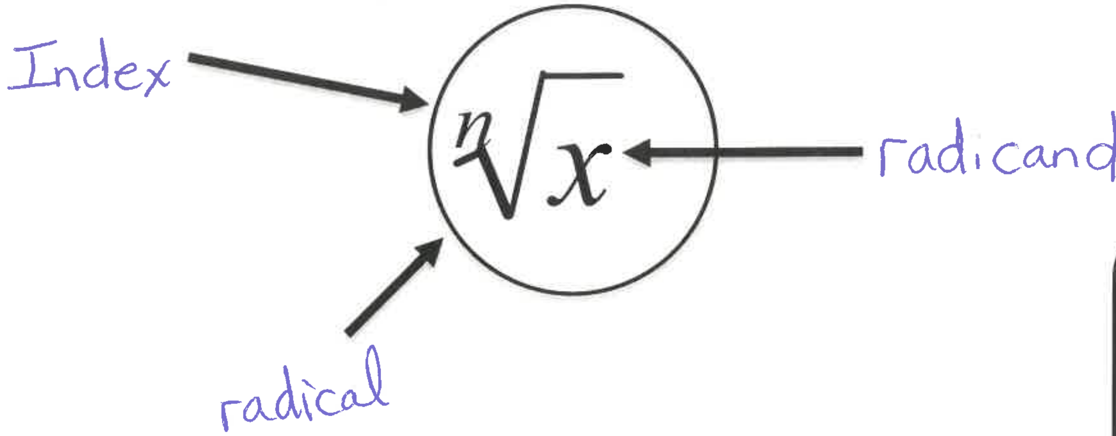
I4.1 Estimating Roots

Parts of the Radical



> To calculate 3^4, you will need to use one of the following on your calculator:
> - 3 ^ 4
> - 3 y^x 4
> - 3 x^y 4
>
> Or use (3)(3)(3)(3)

$3^2 = 9$ because $3 \times 3 = 9$; Then, $\sqrt{9} = 3$

$(-3)^3 = -27$ because $(-3) \times (-3)(-3) = -27$; Then, $\sqrt[3]{-27} = -3$

$3^4 = 81$ because $3 \cdot 3 \cdot 3 \cdot 3 = 81$; Then, $\sqrt[4]{81} = 3$

Example 1 : How would you write 5 as a:

a) square root $5 = \sqrt{5^2} = \sqrt{25}$

b) Cube Root $5 = \sqrt[3]{5^3} = \sqrt[3]{125}$

c) fourth root $5 = \sqrt[4]{5^4} = \sqrt[4]{625}$

Example 2: Without using a calculator, find the exact value or approximate/estimated value of the following:

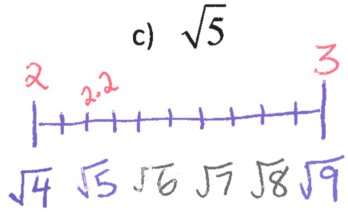
a) $\sqrt{4}$

$= 2$

b) $\sqrt{9}$

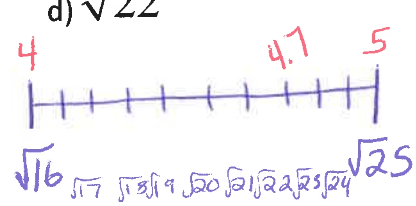
$= 3$

c) $\sqrt{5}$



$\boxed{\sqrt{5} \approx 2.2}$

d) $\sqrt{22}$



$\boxed{\sqrt{22} \approx 4.7}$

Steps for Finding the Approximate Value of a Radical using Benchmarks

1. Draw a number line. Below the number line and at each end place the closest perfect (square, cube , fourth power etc.) depending on the index.
2. Find the root of each of your "answers". Put these root numbers above the number line on the ends using a different colour.
3. Between your root numbers, put 9 tick marks. Label the tick marks (each tick mark is worth .10
4. Look below your number line. This represents the space between your two original "answer" numbers. In between these "answer numbers", place where you think your radicand would appear between the two "answers". Is it right in the middle? Closer to the first answer? Closer to the second answer?
5. Look at the tick marks above where you wrote your radicand. The number that corresponds to your tick mark is your approximate/estimated solution to the root of your radicand.

Example 3: Without using a calculator, find the exact or approximate value of the following:

a) $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}}$

$= \frac{4}{5}$

See step 1 below

b) $\sqrt[4]{16} = \sqrt[4]{2^4}$

$= 2$

c) $\sqrt{0.81}$

$= 0.9$

d) $\sqrt[3]{0.027}$

$= 0.3$

See step 2 below.

Hints for Finding Exact Roots of Fractions or Decimals

1. You can split a fraction in a radical into two separate radicals – one on the top and one on the bottom. Do each radical separately. $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$
2. If you see a radicand that looks like a perfect square root number, cube root number etc except it is after a decimal, chances are it will still work.
   - For a square root, it will work if there are an even number of place values after the decimal. The answer will be the root of the "nice number" with a decimal length of half the length of the original question. Add zero's in front to make the decimal as long as you need it.
   - For a cube root, it will work if the number of place values after the decimal is divisible by 3. The answer will be the root of the "nice number" with a decimal length of a third the length of the original question. Add zero's in front to make the decimal as long as you need it.
   - The same pattern applies for fourth and fifth roots.

