Outcome FM 20.8

Enriched.

6.1 Graphing Linear Inequalities in Two Variables (Concept #1)

Determining solutions of inequalities

Example#1: For which inequalities is (3, 1) a possible solution?

a) $13 - 3x > 4y$

$13 - 3(3) > 4(1)$

$13 - 9 > 4$

$4 > 4$ x Not true

$\therefore$ (3,1) is not a solution

b) $2y - 5 \leq x$

$2(1) - 5 \leq 3$

$2 - 5 \leq 3$

$-3 \leq 3$ ✓ True

$\therefore$ (3,1) is a solution

c) $y \geq 9$

$1 \geq 9$ x False

$\therefore$ (3,1) is not a solution

Graphing Linear Inequalities in two variable

Methods of Graphing:

- table of values
- find x-intercept and y-intercept
- $y = mx + b$ (m = slope, b = y-intercept)

Steps to Graphing Inequalities

1. Initially, graph the boundary line. (ex. y= mx+b)
2. If the inequality is < or > use a ***dotted line*** (the points on the line are NOT included in the solution)
   If the inequality is ≤ or ≥ use a ***solid line*** (the points on the line ARE included in the solution)

When given a domain and range, the solution set is considered :

Continuous – (Real Numbers) (ex. x Є R, y Є R)
Discrete – separate or distinct set of number (Integers, Whole Numbers)(eX. x Є W, y Є W or x Є I, y Є I)
If no domain and range are given, assume the set of Real Numbers.

3. Choose a check point (if possible, choose the origin) and substitute into the original equation. Shade on the appropriate side of the line. Do NOT pick a point that lies on the line.

Example#2: Graph $-2x + 5y \geq 10$

$+2x \quad +2x$

Boundary Line

$\frac{5}{5}y \geq \frac{2x}{5} + \frac{10}{5}$

solid line → $y \geq \frac{2}{5}x + 2$

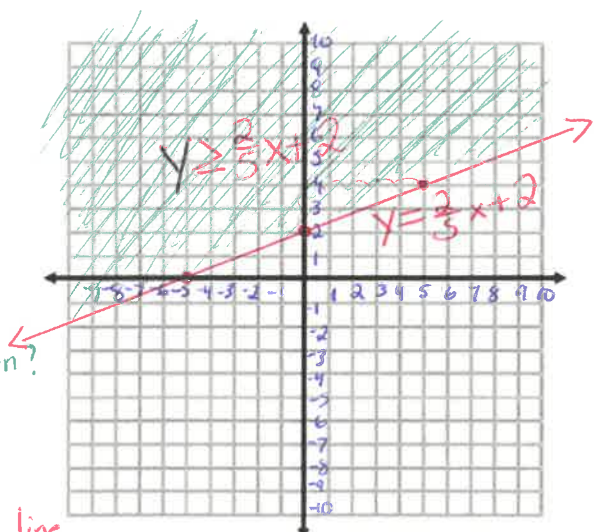
Equation of the boundary line → $y = \frac{2}{5}x + 2$

Test Point: Is (0,0) in the solution region?

$-2(0) + 5(0) \geq 10$

$0 \geq 10$ False

$\therefore$ Shade the region above the boundary line where (0,0) is not included



Question: Is (5,4) a part of the solution set? Is (3,-4) ?

Yes   No

How would the above graph look if the domain and range changed to integers?

