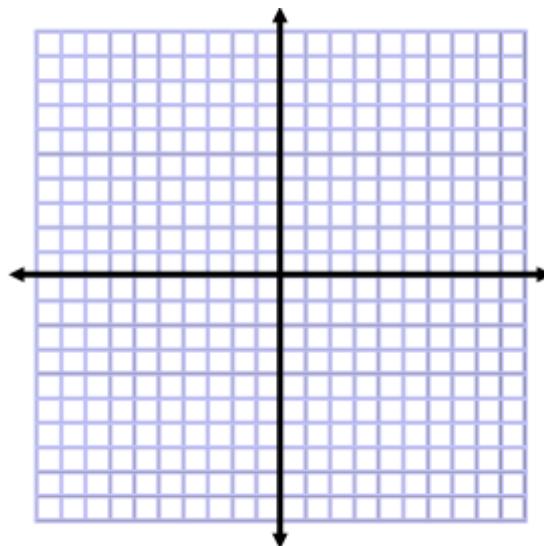
### Investigation B:

- Draw a line going through the origin with slope  $\frac{5}{3}$
- Draw a line going through the origin with slope  $-\frac{3}{5}$

What do you notice between the two lines?

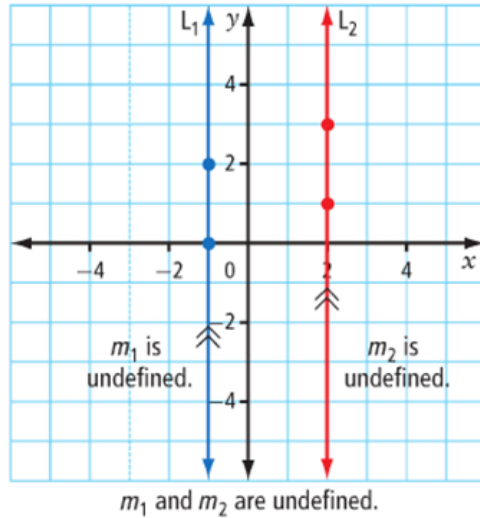
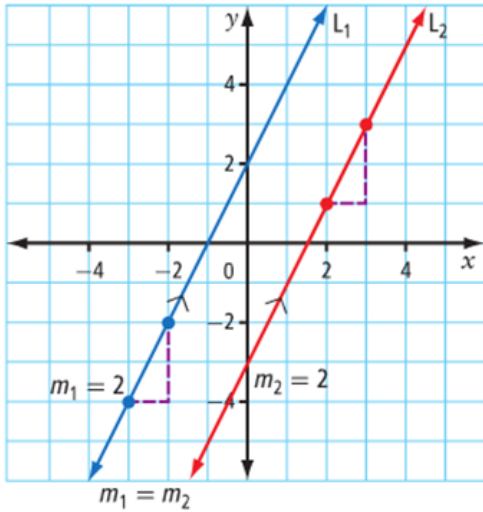
What would you call the lines?

Do you notice a relationship between their slopes?



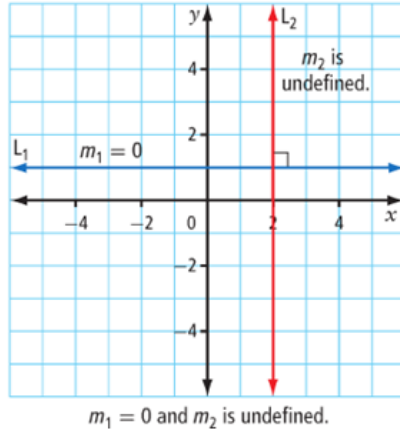
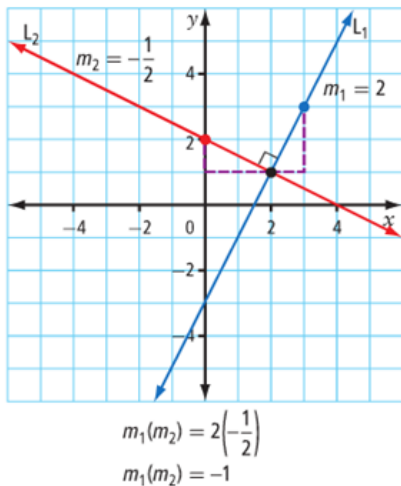
Can you make a general statement about what you have discovered?

