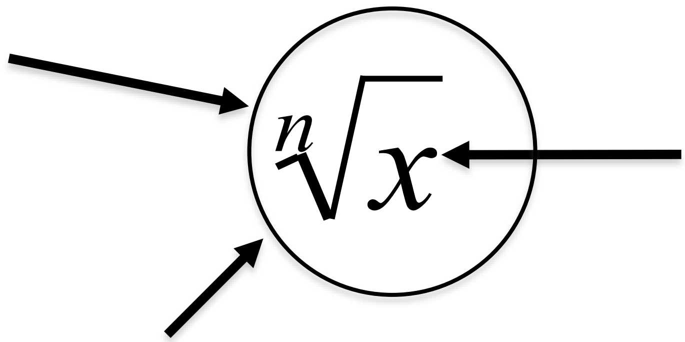
l4.1 Estimating Roots

Parts of the Radical



To calculate 3^4, you will need to use one of the following on your calculator:

- 3 ^ 4
- 3 y^x 4
- 3 x^y 4

Or use (3)(3)(3)(3)

$3^2 = 9$ because ___________________; Then , $\sqrt{9} =$ _____

$(-3)^3 = -27$ because ___________________; Then, $\sqrt[3]{-27} =$ _____

$3^4 = 81$ because ______________________; Then, $\sqrt[4]{81} =$ ______

Example 1 : How would you write 5 as a:

a) square root 5 =

b) Cube Root 5 =

c) fourth root 5 =

Example 2: Without using a calculator, find the exact value or approximate/estimated value of the following:

a) $\sqrt{4}$ b) $\sqrt{9}$ c) $\sqrt{5}$ d) $\sqrt{22}$

Steps for Finding the Approximate Value of a Radical using Benchmarks

1. Draw a number line. **Below** the number line and at each end place the closest perfect (square, cube , fourth power etc.) depending on the index.
2. Find the root of each of your "answers". Put these root numbers **Above** the number line on the ends using a different colour.
3. Between your root numbers, put 9 tick marks. Label the tick marks (each tick mark is worth .10
4. Look below your number line. This represents the space between your two original "answer" numbers. In between these "answer numbers", place where you think your radicand would appear between the two "answers". Is it right in the middle? Closer to the first answer? Closer to the second answer?
5. Look at the tick marks above where you wrote your radicand. The number that corresponds to your tick mark is your approximate/estimated solution to the root of your radicand.

Example 3: Without using a calculator, find the exact or approximate value of the following:

a) $\sqrt{\dfrac{16}{25}}$ b) $\sqrt[4]{16}$ c) $\sqrt{0.81}$ d) $\sqrt[3]{0.027}$

Hints for Finding Exact Roots of Fractions or Decimals

1. You can split a fraction in a radical into two separate radicals – one on the top and one on the bottom. Do each radical separately. $\sqrt[n]{\dfrac{x}{y}} = \dfrac{\sqrt[n]{x}}{\sqrt[n]{y}}$
2. If you see a radicand that looks like a perfect square root number, cube root number etc except it is after a decimal, chances are it will still work.
   - For a square root, it will work if there are an even number of place values after the decimal. The answer will be the root of the "nice number" with a decimal length of half the length of the original question. Add zero's in front to make the decimal as long as you need it.
   - For a cube root, it will work if the number of place values after the decimal is divisible by 3. The answer will be the root of the "nice number" with a decimal length of a third the length of the original question. Add zero's in front to make the decimal as long as you need it.
   - The same pattern applies for fourth and fifth roots.

Example 4: Without using a calculator, find the exact or approximate values of the following:

a) $\sqrt{0.0049}$

b) $\sqrt[3]{0.000216}$

c) $\sqrt[3]{\dfrac{0.008}{0.125}}$

a) $\sqrt{0.85}$

To calculate a cube root or fourth root or higher on a graphing calculator push the MATH buttonthen find $\sqrt[3]{(}$ or $\sqrt[x]{(}$ is a root higher than 3.