Example 4: Without using a calculator, find the exact or approximate values of the following:

a) $\sqrt{0.0049}$

$= 0.07$

b) $\sqrt[3]{0.000216}$

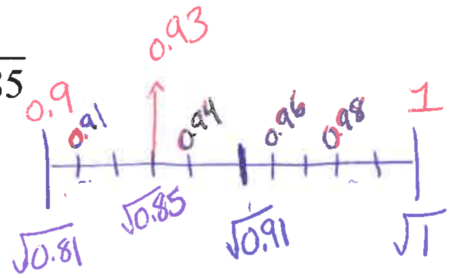
$= \sqrt[3]{0.06}$

c) $\sqrt[3]{\dfrac{0.008}{0.125}} = \dfrac{\sqrt[3]{0.008}}{\sqrt[3]{0.125}}$

$= \dfrac{0.2 \times 10}{0.5 \times 10}$

$= \dfrac{2}{5}$

a) $\sqrt{0.85}$



$\boxed{\sqrt{0.85} \approx 0.93}$

To calculate a cube root or fourth root or higher on a graphing calculator push the MATH buttonthen find $\sqrt[3]{(}$ or $\sqrt[x]{(}$ is a root higher than 3.

Example 5: Using your calculator, find the following answers:

a) $\sqrt{-9} = \varnothing$ No solution.

b) $\sqrt{-25} = \varnothing$ No Solution

c) $\sqrt[3]{-8} = -2$

d) $\sqrt[3]{-125} = -5$

e) $\sqrt[4]{-16} = \varnothing$ No solution

f) $\sqrt[4]{-81} = \varnothing$ No solution

g) $\sqrt[5]{-32} = -2$

h) $\sqrt[5]{-243} = -3$

Explain how you can predict which answers won't have a solution…….

Even indexs' can't have a negative radicand in a radical

Finding Roots with Negative Radicands

If you have a negative radicand with an even index, the answer will be undefined and have no solution

If you have a negative radicand with an odd index, the answer will be negative.

No CALCULATOR Pg 206 #1, 2,3(a,b,c,d,e) #4a, 5, 6

Powers Chart

Number	X^2	X^3	X^4	X^5
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64	256	1024
5	25	125	625	3125
6	36	216	1296	7776
7	49	343	2401	16807
8	64	512	4096	32768
9	81	729	6561	59049
10	100	1000	10000	100000
11	121			
12	144			
13	169			
14	196			
15	225			
16	256			
17	289			
18	324			
19	361			
20	400			
21	441			
22	484			
23	529			
24	576			
25	625			

Pre - AP

Yes = rational
No = irrational

4.2 Irrational Numbers

$\sqrt{100} = 10 = \frac{10}{1}$ Yes

$\sqrt{0.24} \approx 0.48$ No

$\sqrt{0.25} = 0.5 = \frac{5}{10}$ Yes

$\sqrt[3]{8} = 2 = \frac{2}{1}$ Yes

$\sqrt{\frac{9}{64}} = \frac{3}{8}$ Yes

$0.5 = \frac{5}{10}$ Yes

$\sqrt[4]{12} \approx 3.5$ No

$\sqrt{\frac{1}{3}} \approx 0.57$ No

$0.8^2 = 0.64 = \frac{64}{100}$ Yes

$\sqrt{2} \approx$ No

"How are radicals that are rational numbers different from radicals that are irrational?"

Rational Number: Any number that can be written as a fraction such that $\frac{a}{b}$, $b \neq 0$

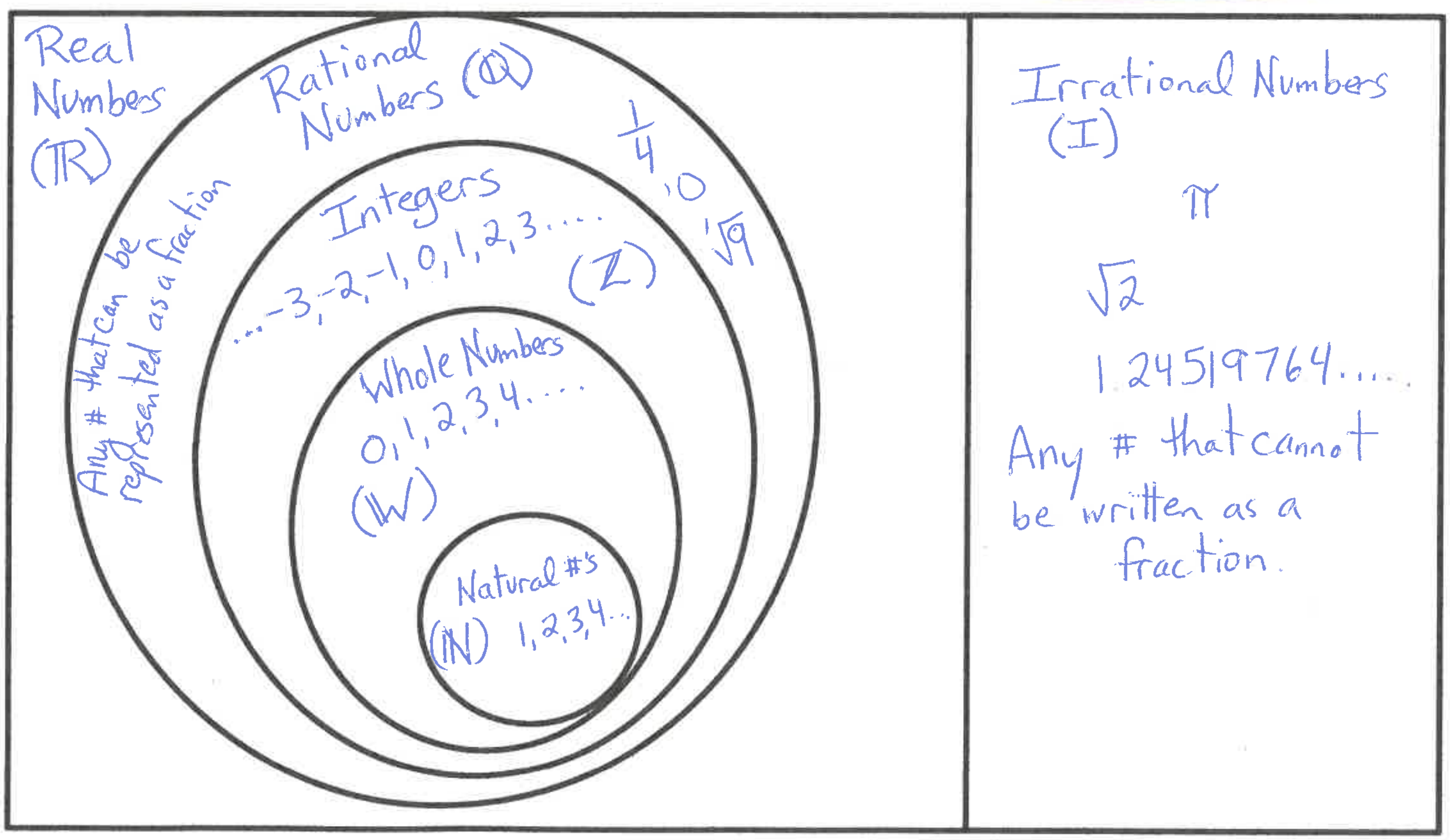
- radicals that are perfect squares, cubes etc.
- decimals that terminate or repeat.

