3.5/3.6 Multiplying Polynomials – Monomials and Binomials (Day1)

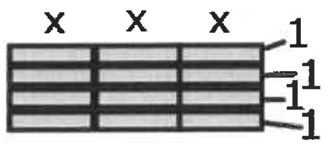
Concept #18: 3.5/3.6 Correctly multiply two binomials **(NC) (Skill)**

The terms of a polynomial can be represented by using Algebra Tiles.

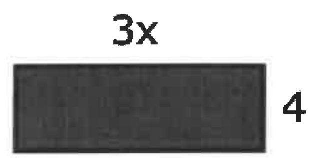
These are what each tile represents
Note: The x can be ANY letter!

This represents the product of the constant 4 and the monomial,3x. We can model the product as 4 rows of three "x" tiles.

4(3x) →

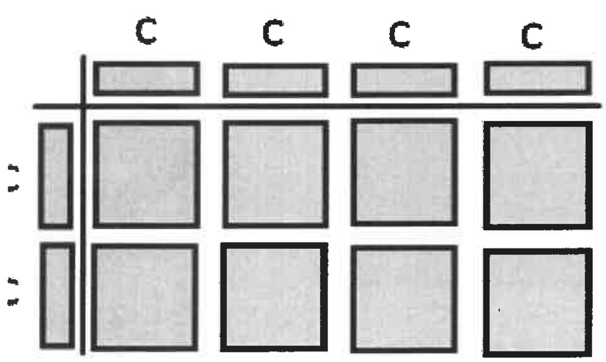


OR



We can Model 4(3x) as the area of a rectangle with dimensions 4 and 3x.

The expression (2c)(4c) is the product of two monomials.
We interpret the product with algebra tiles arranged to form a rectangle with dimensions 2c and 4c.



To help build the rectangle, we place guiding tiles to represent each dimension. Then we fill in the rectangle with tiles.

We need eight c tiles to build the rectangle.
So, $(2c)(4c) = 8c^2$

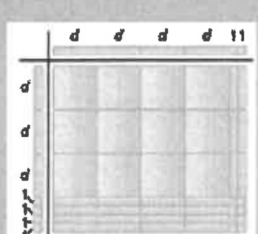
Solving products of all polynomials with degree 1 or less can be represented **concretely** (Using algebra tiles) , **pictorially** (Drawing algebra tiles or rectangles) or **symbolically** (Using numbers and operations).

Here are three strategies to determine the product of binomials

<u>Strategy 1:</u> <u>Use algebra tiles</u>

Expand $(3d + 4)(4d + 2)$

Make a rectangle with dimensions $3d + 4$ and $4d + 2$. Place tiles to represent each dimension, then fill in the rectangle with tiles.



The tiles that form the product are:
12 d^2-tiles, 22 d-tiles, and eight 1-tiles.

So $(3d + 4)(4d + 2) = 12d^2 + 22d + 8$

Strategy 2: Area Model

Expand $(h + 11)(h + 5)$

Sketch a rectangle with dimensions $h + 11$ and $h + 5$. Divide the rectangle into 4 smaller rectangles and calculate the area of each.

	h	11
h	$(h)(h)=h^2$	$(h)(11) = 11h$
5	$(5)(h)=5h$	$(5)(11) = 55$

So, $(h + 11)(h + 5) = h^2 + 5h + 11h + 55$ Combine like terms
$= h^2 + 16h + 55$

Note that $(h + 11)(h + 5) = (h + 5)(h + 11)$ since both products represent the area of the same rectangle.