Example 5: Using your calculator, find the following answers:

a) $\sqrt{-9}$

b) $\sqrt{-25}$

c) $\sqrt[3]{-8}$

d) $\sqrt[3]{-125}$

e) $\sqrt[4]{-16}$

f) $\sqrt[4]{-81}$

g) $\sqrt[5]{-32}$

h) $\sqrt[5]{-243}$

Explain how you can predict which answers won't have a solution…….

Finding Roots with Negative Radicands

If you have a negative radicand with an even index, the answer will be _______________

If you have a negative radicand with an odd index, the answer will be _____________

4.1 Assignment Pg 206 #1, 2,3(a,b,c,d,e) #4a, 5, 6 (No CALCULATOR)

Powers Chart

Number	X^2	X^3	X^4	X^5
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64	256	1024
5	25	125	625	3125
6	36	216	1296	7776
7	49	343	2401	16807
8	64	512	4096	32768
9	81	729	6561	59049
10	100	1000	10000	100000
11	121			
12	144			
13	169			
14	196			
15	225			
16	256			
17	289			
18	324			
19	361			
20	400			
21	441			
22	484			
23	529			
24	576			
25	625			

4.2 Irrational Numbers

$\sqrt{100}$ $\sqrt{0.24}$ $\sqrt{0.25}$ $\sqrt[3]{8}$ $\sqrt{\frac{9}{64}}$

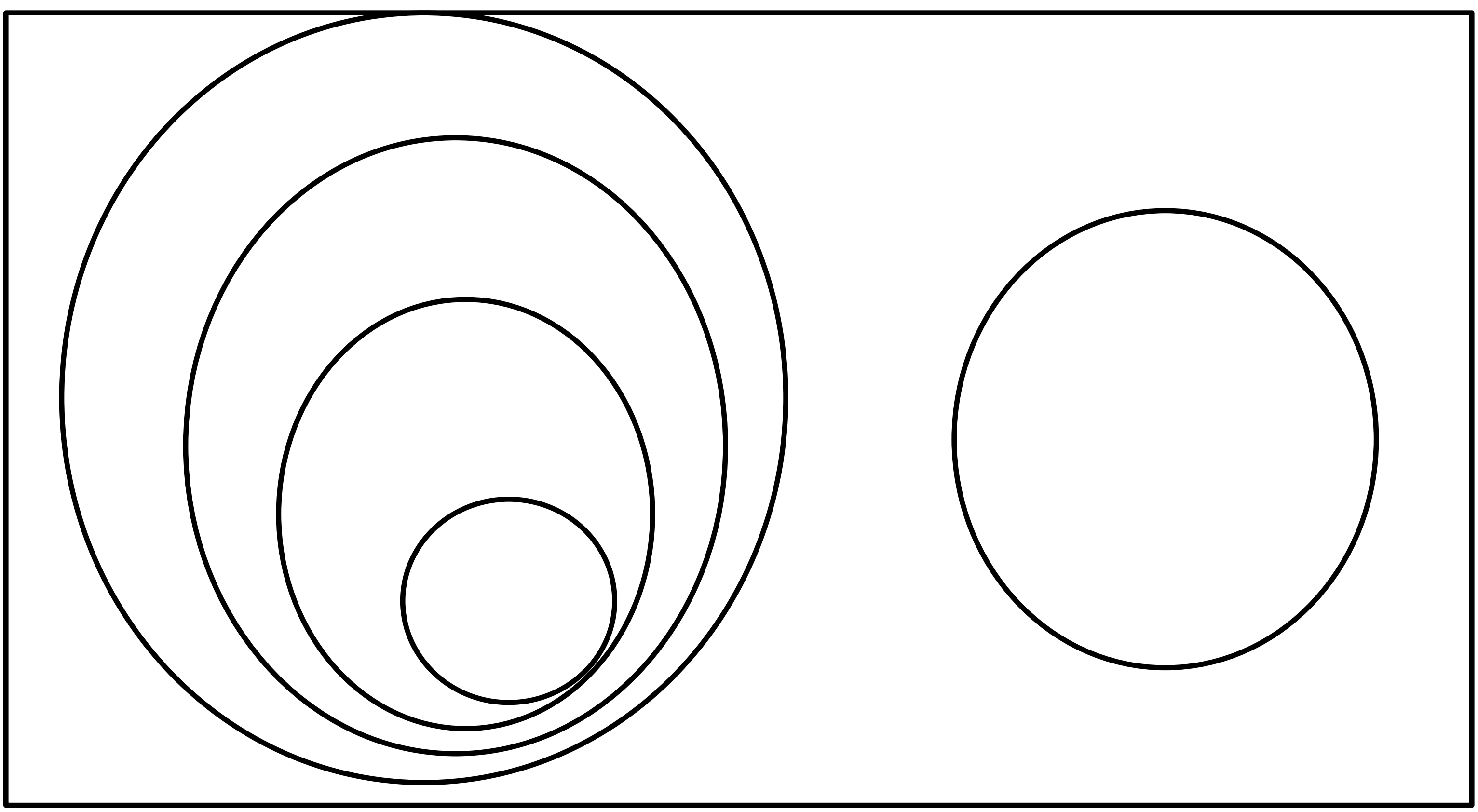
0.7 $\sqrt[4]{12}$ $\sqrt{\frac{1}{3}}$ 0.8^2 $\sqrt{2}$

