

6.1 Graphing Linear Inequalities in Two Variables (Concept #1)

Example#1: For which inequalities is (3, 1) a possible solution?

a) $13 - 3x > 4y$

b) $2y - 5 \leq x$

c) $y \geq 9$

Graphing Linear Inequalities in two variable

Steps to Graphing Inequalities

1. Initially, graph the boundary line. (Ex. $y = mx + b$)
2. If the inequality is $<$ or $>$ use a **dotted line** (the points on the line are NOT included in the solution)
If the inequality is \leq or \geq use a **solid line** (the points on the line ARE included in the solution)

Methods of Graphing:

- table of values
- find x-intercept and y-intercept
- $y = mx + b$ ($m = \text{slope}$, $b = \text{y-intercept}$)

When given a domain and range, the solution set is considered:

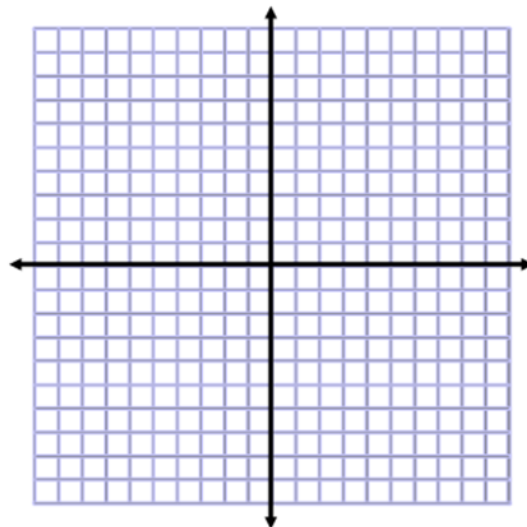
Continuous = if the independent and dependent variables belong to the Real set of Numbers. (ex. $x \in \mathbb{R}$, $y \in \mathbb{R}$)

Discrete = separate or distinct set of numbers (Integers, Whole Numbers, Natural)(ex. $x \in \mathbb{W}$, $y \in \mathbb{W}$)

If no domain and range are given, assume the set of Real Numbers.

3. Choose a check point (if possible, choose the origin) and substitute into the original equation.
Shade on the appropriate side of the line. Do NOT pick a point that lies on the line.

Example#2: a) Graph $-2x + 5y \geq 10$



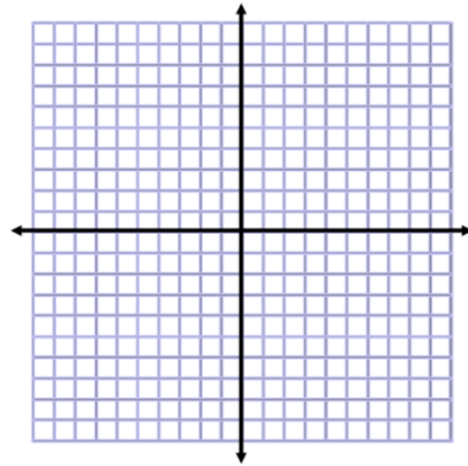
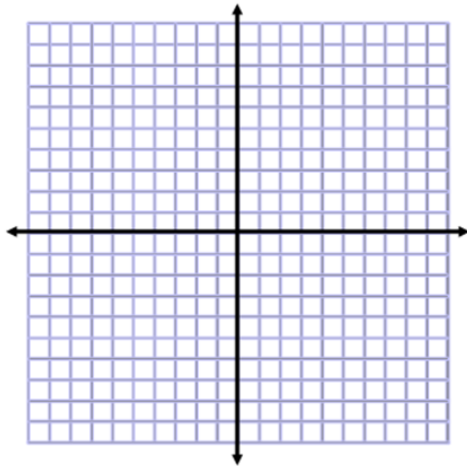
b) Is (5,4) a part of the solution set? Is (3,-4) ?

c) How would the above graph look if the domain and range changed to integers?