Irrational Numbers: Numbers that cannot be written as a fraction $\left(\frac{a}{b}\right)$.

- radicals that are not perfect squares, cubes etc.
- decimals that do not terminate or repeat (ex / π)

Complex #'s
$\sqrt{-1} = i$

Ugg the Caveman Story about the Number Systems



Example 1: Are the following numbers rational or irrational ,explain why ?

a) $\frac{-3}{5}$ Rational ($\mathbb{Q}$)
written as a fraction

b) $\sqrt{14}$ Irrational
Cannot be written as a fraction, it approx equals ≈ 3.7

c) $\sqrt[3]{\frac{8}{27}}$ Rational
$= \frac{2}{3}$ written as a fraction

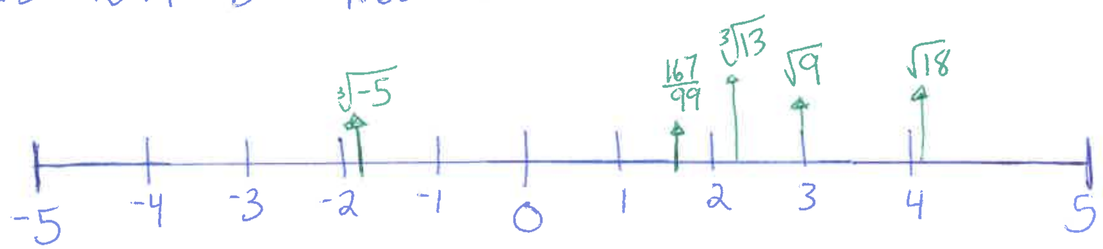
d) $0.\overline{3}$ Rational
$= \frac{1}{3}$ written as a fraction

e) $\sqrt[3]{-30}$ Irrational
30 is not a perfect cube therefore cannot be written as a fraction
≈ -3.1

Example 2: Use a number line to order these from least to greatest?

Note: Show student trick when a # is divided by 99

$\sqrt[3]{13}, \sqrt{18}, \sqrt{9}, \frac{167}{99}, \sqrt[3]{-5}$
$\approx 2.3 \quad \approx 4.1 \quad = 3 \quad \approx 1.\overline{68} \quad \approx -1.7$



Example 3: Write a number that is:

a) a rational number but not an integer 3/4 any positive whole # or fraction

b) a whole number but not a natural number 0

c) an irrational number $\sqrt{7}$ or $\sqrt{2}$

Example 4: Which subsets of the Real Numbers do the following numbers belong to:

a) $\sqrt{7}$
Irrational
Real

b) $\frac{-3}{4}$
Rational
& Real

c) 9
Natural, Whole #'s, Integers, Rational and Real

Assignment Pg 211 #3,5,8,9,12,14,20 (NO CALCULATOR)

4.3Mixed and Entire Radicals (Day 1)

The problem with irrational numbers is that when we put them into a calculator to simplify we get a decimal that never ends – no matter how many decimal numbers we write down we are always rounding the decimal off. Doing that means that our answer is always approximate and never exact. Sometimes it is important that we have the exact answer to an irrational number in radical form. The following section teaches us how to simplify a radical but still leave it exact.

Multiplication Property of Radicals: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ Where n is a natural number and a and b are rational numbers

Example 1: Write each radical in simplest form, if possible.

Mixed Radical = Coefficient multiplied to a radical. ($2\sqrt{3}$)
Entire Radical = Radical w/ no coefficient. ($\sqrt{56}$)

a) $\sqrt{24}$ (Factor the radicand to find the largest perfect square factor)

$= \sqrt{4 \cdot 6}$ (Break it down to the perfect square factor times another number, Leave them under the root sign)

$= \sqrt{4} \cdot \sqrt{6}$ (Use the multi. property of radicals, and write each factor under the root sign and multiply the two radicals)

$= 2 \cdot \sqrt{6}$ (Take the root of any perfect squares, cubes etc. this rational number now becomes the coefficient of your radical)

$= 2\sqrt{6}$

b) $\sqrt{45}$

$= \sqrt{9 \cdot 5}$

$= \sqrt{9} \cdot \sqrt{5}$

$= \boxed{3\sqrt{5}}$

c) $\sqrt{26}$

$= \sqrt{13 \cdot 2}$

There are no perfect square factors so $\sqrt{26}$ cannot be simplified.