Example #3: Graph

a) $\{(x, y) \mid x - 3 > 0; x \in R, y \in R\}$

dashed boundary line

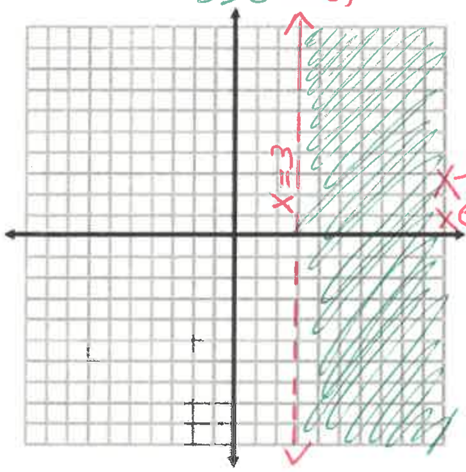
$x - 3 = 0$

$x = 3$ $\quad x > 3$

Test point $(0,0)$

$0 - 3 > 0$

$-3 > 0$ False, ∴ Shade other side



b) $(x, y) \mid x - 2y - 3 < 7 - 2x, x \in I, y \in I\}$

dashed line

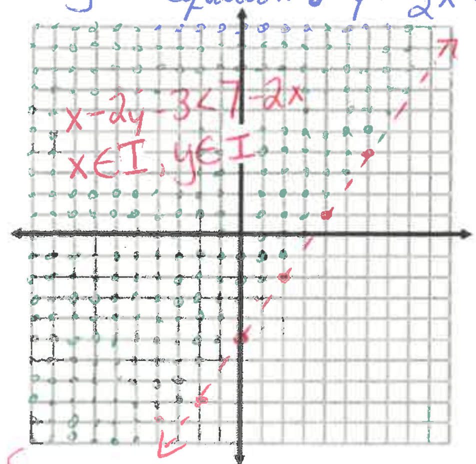
$x - 2y - 3 < 7 - 2x$

$\frac{-2y}{-2} < \frac{10}{-2} \frac{-3x}{-2}$

$y > \frac{3}{2}x - 5$

★ Flip the inequality when you multiply or divide by a negative

boundary line equation: $y = \frac{3}{2}x - 5$



Test Point $(0,0)$

$0 > \frac{3}{2}(0) - 5$

$0 > -5$

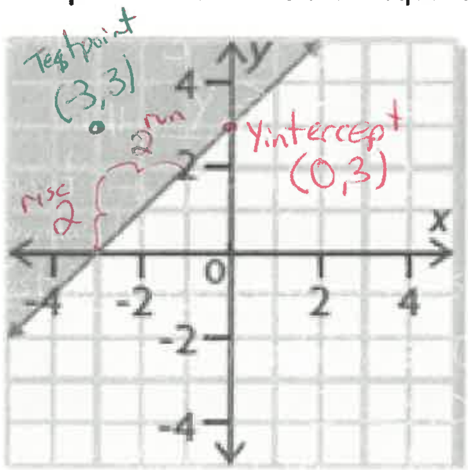
True

The domain and range only include the integers

In graph b is ($\frac{1}{2}$, 0) in the solution set? No, because x is a rational #

Remember: When you divide or multiply both sides of an inequality by a negative number, you must Flip **the inequality sign!!!**

Example#4: Determine the inequality of this graph



$y = x + 3$ boundary line equation

$y \geq x + 3$

check

Pick a point in the shaded region and test it in the inequality.

$(-3,3)$

$y \geq x + 3$

$3 \geq -3 + 3$

$3 \geq 0$ True

∴ $y \geq x + 3$ is the inequality.

EXAMPLE #5: Kolton and Carolyn want to donate some money to a local food pantry. To raise funds, they are selling PI necklaces and earrings that they have made themselves. Necklaces cost \$8 and earrings cost \$5. What is the range of possible sales they could make in order to donate at least \$100?