Example #3: Graph

a) $\{(x, y) \mid x - 3 > 0; x \in \mathbb{R}, y \in \mathbb{R}\}$

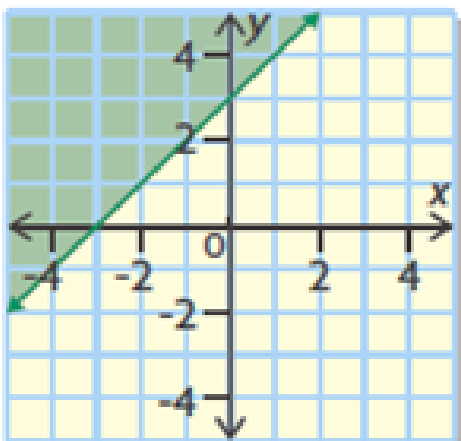
b) $\{(x, y) \mid x - 2y - 3 < 7 - 2x, x \in \mathbb{I}, y \in \mathbb{I}\}$



In graph b is $(\frac{1}{2}, 0)$ in the solution set?

Remember: When you divide or multiply both sides of an inequality by a negative number, you must _____ the inequality sign!!!

Example#4: Determine the inequality of this graph



Example #5: Malia and Raza’s CAP want to donate some money to the local initiative at school. To raise funds, they are selling PI necklaces and earrings that they have made themselves. Necklaces cost \$8 and earrings cost \$5. What is the range of possible sales they could make in order to donate at least \$100?

- a) Define your variables (let statements):

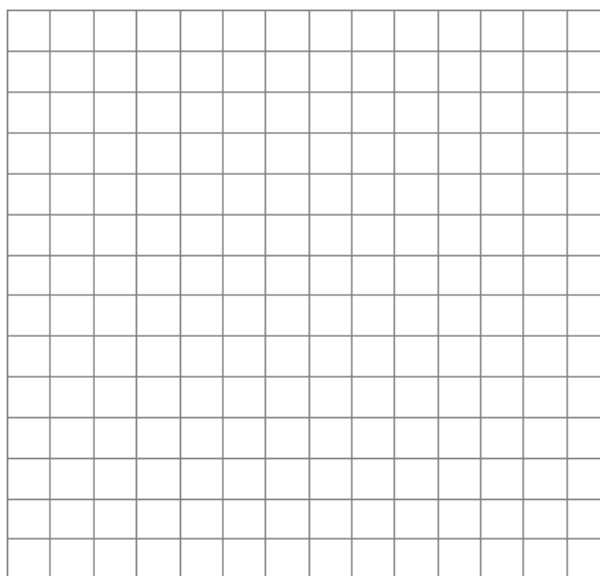
- b) Establish your inequality:

- c) Decide what type of restrictions will be on the domain and range and decide if your graph would include all Real Numbers, Integers or Whole Numbers.

d) Sketch a graph of this situation.

e) Find two points that satisfy this situation.

f) Verify both your points and explain what each point means within the context of this situation.



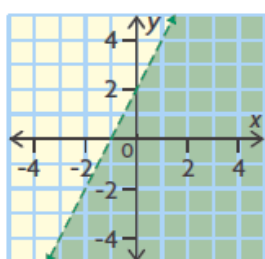
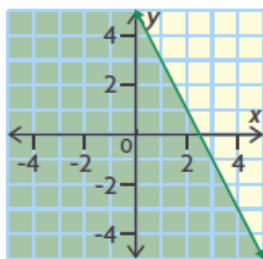
6.1 ASSIGNMENT: TEXTBOOK p303 #3, 4, 5bdc, 6cdef, AT LEAST TWO OF 7-12 or 14

PLUS THE FOLLOWING

1. For $y < 3x + 5$ which of the following points fall within the solution set?

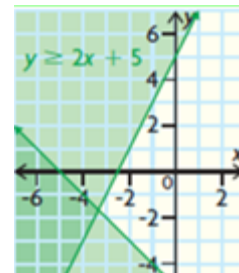
- $(-1, -3), (-1, 2), (-4, 3), (-2, -3), (3, 1), (1, 5), (0, 5), (-1, 3)$