d) $\sqrt[3]{144}$

$= \sqrt[3]{8 \cdot 18}$

$= \sqrt[3]{8} \cdot \sqrt[3]{18}$

$= \boxed{2\sqrt[3]{18}}$

e) $\sqrt[4]{162}$

$= \sqrt[4]{81 \cdot 2}$

$= \boxed{3\sqrt[4]{2}}$

$2^4 = 16$
$3^4 = 81$
$4^4 = 256$

Example 2: Simplify the following radical using prime factorization

a) $\sqrt{189}$

189 = 9 · 21 = 3 · 3 · 3 · 7

$= \sqrt{3 \cdot 3 \cdot 3 \cdot 7}$

$= 3\sqrt{3 \cdot 7}$

$= \boxed{3\sqrt{21}}$

b) $\sqrt{2940}$

2940 = 10 · 294 = 2 · 5 · 3 · 98 = 2 · 5 · 3 · 2 · 49 = 2 · 5 · 3 · 2 · 7 · 7

$= \sqrt{2 \cdot 2 \cdot 3 \cdot 5 \cdot 7 \cdot 7}$

$= 2 \cdot 7\sqrt{3 \cdot 5}$

$= \boxed{14\sqrt{15}}$

c) $\sqrt[4]{18144}$

18144 = 9 · 2016; 2016 = 9 · 224; 224 = 4 · 56; 56 = 8 · 7; 8 = 2 · 4; 4 = 2 · 2

$= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 7}$

$= 2 \cdot 3\sqrt[4]{2 \cdot 7}$

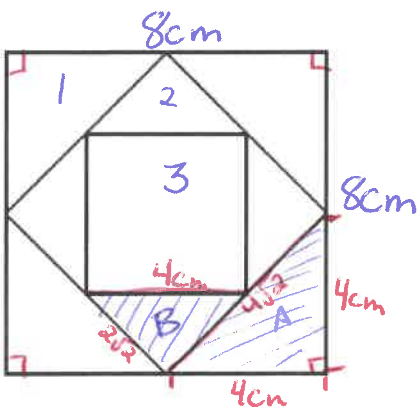
$= \boxed{6\sqrt[4]{14}}$

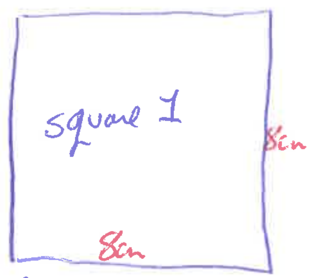
Example 3: Express the side length of the square as a radical in simplest form

$$\sqrt{180}$$
$$=\sqrt{36\cdot 5}$$
$$=\sqrt{36}\cdot\sqrt{5}$$
$$=6\sqrt{5}$$

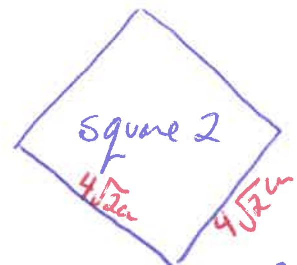
The side length of the square is $6\sqrt{5}$ ft

Example 4: The largest square in this diagram has side length of 8cm. Calculate the side length and area of each of the two smaller squares. Write the radicals in simplest form.

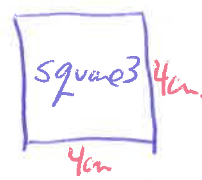




$A=8^2$
$=64cm^2$



$A=(4\sqrt{2})^2$
$A=16\cdot 2$
$A=32cm^2$



$A=4^2$
$A=16cm^2$

Triangle A
$$a^2+b^2=c^2$$
$$4^2+4^2=c^2$$
$$16+16=c^2$$
$$\sqrt{32}=\sqrt{c^2}$$
$$\sqrt{16\cdot 2}=c$$
$$4\sqrt{2}=c$$
Side length square 2 $=4\sqrt{2}$cm

Triangle B → side lengths are $\frac{4\sqrt{2}}{2}=2\sqrt{2}$
$$a^2+b^2=c^2$$
$$(2\sqrt{2})^2+(2\sqrt{2})^2=c^2$$
$$4\cdot 2+4\cdot 2=c^2$$
$$8+8=c^2$$
$$\sqrt{16}=\sqrt{c^2}$$
$$4=c$$
Side length square3 $=4$cm

Assignment Pg 218 #10(a-d),11(a-d),17(a-d),14,19

4.3 Mixed and Entire Radicals Day 2

How to Change a Mixed Radical to an Entire Radical

1. Identify the index (small number outside the root)
2. Take the number outside the radical and put it inside the radical – but repeat it as many times as the index is.
3. Multiply all the numbers inside the radical together.

Example 3: Write each mixed radical as an entire radical

a) $4\sqrt{3}$

$= \sqrt{4^2 \cdot 3}$

$= \sqrt{16 \cdot 3}$

$= \boxed{\sqrt{48}}$

b) $3\sqrt{2}$

$= \sqrt{3^2 \cdot 2}$

$= \sqrt{9 \cdot 2}$

$= \boxed{\sqrt{18}}$

c) $3\sqrt[3]{2}$

$= \sqrt[3]{3^3 \cdot 2}$

$= \sqrt[3]{27 \cdot 2}$

$= \boxed{\sqrt[3]{54}}$

d) $2\sqrt[3]{4}$

$= \sqrt[3]{2^3 \cdot 4}$

$= \sqrt[3]{8 \cdot 4}$

$= \boxed{\sqrt[3]{32}}$

e) $2\sqrt[5]{2}$

$= \sqrt[5]{2^5 \cdot 2}$

$= \sqrt[5]{32 \cdot 2}$

$= \boxed{\sqrt[5]{64}}$

Example 2: Arrange in order from greatest to least.

$8\sqrt{5},\ 6\sqrt{2},\ 7\sqrt{15},\ 2\sqrt{9},\ \sqrt{17}$ =

change to entire radicals or estimate each value.