Concept #5: Classify and order numbers- sort a set of numbers into rational and irrational numbers and describe which subsets of Real numbers it belongs to: natural, whole, integers, rational ,irrational and order them on a number line (NC)

Rational Number:

Irrational Numbers:

Oog the Caveman Story about the Number Systems



Example 1: Are the following numbers rational or irrational ,explain why ?

a) $\frac{-3}{5}$ b) $\sqrt{14}$ c) $\sqrt[3]{\frac{8}{27}}$

d) $0.\overline{3}$ e) $\sqrt[3]{-30}$

Example 2: Use a number line to order these from least to greatest?

$$\sqrt[3]{13}, \sqrt{18}, \sqrt{9}, \frac{167}{99}, \sqrt[3]{-5}$$

Example 3: Write a number that is:

a) a rational number but not an integer

b) a whole number but not a natural number

c) an irrational number

Example 4: Which subsets of the Real Numbers do the following numbers belong to: **Concept # 5**

a) $\sqrt{7}$ b) $\frac{-3}{4}$ c) 9

4.2 Assignment Pg 211 #3,5,8,9,12,14,20 (NO CALCULATOR)

4.3Mixed and Entire Radicals (Day 1)

The problem with irrational numbers is that when we put them into a calculator to simplify we get a decimal that never ends – no matter how many decimal numbers we write down we are always rounding the decimal off. Doing that means that our answer is always approximate and never exact. Sometimes it is important that we have the exact answer to an irrational number in radical form. The following section teaches us how to simplify a radical but still leave it exact.

Entire Radical :

Mixed Radical:

Concept #6: Write a radical as a mixed radical in simplest form and mixed radicals as an entire radical (NC)

Multiplication Property of Radicals: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ **Where n is a natural number and a and b are rational numbers**

Example 1: Write each radical in simplest form, if possible.

a) $\sqrt{24}$ (Factor the radicand to find the largest perfect square factor)

$= \sqrt{4 \cdot 6}$ (Break it down to the perfect square factor times another number, Leave them under the root sign)

$= \sqrt{4} \cdot \sqrt{6}$ (Use the multi. property of radicals, and write each factor under the root sign and multiply the two radicals)

$= 2 \cdot \sqrt{6}$ (Take the root of any perfect squares, cubes etc. this rational number now becomes the coefficient of your radical)