2 Determine the inequality for the following graphs



6.2/3 Solving Systems of Linear Inequalities- Day1 (Concept #2)

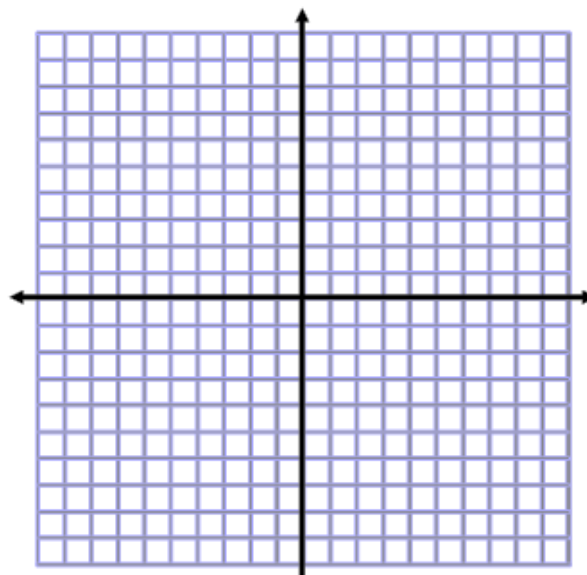
System of Linear Inequalities: A set of two or more linear inequalities that are graphed on the same coordinate plane; the intersection of their solution regions represents the solution set for the system. Example →



EXAMPLE #1:

Graph the system of linear inequalities. Choose two possible solutions from the set. Assume $x \in \mathbb{R}, y \in \mathbb{R}$.

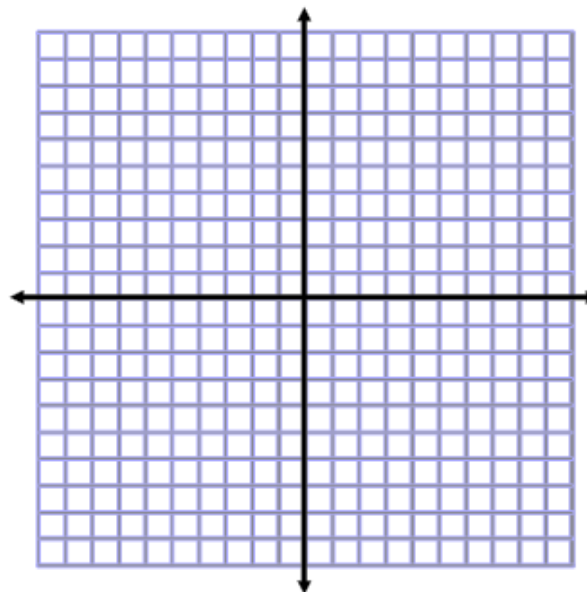
$$2x + 3y \leq 9 \quad \text{and} \quad y - 6x \geq 1$$



EXAMPLE #2:

Graph the system of linear inequalities. Choose two possible solutions from the set. Assume $x \in \mathbb{I}, y \in \mathbb{I}$.

$$-3x - 2y < 6 \quad \text{and} \quad y \leq 3$$

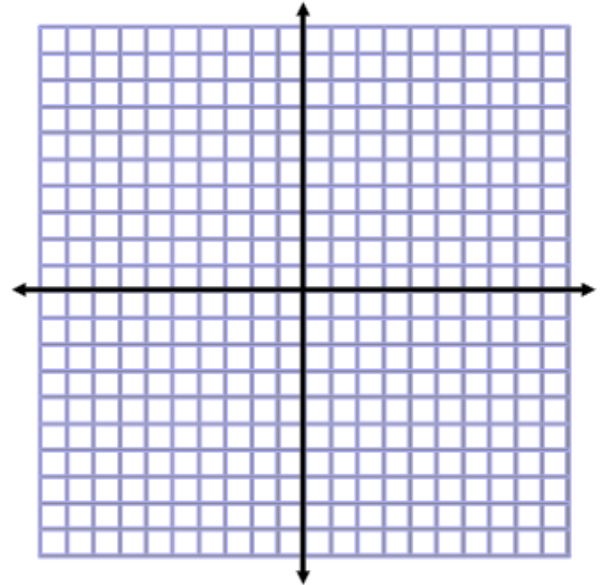


Does the intersection point of the system have an open dot or a closed dot? Explain

EXAMPLE #3:

Graph the system of linear inequalities. Choose two possible solutions from the set. Assume $x \in W, y \in W$.