$= \sqrt{8^2 \cdot 5},\ \sqrt{6^2 \cdot 2},\ \sqrt{7^2 \cdot 15},\ \sqrt{4 \cdot 9},\ \sqrt{17}$

$= \sqrt{64 \cdot 5},\ \sqrt{36 \cdot 2},\ \sqrt{49 \cdot 15},\ \sqrt{36},\ \sqrt{17}$

$= \sqrt{320},\ \sqrt{72},\ \sqrt{735},\ \sqrt{36},\ \sqrt{17}$

(4) (3) (5) (2) (1)

least to greatest

$= \sqrt{17},\ 2\sqrt{9},\ 6\sqrt{2},\ 8\sqrt{5},\ 7\sqrt{15}$

Example 4: a) Can every mixed radical be expressed as an entire radical?

b) Can every entire radical be expressed as a mixed radical? Give examples to support

a) Yes, $3\sqrt[3]{2} = \sqrt[3]{3^3 \cdot 2} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{54}$

b) No, Any radicand that has no factors that are p.squares, cubes etc. (depending on the index) can not be simplified to a mixed radical

Ex./ $\sqrt{21}$

Assignment pg 218 #5a-d,12acegi, 18ac, 20,22ab

4.4 Fractional Exponents and Radicals

Fractional Exponent with a Numerator of 1

1. Square roots can also be written using an exponent of $\frac{1}{2}$. This means that $\sqrt{x} = x^{\frac{1}{2}}$
2. Cube roots can also be written using an exponent of $\frac{1}{3}$. This means that $\sqrt[3]{x} = x^{\frac{1}{3}}$
3. Fourth roots can also be written using an exponent of $\frac{1}{4}$. This means that $\sqrt[4]{x} = x^{\frac{1}{4}}$
4. Fifth roots can also be written using an exponent of $\frac{1}{5}$. This means that $\sqrt[5]{x} = x^{\frac{1}{5}}$

Example 1: Evaluate the following without using a calculator.

a) $16^{\frac{1}{2}}$

$= \sqrt[2]{16}$

$= 4$

b) $(-64)^{\frac{1}{3}}$

$\sqrt[3]{-64}$

$= -4$

c) $16^{\frac{1}{4}}$

$\sqrt[4]{16}$

$= 2$

d) $0.0049^{\frac{1}{2}}$

$\sqrt[2]{0.0049}$

$= 0.07$

Make the connection: Remember your exponent law $(a^m)^n = a^{mn}$. Muliply exponents when raising a power to a power.

So, $8^{\frac{2}{3}} = 8^{\frac{1}{3}\cdot 2}$

$= (\sqrt[3]{8})^2$

$= (2)^2$

$= 4$

or

$8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}}$

$= (64)^{\frac{1}{3}}$

$= \sqrt[3]{64}$

$= 4$

The numerator of a fractional exponent represents a power, and the denominator represents the index of the root. The root and power can be evaluated in any order.

Fractional Exponents

In our previous questions, the numerator of the fractional exponent was 1. Technically, what we actually should have seen for each root was the following: $x^{\frac{1}{2}} = \sqrt{x^1}$ or $\left(\sqrt{x}\right)^1$, $x^{\frac{1}{3}} = \sqrt[3]{x^1}$ or $\left(\sqrt[3]{x}\right)^1$ etc with the number "1" from the numerator appearing as shown. Because anything to the power 1 is itself, writing the 1 was unnecessary. If the numerator is larger than 1, it is necessary to show the number. We will use the following rule to simplify radical with numerators larger than 1:

1. $x^{\frac{m}{2}} = \sqrt{x^m}$ or $\left(\sqrt{x}\right)^m$

"DE- nominator goes in DE- notch"

2. $x^{\frac{m}{3}} = \sqrt[3]{x^m}$ or $\left(\sqrt[3]{x}\right)^m$

3. $x^{\frac{m}{4}} = \sqrt[4]{x^m}$ or $\left(\sqrt[4]{x}\right)^m$

In general, $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $\left(\sqrt[n]{x}\right)^m$

Example 2: Write each power as a radical in two different ways.

a) $15^{\frac{5}{2}} = \left(15^{\frac{1}{2}}\right)^5$ or $\left(15^5\right)^{\frac{1}{2}}$

$= \left(\sqrt[2]{15}\right)^5$ $= \sqrt[2]{15^5}$

b) $82^{\frac{2}{3}} = \left(82^{\frac{1}{3}}\right)^2$ or $\left(82^2\right)^{\frac{1}{3}}$

$= \left(\sqrt[3]{82}\right)^2$ or $\sqrt[3]{82^2}$

Example 3: Write as a power with a fractional exponent.

a) $\sqrt[3]{5^2}$

$= \left(5^2\right)^{\frac{1}{3}}$

$= 5^{\frac{2}{3}}$

b) $\left(\sqrt[4]{7}\right)^3$

$= \left(7^{\frac{1}{4}}\right)^3$

$= 7^{\frac{3}{4}}$

Example 4: Evaluate the following without using a calculator.

a) $16^{\frac{3}{4}}$

$\boxed{\text{without a calculator which method is better??}}$

$= \sqrt[4]{16^3}$ or $\left(\sqrt[4]{16}\right)^3$

$= \sqrt[4]{4096}$ $= 2^3$

$= 8$ $= 8$

b) $27^{\frac{2}{3}}$

$= \left(\sqrt[3]{27}\right)^2$

$= (3)^2$

$= 9$

c) $(0.0049)^{\frac{3}{2}}$

$= \left(\sqrt[2]{0.0049}\right)^3$

$= (0.07)^3$

$\boxed{\text{Note: Need 6 decimal places}}$

$= 0.000343$

Example 5: Evaluate the following to two decimal places. using a calculator, You can punch into calc as a root or power.