Strategy 3: The Distributive Property

Expand $(x - 3)(2x + 1)$

$$\begin{aligned}(x - 3)(2x + 1) &= x(2x + 1) - 3(2x + 1)\\ &= (x)(2x) + (x)(1) + (-3)(2x) + (-3)(1)\\ &= 2x^2 + x - 6x - 3\\ &= 2x^2 - 5x - 3\end{aligned}$$

To multiply polynomials of larger degrees we can use the distributive property and exponent laws

Recall:
Exponent Law - What do you do when you are multiplying same bases?
ie. $a^3 \times a^4$

You add the exponents

$x^m x^n = x^{m+n}$

Example #1) Simplify (Use method/strategy of choice)

a) -3x(2x+2)

$= -3(2x+2)$

$= -3(2x) + 2(-3)$

$= -6x - 6$

b) 5m(-2m-3)

$= 5m(-2m) - 3(5m)$

$= -10m^2 - 15m$

c) $-5x^2y(2x + 3y)$

$= -5x^2y(2x) + 3y(-5x^2y)$

$= -10x^3y - 15x^2y^2$

d) $3m^2n^3p^2(-5m^2n + 2mp^3 - 4n^2p^6)$

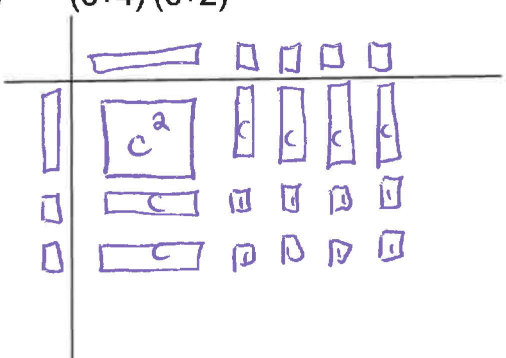
Add exponents of same variables when multiplying

$= 3m^2n^3p^2(-5m^2n) + 2mp^3(3m^2n^3p^2) - 4n^2p^6(3m^2n^3p^2)$

$= -15m^4n^4p^2 + 6m^3p^5n^3 - 12n^5p^8m^2$

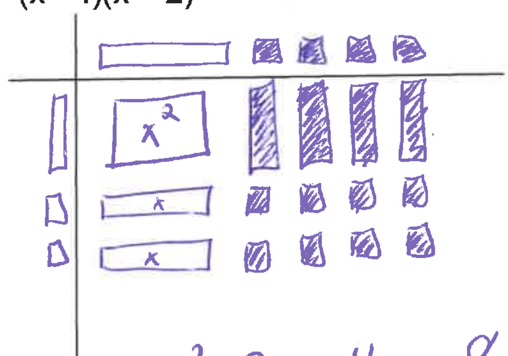
Example #2) Multiply two binomials. Expand and simplify. Use algebra tiles and sketch the tiles you used.

a) (c+4) (c+2)



$= c^2 + 6c + 8$

b) (x - 4)(x + 2)



$= x^2 + 2x - 4x - 8$

$= x^2 - 2x - 8$

Example #3) Expand and simplify. Use the distributive property.

a) (3d + 4)(4d + 2)

$= 3d(4d+2) + 4(4d+2)$

$= 12d^2 + 6d + 16d + 8$

$= 12d^2 + 22d + 8$

b) (-2g + 8)(7 - 3g)

$= -2g(7-3g) + 8(7-3g)$

$= -14g + 6g^2 + 56 - 24g$

$= 6g^2 - 38g + 56$

c) (8 - b)(3 - b)

$= 8(3-b) - b(3-b)$

$= 24 - 8b - 3b + b^2$

$= b^2 - 11b + 24$

Day 1 Assignment: Polynomial Practice Assignment#1- Multiplying Polynomials

3.5/3.6 Multiplying Polynomials – Monomials, Binomials and Trinomials (Day 2)

Concept #19: 3.7 Correctly multiply a binomial by a trinomial and a trinomial by a trinomial **(NC)(Skill)**

The *distributive property* can be used to perform any polynomial multiplication. Each term of one polynomial must be multiplied by each term of the other polynomial.