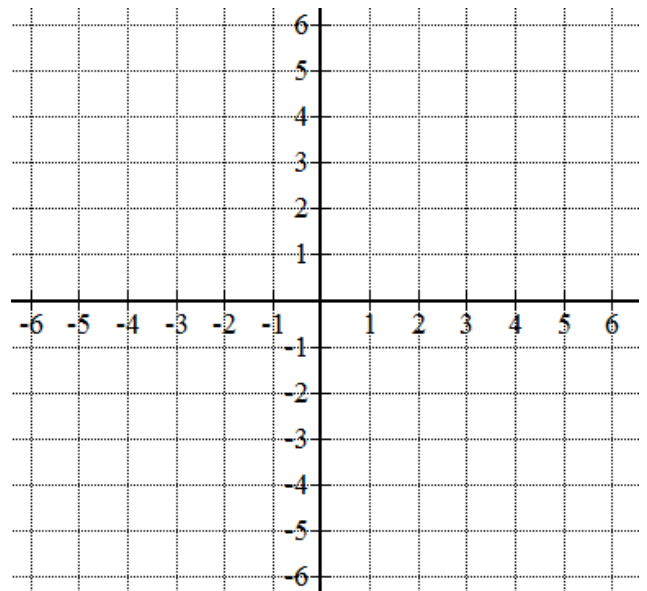
$$y - x < 2 \quad \text{and} \quad x + y \leq 0$$



EXAMPLE #4:

Graph the system of linear inequalities. Choose two possible solutions from the set. Assume $x \in R, y \in R$.

$$2x + 10 > 3y \quad \text{and} \quad 3x - y - 4 \leq 0 \quad \text{and} \quad x \geq 0$$



6.2/3 Solving Situational Problems of Systems of Linear Inequalities – Day2 (Concept #3)

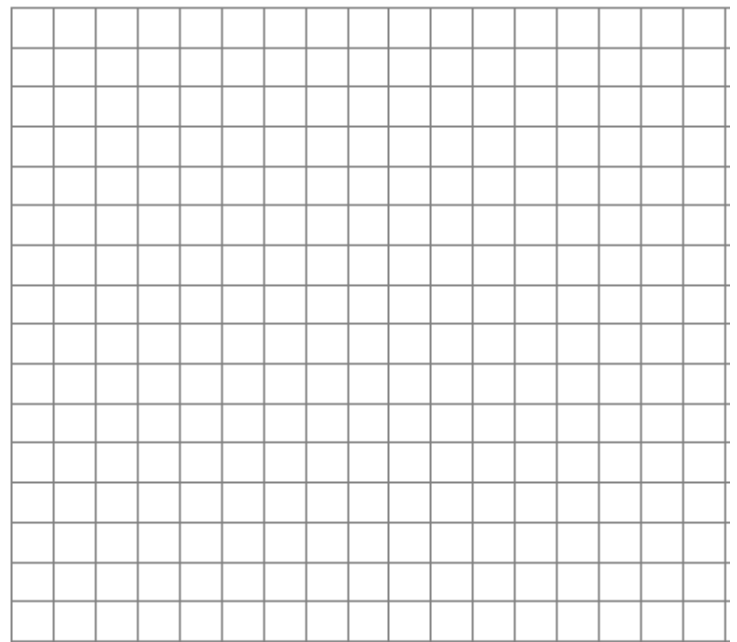
Example #1 A cupcake requires 35 grams of sugar and 50 grams of flour, and a muffin requires 30 grams of sugar and 65 grams of flour. Emily needs to use at least 460 grams of sugar to make cupcakes and muffins, and she wants to use at most 970 grams of flour. Use a graph to display all possible combinations of cupcakes and muffins to meet the inequalities.