a) $\sqrt[3]{6^5} = 6^{\frac{5}{3}}$
$= 19.81$

b) $\left(\sqrt[5]{4}\right)^2 = 4^{\frac{2}{5}}$
$= 1.74$

c) $6^{\frac{3}{4}}$
$= 3.83$

f) $(-3)^{\frac{5}{2}}$
$= \varnothing$ No solution
can't take the square root of a negative

g) $\left(\sqrt{7}\right)^5$
$= 129.64$

Example 6: Change the following exponents from decimal form to fraction form and evaluate. Do not use your calculator!

a) $16^{0.25}$
$= 16^{\frac{1}{4}}$
$= \sqrt[4]{16}$
$= 2$

c) $9^{1.5}$
$= 9^{1\frac{1}{2}}$
$= 9^{\frac{3}{2}}$
$= \left(\sqrt{9}\right)^3$
$= (3)^3$
$= 27$

Steps to Change Decimals into Fractions.

1. Count how many numbers there are after the decimal point. Call this number "a"
2. Rewrite your given decimal number without the decimal point and put it on the top of a fraction.
3. On the bottom of the fraction put the number 1 followed by "a" number of zero's.
4. Reduce this fraction by dividing the numerator and denominator by the same number (if possible)

4.4 Assignment Pg 227 #3aef,4abe,5abc,6abc,7,10abcd,11,12

4.5 ~~4.4~~ Negative Exponents and Reciprocals

Negative Exponents

When we are given a question with negative exponents, we need to change them to positive exponents before we can evaluate or simplify the radical. Here is the rule for changing a negative exponent to a positive exponent:

In general: $x^{-n} = \frac{1}{x^n}$ *and* $\frac{1}{x^{-n}} = x^n$

First, make sure the question itself is a fraction (the exponent does not have to be a fraction but the "big" number does). Put your original term over 1 if it is not a fraction.

The number that has the negative exponent needs to be moved to the opposite part of the fraction (top to bottom, bottom to top). Once you move it there, the exponent becomes positive. The exponent stays exactly the same except changes sign!!!!!

If the bottom of your fraction is now 1, you can remove it. If the top of the fraction is 1, it needs to stay!

Example 1: Write the following with positive exponents then evaluate.

a) $5^{-3} = \frac{1}{5^3} = \boxed{\frac{1}{125}}$

b) $1000^{-2} = \frac{1}{1000^2} = \boxed{\frac{1}{1000000}}$

c) $\frac{1}{3^{-4}} = 3^4 \boxed{= 81}$

Example 2: Evaluate each power (even though it does not say to change to positive exponents, you ALWAYS MUST do this first!)

a) $16^{-\frac{1}{2}} = \frac{1}{16^{1/2}} = \frac{1}{\sqrt{16}} = \boxed{\frac{1}{4}}$

Step 1: Make exponent positive by writing the reciprocal

Step 2: Evaluate

b) $(-8)^{-\frac{1}{3}} = \frac{1}{(-8)^{1/3}} = \frac{1}{\sqrt[3]{-8}} = \boxed{\frac{1}{-2}}$

c) $\frac{1}{81^{\frac{-1}{4}}} = 81^{1/4} = \sqrt[4]{81} \boxed{= 3}$

Negative Exponents when the Base Number is a Fraction

1. First, give the negative exponent to both the number in the numerator and the number in the denominator.
2. Move each number to make the exponent of both numbers positive.
3. Perform the exponential operation on the term in the numerator and the term in t Denominator.
4. Simplify your result (if possible)

$$\left(\frac{2}{5}\right)^{-3} = \frac{(2)^{-3}}{(5)^{-3}} = \frac{5^3}{2^3} = \frac{125}{8}$$

In general: $\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$ *or* $\left(\frac{a}{b}\right)^{-\frac{m}{n}} = \frac{b^{\frac{m}{n}}}{a^{\frac{m}{n}}}$

Example 3: Evaluate each power.

a) $\left(\frac{1}{3}\right)^{-4}$

$= \frac{1^{-4}}{3^{-4}}$

$= \frac{3^4}{1^4}$

$\boxed{= 81}$

b) $\left(\frac{2}{3}\right)^{-3}$

$= \frac{2^{-3}}{3^{-3}}$

$= \frac{3^3}{2^3}$

$\boxed{= \frac{27}{8}}$

c) $\left(\frac{4}{9}\right)^{-\frac{3}{2}}$

$= \frac{4^{-3/2}}{9^{-3/2}}$

$= \frac{9^{3/2}}{4^{3/2}}$

$= \frac{(\sqrt{9})^3}{(\sqrt{4})^3}$

$= \frac{3^3}{2^3} \boxed{= \frac{27}{8}}$

or $= \left(\frac{9}{4}\right)^{3/2}$

$= \left(\sqrt{\frac{9}{4}}\right)^3$

$= \left(\frac{3}{2}\right)^3$

$= \frac{3^3}{2^3}$

$\boxed{= \frac{27}{8}}$

d) $16^{-1.5}$

$= 16^{-3/2}$

$= \frac{1}{16^{3/2}}$

$= \frac{1}{(\sqrt{16})^3}$

$= \frac{1}{4^3}$

$\boxed{= \frac{1}{64}}$

e) $(-0.008)^{-\frac{4}{3}}$

$= \frac{1}{-0.008^{4/3}}$

$= \frac{1}{(\sqrt[3]{-0.008})^4}$

$= \frac{1}{(-0.2)^4}$

$= \frac{1}{+0.0016}$

To leave in simplest form you should not leave a decimal in the denominator or numerator.