Example #1) Using the Distributive Property to Multiply Two Polynomials (NO CALCULATORS)
Expand and simplify

a) $(2h + 5)(h^2 + 3h - 4)$

Distribute

$= 2h(h^2+3h-4) + 5(h^2+3h-4)$

Add like terms

$= 2h^3 + 6h^2 \boxed{-8h} + 5h^2 \boxed{+15h} - 20$

$= 2h^3 + 11h^2 + 7h - 20$

b) $(-3f^2 + 3f - 2)(4f^2 - f - 6)$

$= -3f^2(4f^2-f-6) + 3f(4f^2-f-6) - 2(4f^2-f-6)$

$= -12f^4 + 3f^3 + 18f^2 + 12f^3 - 3f^2 \boxed{-18f} - 8f^2 \boxed{+2f} + 12$

$= -12f^4 + 15f^3 + 7f^2 - 16f + 12$

Example #2) Multiplying Polynomials in More Than One Variable
Expand and Simplify

a) $(2r + 5t)^2$

$= (2r+5t)(2r+5t)$

$= 2r(2r+5t) + 5t(2r+5t)$

$= 4r^2 + 10rt + 10rt + 25t^2$

$= 4r^2 + 20rt + 25t^2$

Check solution for t=2 and r =3

Check $(2r+5t)^2$

$= (2(3)+5(2))^2$

$= (6+10)^2$

$= 16^2$

$= 256$

Check $4r^2 + 20rt + 25t^2$

$= 4(3)^2 + 20(3)(2) + 25(2)^2$

$= 36 + 120 + 100$

$= 256$

equal

Equal ∴ I expanded and simplified correctly

b) $(3x - 2y)(4x - 3y + 5)$

$= 3x(4x-3y+5) - 2y(4x-3y+5)$

$= 12x^2 - 9xy + 15x - 8xy + 6y^2 - 10y$

$= 12x^2 - 17xy + 15x + 6y^2 - 10y$

$= 12x^2 + 6y^2 - 17xy + 15x - 10y$

Example #3) Expand and Simplify

a) $(x+5)^3$

Multiply two binomials first.

$= (x+5)(x+5)(x+5)$

$= [x(x+5) + 5(x+5)](x+5)$

$= [x^2 + 5x + 5x + 25](x+5)$ *combine like terms

$= (x^2 + 10x + 25)(x+5)$ *Multiply trinomial by binomial

$= x(x^2+10x+25) + 5(x^2+10x+25)$

$= x^3 + 10x^2 + 25x + 5x^2 + 50x + 125$

$= x^3 + 15x^2 + 75x + 125$

b) $(2x-3)^3$

$= [(2x-3)(2x-3)](2x-3)$

$= [2x(2x-3) - 3(2x-3)](2x-3)$

$= (4x^2 - 6x - 6x + 9)(2x-3)$

$= (4x^2 - 12x + 9)(2x-3)$

$= 2x(4x^2-12x+9) - 3(4x^2-12x+9)$

$= 8x^3 - 24x^2 + 18x - 12x^2 + 36x - 27$

$= 8x^3 - 36x^2 + 54x - 27$

Example #4) Add or subtract

a) $(5a - 8) - (2a + 3)$ Subtract every term in the second binomial

$= 5a - 8 - 2a - 3$

$= 7a - 11$

b) $(2x^2 + 6x + 5) + (-4x^2 - 3x + 7)$ Add every term in the 2nd trinomial

$= 2x^2 - 4x^2 + 6x - 3x + 5 + 7$

$= -2x^2 + 3x + 12$

c) $(3a^2 - 2a + 6) - 1(-2a^2 + 7a - 9)$

$= 3a^2 - 2a + 6 + 2a^2 - 7a + 9$

$= 5a^2 - 9a + 15$

Example #5) Simplifying Sums and Differences of Polynomial Products

Note: Use order of operations. Multiply before adding and subtracting. Then combine like terms.

Expand and Simplify

a) $(3x - 1)(2x - 4) - (3x + 2)^2$

$= 3x(2x-4) - 1(2x-4) - [(3x+2)(3x+2)]$

$= 6x^2 - 12x - 2x + 4 - [3x(3x+2) + 2(3x+2)]$

$= 6x^2 - 14x + 4 - (9x^2 + 6x + 6x + 4)$

$= 6x^2 - 14x + 4 - (9x^2 + 12x + 4)$

$= 6x^2 - 14x + 4 - 9x^2 - 12x - 4$

$= -3x^2 - 26x$

b) $2b(2b - c)(b + c)$

$= (4b^2 - 2bc)(b + c)$

$= 4b^2(b+c) - 2bc(b+c)$

$= 4b^3 + 4b^2c - 2b^2c - 2bc^2$

$= 4b^3 + 2b^2c - 2bc^2$

Day 2 Assignment: Polynomial Practice Assignment #2 – Multiplying Polynomials

3.3 Greatest Common Factors(GCF) of Polynomials (Day3)

Greatest common factor of variables: The smallest common exponent of each variable in each term Ex. $x^5y^3 and x^8y$ The GCF = x^5y between the two monomials

Add in notes (Concept #20: 3.3 Correctly factor polynomials with a GCF

Note: Take a negative GCF out if the first term is negative.