Step 1: Define your variables let C =
let M =

Step 2: State the domain and range and any restrictions.

Step 3: Write a system of inequalities. (Also known as constraints)

Step 4: Graph the inequalities.
Be sure it is fully labelled.



Step 5: Find two coordinates that satisfy both inequalities.

Questions:

a) Verify one point and explain what the point means within the context of this situation.

b) What is the minimum amount of cupcakes and muffins that she can bake that satisfy both inequalities?

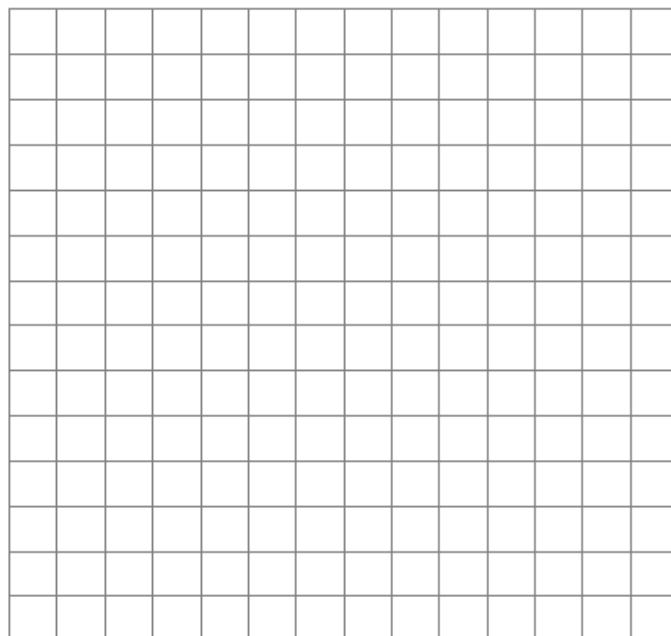
Example #2 A parkade can fit at most 100 cars and trucks on its lot. A car covers 100 ft^2 and a truck covers 200 ft^2 . The lot has $12,000 \text{ ft}^2$ of space. Use a graph to display all possible combinations of trucks and cars that meet the constraints.

Step 1: Define the variables

Step 2: State the domain and range and any restrictions

Step 3: Write the inequalities (Note: in the future these will be called the constraint inequalities)

Step 4: Graph the system of inequalities within the restrictions. Fully label the graph!



Step 5: Determine two possible combinations of trucks and cars possible on the lot.

Question:

a) What do you think the points of intersection on the graph mean?

Assignment Pg 318 #6,8,12 Pg 323 #7

NOTE: The question below will be considered fully answered only by following all of the five steps in the examples in today's lesson.

1. Your company makes Ipods and MP3 players. Each one must be processed by 2 machines. An Ipod takes 1 hour at the moulding station and 1 hour at the wiring station. An MP3 player it takes 2 hours at the moulding station and 1 hour at the wiring station. The moulding station is available for 16 hours and the wiring for 10 hours. What combinations of each music item can be made to meet the constraints?

6.4 Creating the model for Optimization Problems (Concept #4)

KEY NEW IDEAS FOR THIS LESSON

You will be able to develop algebraic and graphical reasoning by solving optimization problems using linear programming

OPTIMIZATION PROBLEM: A problem where a quantity must be maximized or minimized following a set of guidelines or conditions.

CONSTRAINT: A limiting condition of the optimization problem being modelled, represented by a linear inequality.

OBJECTIVE FUNCTION: In an optimization problem, the equation that represents the relationship between the two variables in the system of linear inequalities and the quantity to be optimized

FEASIBLE REGION: The solution region for a system of linear inequalities that is modelling an optimization problem.

OPTIMAL SOLUTION: A point in the solution set that represents the maximum or minimum value of the objective function.