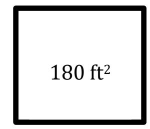
$= 2\sqrt{6}$

b) $\sqrt{45}$ c) $\sqrt{26}$ d) $\sqrt[3]{144}$ e) $\sqrt[4]{162}$

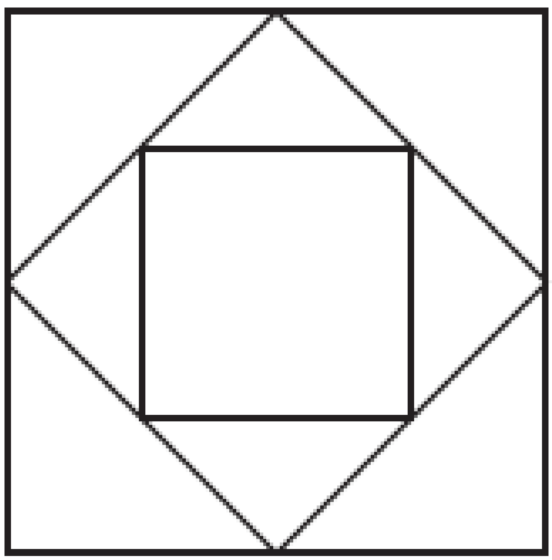
Example 2: Simplify the following radical using prime factorization

a) $\sqrt{189}$ b) $\sqrt{2940}$ c) $\sqrt[4]{18144}$

Example 3: Express the side length of the square as a radical in simplest form



Example 4: The largest square in this diagram has side length of 8cm. Calculate the side length and area of each of the two smaller squares. Write the radicals in simplest form.



4.3 Day 1 Assignment Pg 218 #10(a-d),11(a-d),17(a-d),14,19 (NO CALCULATOR)

4.3 Mixed and Entire Radicals Day 2

Concept #6: Write a radical as a mixed radical in simplest form and mixed radicals as an entire radical(NC)

How to Change a Mixed Radical to an Entire Radical

1. Identify the index (small number outside the root)
2. Take the number outside the radical and put it inside the radical – but repeat it as many times as the index is.
3. Multiply all the numbers inside the radical together.

Example 3: Write each mixed radical as an entire radical

a) $4\sqrt{3}$ b) $3\sqrt{2}$ c) $3\sqrt[3]{2}$

d) $2\sqrt[3]{4}$ e) $2\sqrt[5]{2}$

Example 2: Arrange in order from greatest to least.

$8\sqrt{5},\ 6\sqrt{2},\ 7\sqrt{15},\ 2\sqrt{9},\ \sqrt{17}$

Example 4: a) Can every mixed radical be expressed as an entire radical?

b) Can every entire radical be expressed as a mixed radical? Give examples to support

4.3 Day2 Assignment pg 218 #5a-d,12f-i, 18ac, 20,22ab (No Calculator)

4.4 – FRACTIONAL EXPONENTS AND RADICALS

A. Work with a partner and complete each table. Use a calculator to complete the second column.

x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} =$
4	$4^{\frac{1}{2}} =$
9	
16	
25	

x	$x^{\frac{1}{3}}$
1	
8	
27	
64	
125	

Continue the pattern. Write in the next 3 lines in each table.

B. For each table:

- What do you notice about the numbers in the first column? Compare the numbers in the first and second columns. What conclusions can you make?
- What do you think the exponent $\frac{1}{2}$ means? Confirm your prediction by trying other examples on a calculator.
- What do you think the exponent $\frac{1}{3}$ means? Confirm your prediction by trying other examples on a calculator.

C. What do you think $a^{\frac{1}{4}}$ and $a^{\frac{1}{5}}$ mean? Use a calculator to test your predictions for different values of a.