Example #1) Factor out the GCF from a polynomial by dividing each term by the GCF.

a) $\frac{6n}{3} + \frac{9}{3}$

$= 3(2n+3)$

b) $\frac{-c^2}{-c} + \frac{4c}{-c}$

$= -c(c-2)$

c) $\frac{5}{-5} - \frac{10z}{-5} - \frac{5z^2}{-5}$

$= -5(z^2+2z-1)$

Example #2) Factor. Verify that the factors are correct by expanding.

a) $\frac{-12x^3y}{-4xy} - \frac{20xy^2}{-4xy} - \frac{16x^2y^2}{-4xy}$

$= -4xy(3x^2+5y+4xy)$

Check

$= -4xy(3x^2) - 4xy(5y) - 4xy(4xy)$

$= -12x^3y - 20xy^2 - 16x^2y^2$ ✓

b) $\frac{-20c^4d}{-5cd} - \frac{30c^3d^2}{-5cd} - \frac{25cd}{-5cd}$

$= -5cd(4c^3+6c^2d+5)$

Check

$-5cd(4c^3) - 5cd(6c^2d) - 5cd(5)$

$= -20c^4d - 30c^3d^2 - 25cd$ ✓

Example #3) Factor.

a) $\frac{16x^2y}{8x^2y} + \frac{24x^2y^3}{8x^2y}$

$= 8x^2y(2+3y^2)$

b) $\frac{7a^2b}{7ab} - \frac{28ab}{7ab} + \frac{14ab^2}{7ab}$

$= 7ab(a-4+2b)$

c) $\frac{-16x^2y^2}{-8x^2y^2} + \frac{24x^3y^3}{-8x^2y^2}$

$= -8x^2y^2(2-3xy)$

Day 3 Assignment: Polynomial Practice Assignment #3 - Factoring Polynomials with a GCF

3.5 Factoring Polynomials of the form x^2+bx+c and GCF (Day 4)

Concept # ~~20~~ 21- 3.5 Correctly factor polynomials with a GCF; correctly factor polynomials of the form $x^2 + bx + c$; and factor trinomials with an initial GCF resulting in the form $x^2 + bx + c$ (by method of choice) **(NC) (Skill)**

$(2x + 3)(x + 1)$ ⟵ FACTORS
$2x^2 + 5x + 3$ ⟵ EXPANDED FORM

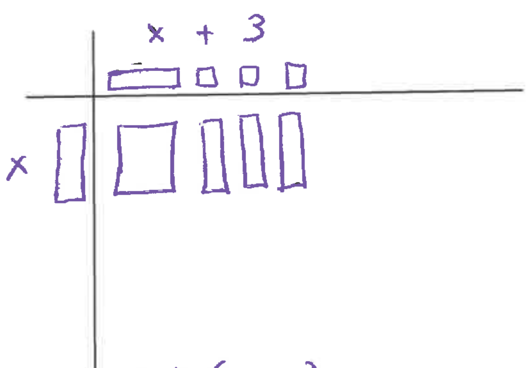
Factoring and multiplying/expanding are inverse processes. We can use this to factor a trinomial.