**Parallel Lines** have the same slope but different intercepts. This includes horizontal lines, which have a slope of zero. Vertical lines, which have an undefined slope, are also parallel.



Parallel Lines: Each have the same slope  
 .....If two lines have the same slope, they are parallel.....

The slopes of **perpendicular lines** are negative reciprocals of each other. The product of negative reciprocals is -1. A vertical line, which has an undefined slope, and a horizontal line, which has a slope of 0, are perpendicular to each other.



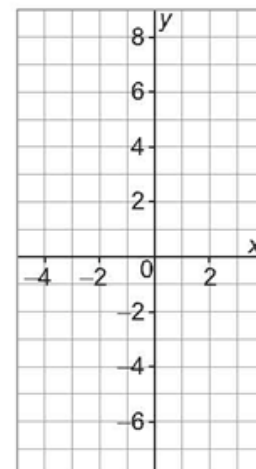
**Example #1** We are given the following three lines:

Line AB passes through A (-4, -3) and B (0, 3)

Line CD passes through C(-3, -5) and D(2, 3)

Line EF passes through E(0,-3) and F(4, 3).

- Graph the lines to determine if they are parallel
- Use the slope formula to determine if they are parallel.



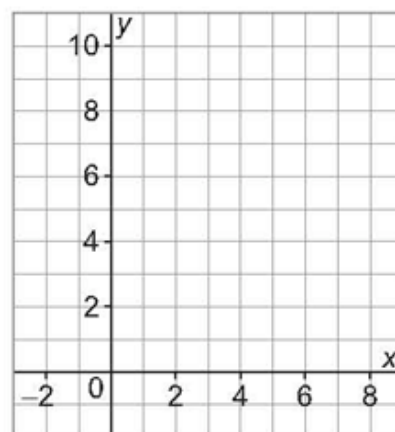
**Example #2** We are given the following three lines:

Line AB passes through A (-1, -1) and B (2, 8)

Line CD passes through C(6, -3) and D(0, -1)

Line EF passes through E(4, 6) and F(2, 1).

- Graph the lines to determine if they are perpendicular
- Use the slope formula to determine if they are perpendicular.



**Example #3**

A line,  $k$ , has a slope  $\frac{4}{3}$ . What is the slope of the line that is a) parallel to line  $k$  b) perpendicular to line  $k$

**Example #4**

Line PQ passes through P(-7, 2) and Q(-2, 10).

Line RS passes through R(-3, -4) and S(5, 1).

a) Are these two lines parallel, perpendicular, or neither?

**Example #5:**

State whether the lines in each pair are parallel, perpendicular or neither

a)  $y = 3x - 6$

b)  $y = 2x + 6$

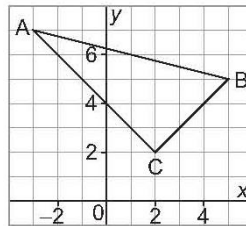
c)  $y = 4x - 5$

$$y = -\frac{1}{3}x + 4$$

$$y = -2x + 5$$

$$y = 4x + 3$$

**Example #6:** Is ABC a right triangle?

**Topic 7 (Day4) - 6.2 ASSIGNMENT**

Page 349 #3, 4, 5, 6, 8, 9, 17 AND the following questions:

- Sheldon was asked if line segment AB with A(-9, 2) and B(-3, 4) is parallel to line segment CD with C(-7, -7) and D(1, -3). He sketches a graph of the two line segments and concludes that they appear parallel.
  - Is it correct to assume from a sketch that the two line segments are parallel? Explain.
  - How could you prove that two lines segments are parallel?
  - Is line segment AB parallel to line segment CD? Justify your answer.
- Is the following statement always true, sometimes true, or never true? "The slopes of perpendicular lines are always negative reciprocals of each other." Explain your reasoning.

**Answers:** 1. a) answers may differ b) find their slopes using the two point slope formula c) no they are not parallel 2. Answers may vary. (Always true)