D. What does a mean? Explain your reasoning.

4.4 Fractional Exponents and Radicals

Fractional Exponent with a Numerator of 1

1. Square roots can also be written using an exponent of $\frac{1}{2}$. This means that $\sqrt{x} = x^{\frac{1}{2}}$
2. Cube roots can also be written using an exponent of $\frac{1}{3}$. This means that $\sqrt[3]{x} = x^{\frac{1}{3}}$
3. Fourth roots can also be written using an exponent of $\frac{1}{4}$. This means that $\sqrt[4]{x} = x^{\frac{1}{4}}$
4. Fifth roots can also be written using an exponent of $\frac{1}{5}$. This means that $\sqrt[5]{x} = x^{\frac{1}{5}}$

Concept #7: Express powers with rational exponents as radicals and vice versa (NC)

Example 1: Evaluate the following without using a calculator.

a) $16^{\frac{1}{2}}$ b) $(-64)^{\frac{1}{3}}$ c) $16^{\frac{1}{4}}$ d) $0.0049^{\frac{1}{2}}$

Make the connection:

So, $8^{\frac{2}{3}} = 8^{\frac{1}{3} \cdot 2}$ or $8^{\frac{2}{3}} = 8^{2 \cdot \frac{1}{3}}$

Fractional Exponents

In our previous questions, the numerator of the fractional exponent was 1. Technically, what we actually should have seen for each root was the following: $x^{\frac{1}{2}} = \sqrt{x^1}$ *or* $\left(\sqrt{x}\right)^1$, $x^{\frac{1}{3}} = \sqrt[3]{x^1}$ *or* $\left(\sqrt[3]{x}\right)^1$ etc with the number "1" from the numerator appearing as shown. Because anything to the power 1 is itself, writing the 1 was unnecessary. If the numerator is larger than 1, it is necessary to show the number. We will use the following rule to simplify radical with numerators larger than 1:

1. $x^{\frac{m}{2}} = \sqrt{x^m}$ *or* $\left(\sqrt{x}\right)^m$ **"DE- nominator goes in DE- notch"**
2. $x^{\frac{m}{3}} = \sqrt[3]{x^m}$ *or* $\left(\sqrt[3]{x}\right)^m$
3. $x^{\frac{m}{4}} = \sqrt[4]{x^m}$ *or* $\left(\sqrt[4]{x}\right)^m$

In general, $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ *or* $\left(\sqrt[n]{x}\right)^m$

Example 2: Write each power as a radical in two different ways.

a) $15^{\frac{5}{2}}$

b) $82^{\frac{2}{3}}$

Example 3: Write as a power with a fractional exponent.

a) $\sqrt[3]{5^2}$

b) $\left(\sqrt[4]{7}\right)^3$

Example 4: Evaluate the following without using a calculator.

a) $16^{\frac{3}{4}}$

b) $27^{\frac{2}{3}}$

c) $(0.0049)^{\frac{3}{2}}$

Example 5: Evaluate the following to two decimal places, using a calculator.

a) $\sqrt[3]{6^5}$

b) $\left(\sqrt[5]{4}\right)^2$

c) $6^{\frac{3}{4}}$

f) $(-3)^{\frac{5}{2}}$

g) $\left(\sqrt{7}\right)^5$

Example 6: Change the following exponents from decimal form to fraction form and evaluate. Do not use your calculator!

a) $16^{0.25}$

c) $9^{1.5}$

Steps to Change Decimals into Fractions.

1. Count how many numbers there are after the decimal point. Call this number "a"
2. Rewrite your given decimal number without the decimal point and put it on the top of a fraction.
3. On the bottom of the fraction put the number 1 followed by "a" number of zero's.
4. Reduce this fraction by dividing the numerator and denominator by the same number (if possible)

4.4 Assignment Pg 227 #3aef,4abe,5abc,6abc,7,10abcd,11,12 (NO CALCULATOR)

4.5 Negative Exponents and Reciprocals

May work with a partner. You will need a 16 x 16 grid paper. May use scissors if needed.
Step 1: Determine the area of the grid in square units and as a power of 2. Record your results in the table below.
Step 2: Fold or cut the gird in half. In the table, record the area of the remaining piece in square units and as a power of 2.
Step 3: Repeat until it cannot be folded or cut anymore.
Step4: Use the patterns to extend the second and third columns of the table to fold/cut 13.