$= 1 \div \frac{16 \div 2}{10\,000 \div 2}$ Reduce

$= 1 \div \frac{8}{5000}$

$= 1 \times \frac{5000}{8}$

$= 1 \times 625$

$\boxed{= 625}$

$$\begin{array}{r} 625 \\ 8\overline{)5000} \\ -48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

OR example (e)

4.5 Assignment: Page 233 #3, 8, 9, 13,

e) $(-0.008)^{-4/3}$

$= \left(-\frac{8}{1000}\right)^{-4/3}$

$= \left(-\frac{1000}{8}\right)^{4/3}$

$= \left(\sqrt[3]{-\frac{1000}{8}}\right)^4$

$= \left(-\frac{10}{2}\right)^4$

$= (-5)^4 \boxed{= 625}$

4.6 Applying the Exponent Laws (Day 1)

Review of Exponent Laws

1. When two powers with the same base are multiplied, add the exponents.

$$\boxed{(a^m)(a^n) = a^{m+n}}$$

ex: $(2^5)(2^7) = 2^{5+7}$
$= 2^{12}$
$= 4096$

2. When two powers with the same base are divided, subtract the exponents.

$$\boxed{\frac{a^m}{a^n} = a^{m-n}}$$

ex: $\frac{3^8}{3^5} = 3^{8-5}$
$= 3^3$
$= 27$

3. When you are taking a "power of a power", multiply the exponents.

$$\boxed{(a^m)^n = a^{(m)(n)}}$$

ex: $(2^3)^2 = 2^{3 \times 2}$
$= 2^6$
$= 64$

4. When you are taking a "power of a product", give the exponent to each product.

$$\boxed{(ab)^m = (a^m)(b^m)}$$

ex: $[(2)(5)]^3 = (2^3)(5^3)$
$= (8)(125)$
$= 1000$

5. When you are taking a "power of a quotient (division)", give the exponent to both the numerator and denominator.

$$\boxed{\left(\frac{a}{b}\right)^n = \frac{(a^n)}{(b^n)}}$$

ex: $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$
$= \frac{16}{81}$

Example 1: Simplify each expression by writing it as a power with a positive exponent.

REMEMBER: Questions always need to be left with answers that only contain positive exponents to be in simplest form.

a) $(3^{-7})(3^3) = 3^{-7+3}$
$= 3^{-4} = \boxed{\frac{1}{3^4}}$

b) $(5^{-7})^2(5^{-2})^3$
$= (5^{-14})(5^{-6}) = 5^{-14+-6}$
$= 5^{-20}$
$= \boxed{\frac{1}{5^{20}}}$

Example 2: Simplify each expression by writing it as a **single power** with a positive exponent. ~~Continued~~.... and Evaluate.

c) $\frac{(6^{-8})(6^4)}{(6^{-3})}$
$= \frac{6^{-4}}{6^{-3}}$
$= 6^{-1}$
$= \boxed{\frac{1}{6}}$

d) $(2^{-4})(2^{-3})$
$= 2^{-7}$
$= \frac{1}{2^7}$
$= \boxed{\frac{1}{128}}$

e) $(3^{-2} \cdot 3^4)^{-2}$
$= (3^2)^{-2}$
$= 3^{-4}$
$= \frac{1}{3^4}$
$= \boxed{\frac{1}{81}}$

Ex. 2 continued...

f) $\dfrac{11^{-2}}{11^{4}\cdot 11^{-6}}$

$= \dfrac{11^{-2}}{11^{-2}}$

$= 11^{0}$

$\boxed{= 1}$

g) $\left[\left(\dfrac{2}{3}\right)^{3}\right]^{-2}$

$= \left[\dfrac{2^3}{3^3}\right]^{-2}$

$= \left[\dfrac{8}{27}\right]^{-2}$

$= \left[\dfrac{27}{8}\right]^{2}$

$= \boxed{\dfrac{729}{64}}$

Example 3: Simplify each expression by writing it as a power with a positive exponent.

a) $3^{\frac{1}{2}}\cdot 3^{\frac{1}{4}}$

$= 3^{\frac{1}{2}+\frac{1}{4}}$

$= \dfrac{1\times 2}{2\times 2}+\dfrac{1}{4}$

$= \dfrac{2}{4}+\dfrac{1}{4}$

$= \dfrac{3}{4}$

$= \boxed{3^{3/4}}$

b) $2^{-\frac{1}{3}}\cdot\left(2^{-2}\right)^{\frac{1}{2}}$

$= 2^{-\frac{1}{3}}\cdot 2^{-1}$

$= 2^{-4/3}$

$= \boxed{\dfrac{1}{2^{4/3}}}$

$= -\dfrac{1}{3} + -\dfrac{1\times 3}{1\times 3}$

$= -\dfrac{1}{3} + -\dfrac{3}{3}$

$= \dfrac{-4}{3}$

c) $\dfrac{\left(5^{-0.5}\right)\left(5^{1.5}\right)}{5^{0.5}}$

$= \dfrac{\left(5^{-\frac{1}{2}}\right)\left(5^{3/2}\right)}{5^{1/2}}$

$= -\dfrac{1}{2}+\dfrac{3}{2}$

$= \dfrac{2}{2} = 1$

$= \dfrac{5^1}{5^{1/2}}$

$= \boxed{5^{1/2}}$

d) $\left(4^{\frac{1}{2}}\cdot 4^{-\frac{1}{4}}\right)^{3}$

$= \left(4^{\frac{1}{2}+-\frac{1}{4}}\right)^{3}$

$= \dfrac{1\times 2}{2\times 2} + -\dfrac{1}{4}$

$= \dfrac{2}{4} + -\dfrac{1}{4}$

$= \dfrac{1}{4}$

$= \left(4^{1/4}\right)^{3}$

$\boxed{= 4^{3/4}}$

e) $\left[\left(-\dfrac{4}{5}\right)^{2}\right]^{-3} \div \left[\left(-\dfrac{4}{5}\right)^{4}\right]^{-5}$