LINEAR PROGRAMMING: A mathematical technique used to determine which solutions in the feasible region result in the optimal solutions of the objective function.

Steps to Solving an Optimization Problem

- The solution to an optimization problem is usually found at one of the vertices of the feasible region.
- To determine the optimal solution to an optimization problem using linear programming, follow these steps:
 - Step 1.** Create an algebraic model that includes:
 - a defining statement of the variables used in your model
 - the restrictions on the variables
 - a system of linear inequalities that describes the constraints
 - an objective function that shows how the variables are related to the quantity to be optimized
 - Step 2.** Graph the system of inequalities to determine the coordinates of the vertices of its feasible region.
 - Step 3.** Evaluate the objective function by substituting the values of the coordinates of each vertex.
 - Step 4.** Compare the results and choose the desired solution.
 - Step 5.** Verify that the solution(s) satisfies the constraints of the problem situation.

Note: In optimization problems, any restrictions on the variables are considered constraints. For example, if you are working with positive real numbers, $x \geq 0$ and $y \geq 0$ are constraints and should be included in the system of linear inequalities

Example #1

Three teams are travelling to a basketball tournament in cars and minivans.

- * Each team has no more than 2 coaches and 14 athletes
- * Each car can take 4 team members, and each minivan can take 6 team members.
- * No more than 4 minivans and 12 cars are available.

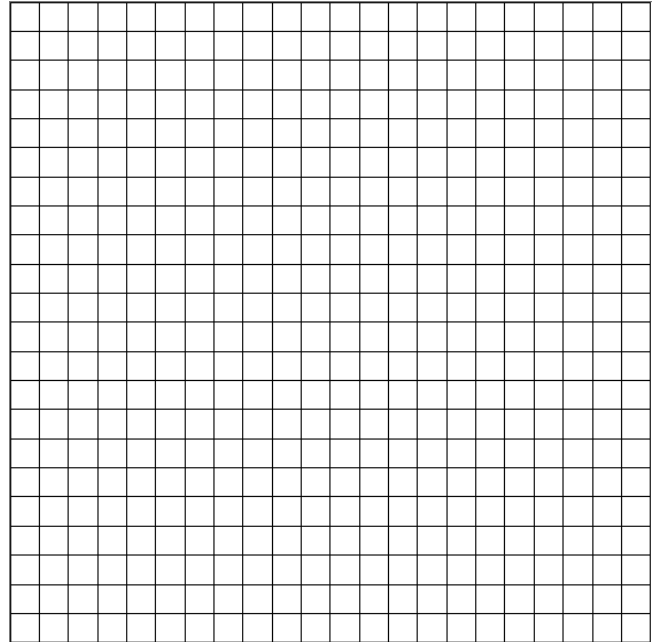
The school wants to know the combination of cars and minivans that will require the minimum and maximum number of vehicles. Create a model to represent this situation.

STEP 1:

Define Variables:

Restrictions:

Create a system of linear inequalities that describes the constraints:



Step 2: Graph the above Constraint Inequalities

within the restrictions:

Step 3: State the Objective Function to

be Maximized/Minimized:

Step 3/4: Evaluate the objective Function with the coordinates of the vertices to determine the min/max value(s).

Step 5: CHECK. Make sure the coordinates meet all the constraints

Example #2:

A refinery produces oil and gas.

- * At least 2 L of gasoline is produced for each litre of heating oil
- * The refinery can produce up to 9 million litres of heating oil and 6 million litres of gasoline each day.
- * Gasoline is projected to sell for \$1.10 per litre.
- * Heating oil is projected to sell for \$1.75 per litre.

The company needs to determine the daily combination of gas and heating oil that must be produced to maximize revenue. Create a model to represent this situation.