Cut/fold	Area (units $^{2)}$	Area as a power of 2
Start	256	

Cut/fold	Area (units $^{2)}$	Area as a power of 2

Compare the areas for each pair of powers in the table: $2^{-2}\,and\,2^{2}$ $2^{-1}\,and\,2^{1}$ $2^{-3}\,and\,2^{3}$
What relationship do you notice?

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Cut/fold	Area (units $^{2)}$	Area as a power of 2

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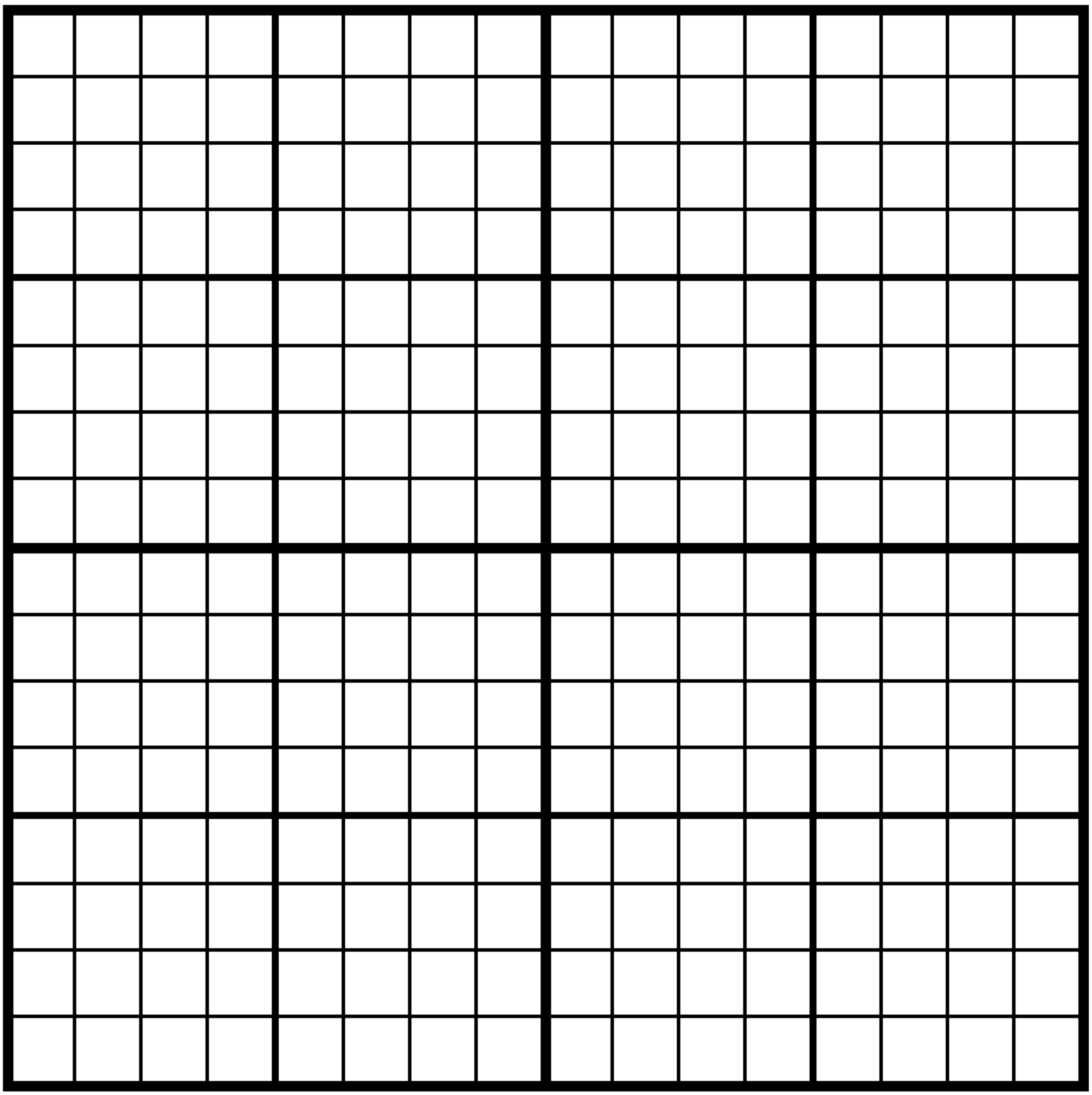
What relationship do you notice?