Method #1 – Using Algebra Tiles concretely and pictorially factor binomials and trinomials

Step 1 – Get a bag of algebra tiles
Step 2 – From your bag, collect tiles that represent the given polynomial
Step 3 – Rearrange the collected tiles into a rectangle (draw the rectangle)
Step 4 – Determine the dimensions of the rectangle (These are your factors)

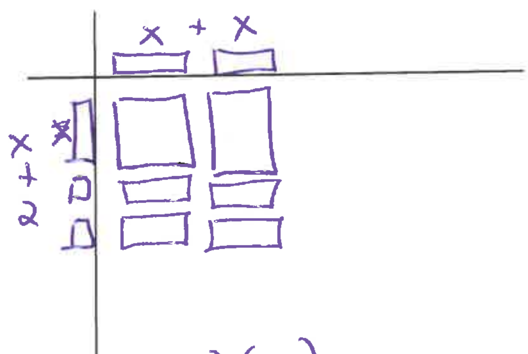
Example #1) Factor using algebra tiles.

a) $x^2 + 3x$



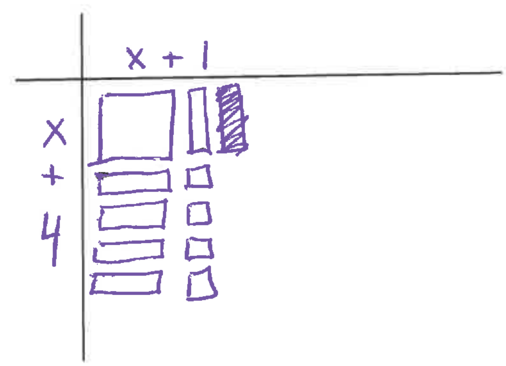
$= (x)(x+3)$

b) $2x^2 + 4x$



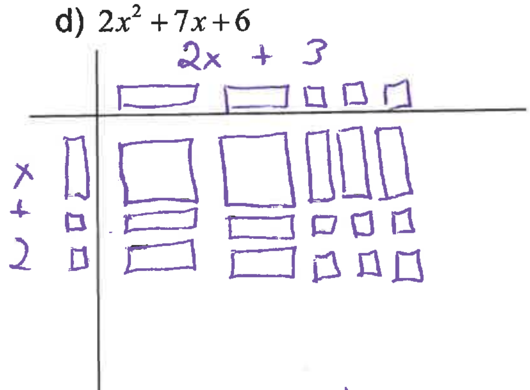
$= (x+2)(2x)$

c) $x^2 + 5x + 4$



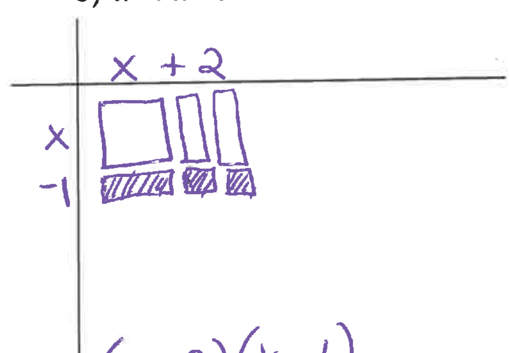
$= (x+1)(x+4)$

d) $2x^2 + 7x + 6$



$= (2x+3)(x+2)$

e) $x^2 + x - 2$



$= (x+2)(x-1)$

Method #2 – Symbolically Factor Binomials and Trinomials

Note: 1) REMEMBER TO ALWAYS LOOK TO FACTOR OUT A GCF FIRST
2) Rearrange polynomials in descending order
3) There are other methods when factoring a trinomial. If you'd like to try a different method let me know.

Example #2) Factor by guess and check (a.k.a Window method)

a) $x^2 - 2x - 8$

$\boxed{\text{Step 1}}$ Is their a GCF ??

$x^2 - 2x - 8$

Two factors of first term: x, x — Two factors of last term: -4, 2

Multiply across

$x \quad -4 \quad -4x$
$x \quad 2 \quad 2x$

need to add to middle term

$\boxed{-2x}$ ← middle term ✓

$\therefore (x-4)(x+2)$ are the factors

b) $z^2 - 12z + 35$

$z \quad -5 \quad -5z$
$z \quad -7 \quad -7z$
$-12z$ ✓

$= (z-5)(z-7)$

b) **Factor** (Note: Show factoring in ascending vs descending order)

$-24 - 5d + d^2$

$-8 \quad d \quad 3d$
$3 \quad d \quad -8d$
$-5d$ ✓

$= (-8+d)(3+d)$

$d^2 - 5d - 24$

$d \quad -8 \quad -8d$
$d \quad 3 \quad +3d$
$-5d$ ✓

$= (d-8)(d+3)$

d) Factor and verify your answer

$m^2 - 7m - 60$

$m \quad -12 \quad -12m$
$m \quad 5 \quad 5m$
$-7m$ ✓

$= (m-12)(m+5)$

Does the order we write the terms of the binomial matter? No

ALWAYS LOOK FOR GCF FIRST!!!