Step1- Define Variables:

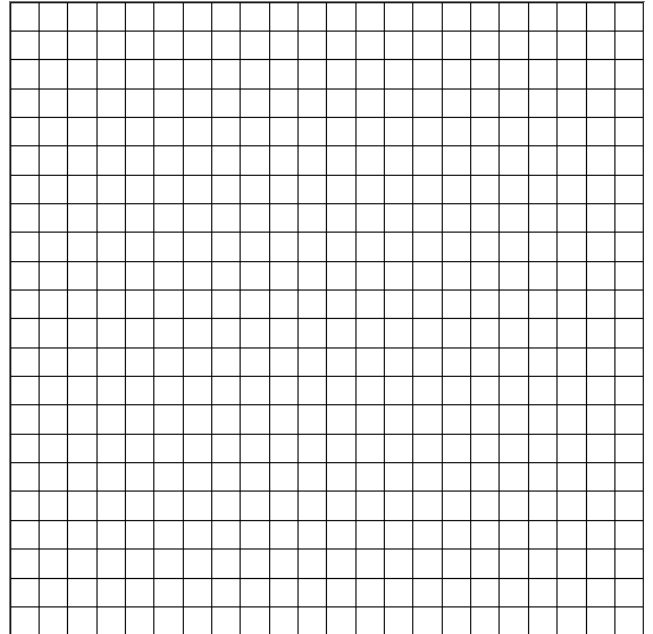
Restrictions:

Constraint Inequalities:

Step 2- Graph:

Step 3- State the Objective Function to be

Maximized/Minimized:



Step 3/4: Evaluate the objective Function with the coordinates of the vertices to determine the min/max value(s).

Step 5: CHECK. Make sure the coordinates meet all the constraints

KEY IDEAS

- * To solve an optimization problem, you need to determine which combination of values of two variables results in a maximum or minimum value of a related quantity.
 - * When creating a model, the first step is to represent the situation algebraically. An algebraic model includes these parts:
 - a defining statement of the variables used in your model
 - a statement describing the restrictions on the variables
 - a system of linear inequalities that describes the constraints
 - an objective function that shows how the variables are related to the quantity to be optimized
 - * The second step is to represent the system of linear inequalities graphically
 - * In optimization problems, any restrictions on the variables are considered constraints. For example, if you are working with positive real numbers, $x \geq 0$ and $y \geq 0$ are constraints and should be included in the system of linear inequalities.
-

6.4 Assignment

Note: Please do each question on one side of a page. Please make sure that there are at least 10 lines that are blank on the bottom of each page as we will be doing a bit more work on these questions tomorrow!

TEXTBOOK P 330 # 1-7

6.5/6.6 Optimization Problems and Exploring Solutions (Concept #4)

WHAT IS OPTIMIZATION?

Optimization in real life is a method of making the best of anything. In mathematics it is a mathematical technique for finding a maximum or minimum value of a function of several variables subject to a set of constraints. We can also describe it as is the selection of a best element (with regard to some criterion) from some set of available alternatives. Often this is used to find the minimum cost to produce an item or the maximum profit that can be obtained. The method we use to find the maximum or minimum is called **Linear Programming**.

Example #1 Find the maximum and minimum to each objective function below.:

a) Model A

Restrictions:

$$x \in \mathbb{R}, y \in \mathbb{R}$$

Constraints:

$$x > -4$$

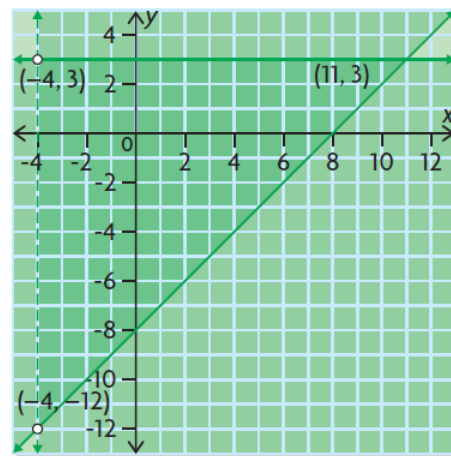
$$x - y \leq 8$$

$$y \leq 3$$

Objective function:

$$T = 2x + 5y$$

Graph of Model A:



b) Model B

Restrictions:

$$x \in \mathbb{W}, y \in \mathbb{W}$$

Constraints:

$$x \geq 0$$

$$y \geq 0$$

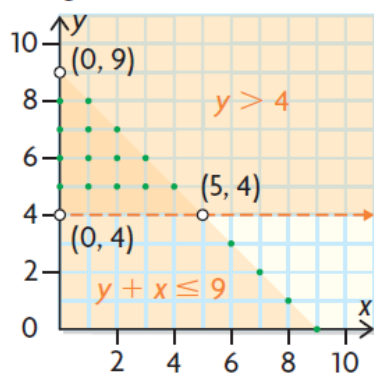
$$y > 4$$

$$y + x \leq 9$$

Objective function:

$$N = 3x - 2y$$

Graph of Model B:



Example #2 L&G Construction is competing for a contract to build a fence.

- The fence will be no longer than 50 yd and will consist of narrow boards that are 6 in. wide and wide boards that are 8 in. wide.
- There must be no fewer than 100 wide boards and no more than 80 narrow boards.
- The narrow boards cost \$3.56 each, and the wide boards cost \$4.36 each.

Determine the maximum and minimum costs for the lumber to build the fence.

Note: In questions like these, you won't always be asked for a series of steps. In order to completely answer the question you will be expected to know to do the following:

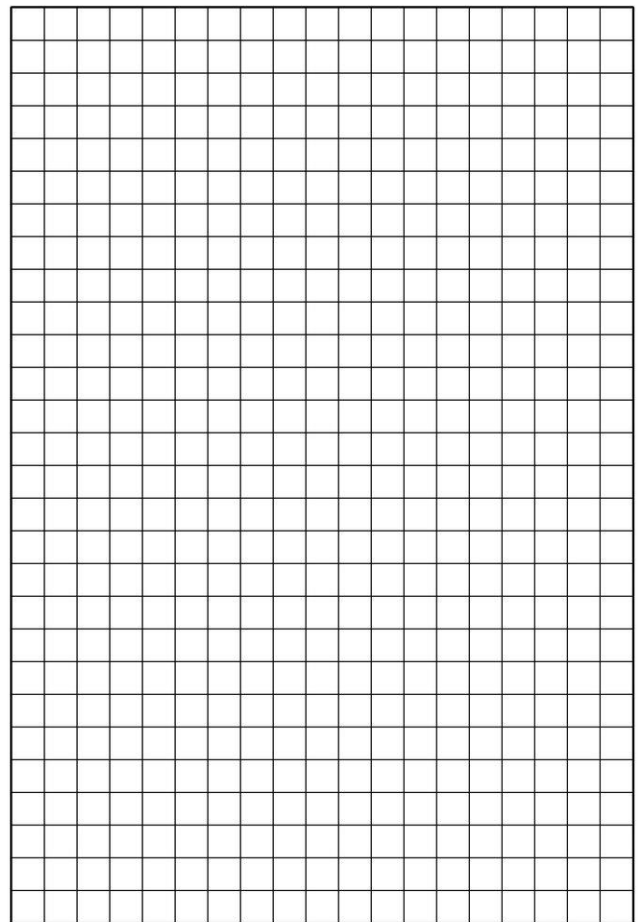
Step 1 - Define the variables and the restrictions. Write a system of linear inequalities (constraint inequalities) to model the situation.

Step 2 - Sketch the solution and label your graph. Decide if the boundaries are part of the solution

Step 3 - Find the optimal equation (An equation that requires finding a maximum or minimum – often a PROFIT equation or a COST equation). Find the vertices of the feasible region

Step 4 - Substitute all ordered pairs from the vertices into the optimal equation, the largest answer is your MAXIMUM and the smallest answer is your MINIMUM.

Step 5- Check you max or min with all constraints



www.GreatMinds.com

6.5/6.6 Assignment

Finish Step 4 and 5 on 6.4 assignment Pg 330 #1-7

Pg 345 Pick Two Questions from #11,12, 13, 14 or 15