4.5 Negative Exponents and Reciprocals

Negative Exponents

When we are given a question with negative exponents, we need to change them to positive exponents before we can evaluate or simplify the radical. Here is the rule for changing a negative exponent to a positive exponent:

In general: $x^{-n} = \dfrac{1}{x^n}$ *and* $\dfrac{1}{x^{-n}} = x^n$

First, make sure the question itself is a fraction (the exponent does not have to be a fraction but the "big" number does). Put your original term over 1 if it is not a fraction.

The number that has the negative exponent needs to be moved to the opposite part of the fraction (top to bottom, bottom to top). Once you move it there, the exponent becomes positive. The exponent stays exactly the same except changes sign!!!!!

If the bottom of your fraction is now 1, you can remove it. If the top of the fraction is 1, it needs to stay!

Example 1: Write the following with positive exponents then evaluate.

a) 5^{-3} b) 1000^{-2} c) $\dfrac{1}{3^{-4}}$

Example 2: Evaluate each power (even though it does not say to change to positive exponents, you ALWAYS MUST do this first!)

a) $16^{-\frac{1}{2}}$ b) $(-8)^{-\frac{1}{3}}$ c) $\dfrac{1}{81^{\frac{-1}{4}}}$

Negative Exponents when the Base Number is a Fraction

1. First, give the negative exponent to both the number in the numerator and the number in the denominator.
2. Move each number to make the exponent of both numbers positive.
3. Perform the exponential operation on the term in the numerator and the term in t Denominator.
4. Simplify your result (if possible)

$$\left(\frac{2}{5}\right)^{-3} = \frac{(2)^{-3}}{(5)^{-3}} = \frac{5^3}{2^3} = \frac{125}{8}$$

In general: $\left(\dfrac{a}{b}\right)^{-n} = \dfrac{b^n}{a^n}$ *or* $\left(\dfrac{a}{b}\right)^{-\frac{m}{n}} = \dfrac{b^{\frac{m}{n}}}{a^{\frac{m}{n}}}$

Concept #8: Evaluate powers with negative integer exponents, negative rational exponents or an exponent of zero **(NC)**

Example 3: Evaluate each power.

a) $\left(\frac{1}{3}\right)^{-4}$

b) $\left(\frac{2}{3}\right)^{-3}$

c) $\left(\frac{4}{9}\right)^{-\frac{3}{2}}$

d) $16^{-1.5}$

e) $(-0.008)^{-\frac{4}{3}}$

4.5 Assignment: Page 233 #3, 8, 9, 13 (No CALCULATOR)

4.6 Applying the Exponent Laws (Day 1)

Concept #9: Simplify expressions by applying the exponent laws (including expressions variable bases) **(NC)**

Review of Exponent Laws

1. When two powers with the same base are multiplied, add the exponents.

$$\boxed{(a^m)(a^n) = a^{m+n}}$$

ex: $\left(2^5\right)\left(2^7\right) = 2^{5+7}$
$= 2^{12}$
$= 4096$

2. When two powers with the same base are divided, subtract the exponents.

$$\boxed{\frac{a^m}{a^n} = a^{m-n}}$$

ex: $\frac{3^8}{3^5} = 3^{8-5}$
$= 3^3$
$= 27$

3. When you are taking a "power of a power", multiply the exponents.

$$\boxed{\left(a^m\right)^n = a^{(m)(n)}}$$

ex: $\left(2^3\right)^2 = 2^{3 \times 2}$
$= 2^6$
$= 64$

4. When you are taking a "power of a product", give the exponent to each product.

$$\boxed{\left(ab\right)^m = (a^m)(b^m)}$$

ex: $\left[(2)(5)\right]^3 = \left(2^3\right)\left(5^3\right)$
$= (8)(125)$
$= 1000$