$= \left(-\dfrac{4}{5}\right)^{-6} \div \left(-\dfrac{4}{5}\right)^{-20}$

$= \left(-\dfrac{4}{5}\right)^{-6-(-20)}$

$\boxed{= \left(-\dfrac{4}{5}\right)^{14}}$

f) $\dfrac{9^{\frac{5}{4}}\cdot 9^{-\frac{1}{4}}}{9^{\frac{3}{4}}}$

$= \dfrac{9^{\frac{5}{4}+-\frac{1}{4}}}{9^{3/4}}$

$\dfrac{5}{4} - \dfrac{1}{4} = \dfrac{4}{4} = 1$

$= \dfrac{9^1}{9^{3/4}}$

$1 - \dfrac{3}{4}$

$= \dfrac{4}{4} - \dfrac{3}{4}$

$= \dfrac{1}{4}$

$\boxed{= 9^{1/4}}$

4.6 Assignment 1: Page 242 # 4, 7, 10

4.6 Applying the Exponents Laws (Day 2)

Review of exponent Laws using variables:
Pg 241 #3, 5, 6 (Complete Orally as a class)

Example 1: Simplify. Leave all variables with positive exponents and evaluate all coefficients (numbers in front of the variables)

a) $3a^2 \cdot a^{-5} \cdot a^4$

$= 3a^{2+-5+4}$

$= 3a^{-3+4}$

$\boxed{= 3a}$

b) $(2x^2 \cdot 3x^{-5})^3$ $= (2 \cdot 3x^{2+-5})^3$

$= (6x^{-3})^3$

$= 6^3 x^{-9}$

$\boxed{= \frac{216}{x^9}}$

c) $\frac{12a^2}{3a^{-3}} = 4a^{2-(-3)}$

$\boxed{= 4a^5}$

d) $x^{\frac{3}{2}} \cdot x^{-1}$

$= x^{\frac{3}{2}+(-1)}$

$= x^{\frac{3}{2}+(-\frac{2}{2})}$

$\boxed{= x^{\frac{1}{2}}}$

e) $\frac{10a^{\frac{9}{4}}}{8a^3} = \frac{5}{4}a^{\frac{9}{4}-3}$

$= \frac{5}{4}a^{\frac{9}{4}-\frac{12}{4}}$

$= \frac{5}{4}a^{-\frac{3}{4}}$ ← not the exponent is only being applied to the "a". So we only flip the "a".

$\boxed{= \frac{5}{4a^{3/4}}}$

f) $m^4n^{-2} \cdot m^2n^3$ $= m^4 \cdot m^2 \; n^{-2} \cdot n^3$

$= m^{4+2} \cdot n^{-2+3}$

$\boxed{= m^6 n^1}$

g) $\frac{6x^4y^{-3}}{14xy^2}$

$= \frac{3x^{4-1}y^{-3-2}}{7}$

$= \frac{3x^3y^{-5}}{7}$

$= \boxed{\frac{3x^3}{7y^5}}$

h) $(25a^4b^2)^{\frac{3}{2}}$

$= (\sqrt{25a^4b^2})^3$

$= (5a^2b)^3$

$= 5^3 a^{2 \cdot 3} b^{1 \cdot 3}$

$= \boxed{125a^6b^3}$

Note: $\sqrt{a^4} = a^2$ because $a^2 \cdot a^2 = a^4$

$\sqrt{b^2} = b$ because $b \cdot b = b^2$

$\frac{1}{4}$ -16

$\frac{1}{4}$ $\frac{1}{16}$

i) $\left(x^3y^{-\frac{3}{2}}\right)\left(x^{-1}y^{\frac{1}{2}}\right)$

$= x^{3+-1}\,y^{-\frac{3}{2}+\frac{1}{2}}$

$= x^2y^{-\frac{2}{2}}$

$= x^2y^{-1}$

$= \boxed{\dfrac{x^2}{y}}$

j) $\dfrac{12x^{-5}y^{\frac{5}{2}}}{3x^{\frac{1}{2}}y^{-\frac{1}{2}}}$

$= 4x^{-5-\frac{1}{2}}\,y^{\frac{5}{2}-\left(-\frac{1}{2}\right)}$

$= 4x^{\frac{-10}{2}-\frac{1}{2}}\,y^{\frac{6}{2}}$

$= 4x^{\frac{-11}{2}}\,y^3$

$= \boxed{\dfrac{4y^3}{x^{11/2}}}$

k) $\left(\dfrac{50x^2y^4}{2x^4y^7}\right)^{\frac{1}{2}}$

$= \left(25x^{2-4}\,y^{4-7}\right)^{\frac{1}{2}}$

$= \left(25x^{-2}y^{-3}\right)^{\frac{1}{2}}$

$= \left(\dfrac{25}{x^2y^3}\right)^{\frac{1}{2}}$

$= \dfrac{25^{\frac{1}{2}}}{x^{2\cdot\frac{1}{2}}\,y^{3\cdot\frac{1}{2}}}$

$= \boxed{\dfrac{5}{x\,y^{3/2}}}$

l) $\dfrac{(2a^{-2}b^4c^{-3})^{-2}}{(4a^2bc^{-4})^2}$

$= \dfrac{2^{-2}\cancel{a^4}\,b^{-8}\,c^{+6}}{4^2\cancel{a^4}\,b^2\,c^{-8}}$

$= \dfrac{1\,b^{-10}c^{14}}{4\cdot 16}$

$= \boxed{\dfrac{c^{14}}{64b^{10}}}$

4.6 Assignment #2: Page 242 8, 11cd, 14, 16 , 21

(no need to explain)

(Just simplify)

17, 19