Example #3) Factor $-4t^2 - 16t + 128$

$= -4(t^2 + 4t - 32)$

$= -4(t+8)(t-4)$

$t \quad +8 \quad 8t$
$t \quad -4 \quad -4t$
$4t$

b) $-5h^2 - 20h + 60$

$= -5(h^2 + 4h - 12)$

$= -5(h+6)(h-2)$

$h \quad +6 \quad 6h$
$h \quad -2 \quad -2h$
$4h$ ✓

Do you see any patterns? Did anyone find a more efficient/quicker way to factor the previous trinomials? Can you use this explanation to show how the following questions can be quickly factored?

$x^2+8x+12$ $=(x+6)(x+2)$	$x^2+7x+10$ $=(x+5)(x+2)$	$x^2+8x+15$ $(x+5)(x+3)$
q^2+6q+8 $=(q+4)(q+2)$	$m^2+7m+12$ $=(m+3)(m+4)$	$r^2+13m+12$ $=(r+12)(r+1)$
$49+14h+h^2$ $=(7+h)(7+h)$	$144+24x+x^2$ $=(12+x)(12+x)$	$81+18w+w^2$ $=(9+w)(9+w)$
$m^2-7m+12$ $(m-4)(m-3)$	$x^2-8m+12$ $(x-6)(x-2)$	$u^2-12u+27$ $(u-9)(u-3)$

Day 4 Assignment: Polynomial Practice Assignment #4-Factoring Polynomials of the form x2+bx+c

3.6 Factoring Polynomials ax²+bx+c and GCF (Day 5)

Concept # ~~21~~ 22 3.6 Correctly factor using GCF and a trinomial ax^2+bx+c, where $a > 1$ by method of choice **(NC)(Skill)**

ALWAYS LOOK FOR A GREATEST COMMON FACTOR (GCF) BETWEEN ALL TERMS FIRST

Example #1) Factor

a) $4h^2+20h+9$

① 4h 3 / h 3

Multiply to first term — multiply to last term

$4h \times 3 = 3h$, $h \times 3 = 12h$: $15h$ middle term ✗ Try different factors.

② $2h \times 9 = 18h$, $2h \times 1 = 2h$: $20h$ ✓

$\boxed{=(2h+9)(2h+1)}$

b) $\frac{12k^2}{2} - \frac{22k}{2} - \frac{70}{2}$ GCF

$=2(6k^2-11k-35)$

$\boxed{=2(2k-7)(3k+5)}$

$2k \times 5 = 10k$, $3k \times -7 = -21k$: $-11k$ ✓

c) $4g^2+11g+6$

$4g \times 2 = 8g$, $g \times 3 = 3g$: $11g$ ✓

$\boxed{=(4g+3)(g+2)}$

d) $\frac{18m^3}{3m} - \frac{21m^2}{3m} - \frac{30m}{3m}$

$=3m(6m^2-7m-10)$

$\boxed{=3m(6m+5)(m-2)}$

$6m \times -2 = -12m$, $m \times 5 = 5m$: $-7m$ ✓

Example #2) a) Evaluate $3x^2-14x+15$ **when x = 2**

$3(2)^2-14(2)+15$

$=3(4)-28+15$

$=12-28+15$

$=-16+15$

$=-1$ ✓

b) Factor $3x^2-14x+15$**, then evaluate x = 2**

$3x \quad -5 \quad -5x$

$x \quad -3 \quad -9x$

$-14x$ ✓

$=(3x-5)(x-3)$

Evaluate $x=2$

$(3(2)-5)(2-3)$

$=(6-5)(-1)$

$=(1)(-1)$

$=-1$ ✓

c) Did you get the same answer for a & b? Why or why not?