5. When you are taking a "power of a quotient (division)", give the exponent to both the numerator and denominator.

$$\boxed{\left(\frac{a}{b}\right)^n = \frac{(a^n)}{(b^n)}}$$

ex: $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$
$= \frac{16}{81}$

Example 1: Simplify each expression by writing it as a power with a positive exponent.

REMEMBER: Questions always need to be left with answers that only contain positive exponents to be in simplest form.

a) $\left(3^{-7}\right)\left(3^3\right)$

b) $\left(5^{-7}\right)^2\left(5^{-2}\right)^3$

Example 2: Simplify each expression by writing it as a **single power** with a positive exponent and evaluate.

c) $\dfrac{\left(6^{-8}\right)\left(6^4\right)}{\left(6^{-3}\right)}$

d) $\left(2^{-4}\right)\left(2^{-3}\right)$

e) $\left(3^{-2} \bullet 3^4\right)^{-2}$

Ex. 2 continued… f) $\dfrac{11^{-2}}{11^{4}\bullet 11^{-6}}$ g) $\left[\left(\dfrac{2}{3}\right)^{3}\right]^{-2}$

Example 3: Simplify each expression by writing it as a power with a positive exponent.

a) $3^{\frac{1}{2}}\cdot 3^{\frac{1}{4}}$

b) $2^{-\frac{1}{3}}\bullet\left(2^{-2}\right)^{\frac{1}{2}}$

c) $\dfrac{\left(5^{-0.5}\right)\left(5^{1.5}\right)}{5^{0.5}}$

d) $\left(4^{\frac{1}{2}}\bullet 4^{-\frac{1}{4}}\right)^{3}$

e) $\left[\left(-\dfrac{4}{5}\right)^{2}\right]^{-3}\div\left[\left(-\dfrac{4}{5}\right)^{4}\right]^{-5}$

f) $\dfrac{9^{\frac{5}{4}}\bullet 9^{-\frac{1}{4}}}{9^{\frac{3}{4}}}$

4.6 Assignment 1: Page 242 # 4, 7, 10 (NO CALCULATOR)

4.6 Applying

Concept #9: Simplify expressions by applying the exponent laws (including expressions variable bases) **(NC)**

Review of exponent Laws using variables:
Pg 241 #3, 5, 6 (Complete Orally as a class)

Example 1: Simplify. (Means to leave all variables with positive exponents and evaluate all coefficients)

a) $3a^2 \cdot a^{-5} \cdot a^4$

b) $\left(2x^2 \cdot 3x^{-5}\right)^3$

c) $\dfrac{12a^2}{3a^{-3}}$

d) $x^{\frac{3}{2}} \cdot x^{-1}$

e) $\dfrac{10a^{\frac{9}{4}}}{8a^3}$

f) $m^4 n^{-2} \cdot m^2 n^3$

g) $\dfrac{6x^4 y^{-3}}{14xy^2}$

h) $\left(25a^4 b^2\right)^{\frac{3}{2}}$

i) $\left(x^{3}y^{-\frac{3}{2}}\right)\left(x^{-1}y^{\frac{1}{2}}\right)$

j) $\dfrac{12x^{-5}y^{\frac{5}{2}}}{3x^{\frac{1}{2}}y^{-\frac{1}{2}}}$

k) $\left(\dfrac{50x^{2}y^{4}}{2x^{4}y^{7}}\right)^{\frac{1}{2}}$

l) $\dfrac{(2a^{-2}b^{4}c^{-3})^{-2}}{(4a^{2}bc^{-4})^{2}}$

4.6 Assignment #2: Page 242 8, 11 (Just simplify), 14(just simplify), 16, 17,19 21(NO CALCULATOR)