Yes, when you evaluate in trinomial form or factored form you get the same answer because the forms are equivalent to one another.

Example #3) Factor

a) x^2-25

$x \quad -5 \quad -5x$

$x \quad 5 \quad 5x$

$0x$

$=(x-5)(x+5)$

b) $4x^2-49$

$=(2x-7)(2x+7)$

c) $25x^2-16$

$=(5x-4)(5x+4)$

d) $9m^2-100n^2$

$=(3m-10n)(3m+10n)$

Can you find an easier way to factor the above "DIFFERENCE OF SQUARES" questions? What criteria must be present in order for this method to work?

Yes – the middle term must be "zero"

– the coefficient of the x^2 term must be a perfect square

– the constant term must be a perfect square

$4x^2-25$

square root

square root

$=(2x-5)(2x+5)$

one "+" and one "–"

Example #4) Use your new method to factor the following. If you can't factor them – explain why?

a) $36x^2 - 25y^2$

$= (6x-5y)(6x+5y)$

b) $x^2 + 81$ not a difference of squares.

prime

c) $-100 + r^2$ rewrite as r^2-100

$= r^2 - 100$

$= (r-10)(r+10)$

d) $\frac{4}{9}x^2 - 25$

$= (\frac{2}{3}x - 5)(\frac{2}{3}x + 5)$

e) $1.21x^2 - 0.36$

$= (1.1x - 0.6)(1.1x + 0.6)$

$=$

f) $\frac{x^2}{16} - \frac{y^2}{49}$

$= (\frac{x}{4} - \frac{y}{7})(\frac{x}{4} + \frac{y}{7})$

g) $2x^2 - 14$

$= 2(x^2 - 7)$

$= 2(x^2 - 7)$

Not a perfect square.

h) $81a^4 - 16b^4$

Difference of squares again

$= (9a^2 - 4b^2)(9a^2 + 4b^2)$

$= (3a-2b)(3a+2b)(9a^2+4b^2)$

i) $49a^2b^2 - 1$

$= (7ab-1)(7ab+1)$

Note: A binomial is only factorable if there is a GCF **or the bionomial is a** difference of squares**.**

Example #5)

Create a binomial that is PRIME (Not factorable): $x^2 - 5$

Create a binomial that is a difference of squares and factor it: $16x^2 - 25y^2$

Day 5 Assignment: Polynomial Practice Assignment #5 - Factoring Polynomials

3.8 Factoring ALL Polynomials (Day 6)

Concept # ~~22~~ 23: 3.8 Factoring using GCF and/or all of the above (including perfect square trinomials, trinomials in two variables, difference of squares) **(NC)(Skill)**

Ex1): Factor $20r^2 + 70r + 60$

$$\frac{20r^2}{10} \quad \frac{70r}{10} \quad \frac{60}{10}$$

$= 10(2r^2+7r+6)$

$= 10(2r+3)(r+2)$

$2r$	3	$3r$
r	2	$4r$
		$7r$ ✓

Ex2): Factor $-5h^2 - 20h + 60$

$$\frac{-5h^2}{-5} \quad \frac{20h}{-5} \quad \frac{60}{-5}$$

$= -5(h^2+4h-12)$

$= -5(h+6)(h-2)$

h	6	$6h$
h	-2	$-2h$
		$4h$ ✓

Ex3): Factor $x^2 + 5x + 1$

Prime

x	1
x	1

Not factorable.

Ex4): Factor $4x^2+12x+9$

$= (2x+3)(2x+3)$

$= (2x+3)^2$

$4x \quad 3 \quad 3x$
$x \quad 3 \quad 12x$
$15x$ ✗

$2x \quad 3 \quad 6x$
$2x \quad 3 \quad 6x$
$12x$ ✓

Perfect square trinomial

Ex5): Factor $x^4 - 81y^4$ *Difference of squares*

$= (x^2-9y^2)(x^2+9y^2)$

$= (x-3y)(x+3y)(x^2+9y^2)$

Day 6 Assignment : Polynomial Practice Assignment #6- Factoring Polynomials(All types)