a) Assign your variables: Let x = # of necklaces
Let y = # of earrings

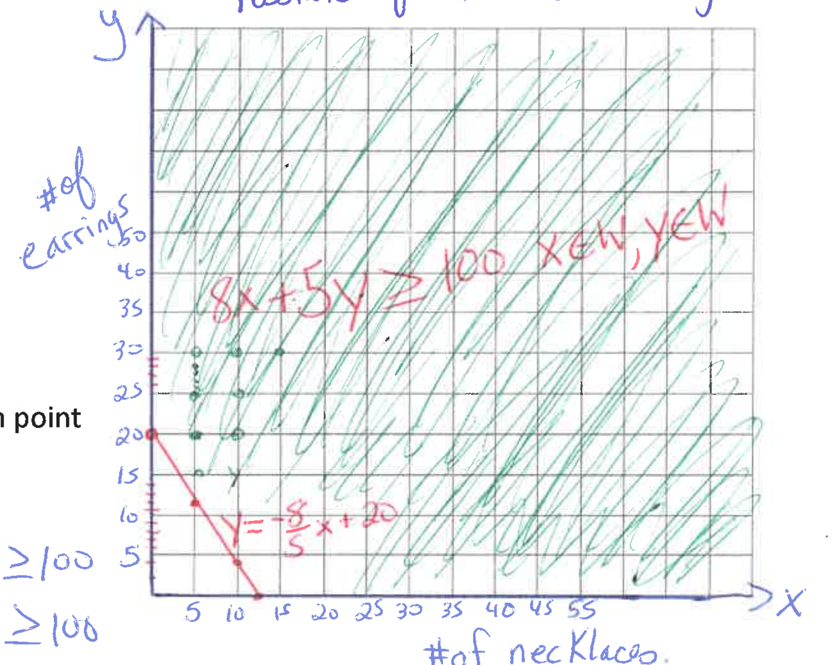
b) Establish your inequality: $8x + 5y \geq 100$

c) Decide what type of restrictions will be on the domain and range and decide if your graph would include all Real Numbers, Integers or Whole Numbers. Possible # of necklaces and earrings to sell

$x \in W \quad y \in W$

d) Sketch a graph of this situation.

$$\frac{5y}{5} \geq \frac{-8x}{5} + \frac{100}{5}$$
$$y \geq -\frac{8}{5}x + 20$$



e) Find two points that satisfy this situation.

(15,15) (20,25)

f) Verify both your points and explain what each point means within the context of this situation.

(15,15)
$8(15)+5(15) \geq 100$
$120 + 75 \geq 100$
$195 \geq 100$
True

(20,25)
$8(20)+5(25) \geq 100$
$160 + 125 \geq 100$
$285 \geq 100$
True

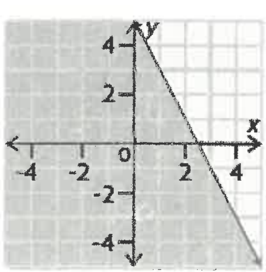
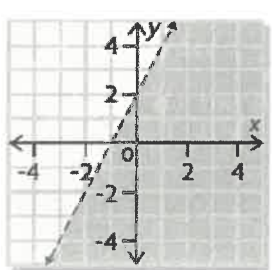
ASSIGNMENT: TEXTBOOK p303 #3, 4, 5, 6, AT LEAST TWO OF 7-12 & 14

PLUS THE FOLLOWING

1. For $y < 3x + 5$ which of the following points fall within the solution set?

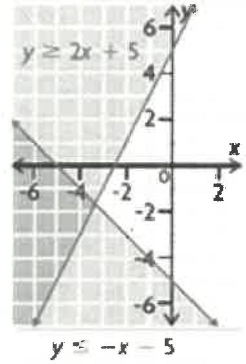
$(-1,-3), (-1,2), (-4,3), (-2,-3), (3,1), (1,5), (0,5), (-1,3)$

2 **Determine the inequality for the following graphs**

6.2/3 Solving Systems of Linear Inequalities- Day1 (Concept #2)

System of Linear Inequalities: A set of two or more linear inequalities that are graphed on the same coordinate plane; the intersection of their solution regions represents the solution set for the system. Example →



EXAMPLE #1:

Graph the system of linear inequalities. Choose two possible solutions from the set. Assume $x \in R$, $y \in R$.

$2x + 3y \leq 9$ and $y - 6x \geq 1$

$-2x$ $-2x$ $+6x$ $+6x$

$\frac{3y}{3} \leq \frac{-2x}{3} + \frac{9}{3}$

$y \leq -\frac{2}{3}x + 3$

slope y-int.

→ boundary line is solid because ≤

→ shade below the line

Boundary Line Equation ⇒ $y = -\frac{2}{3}x + 3$

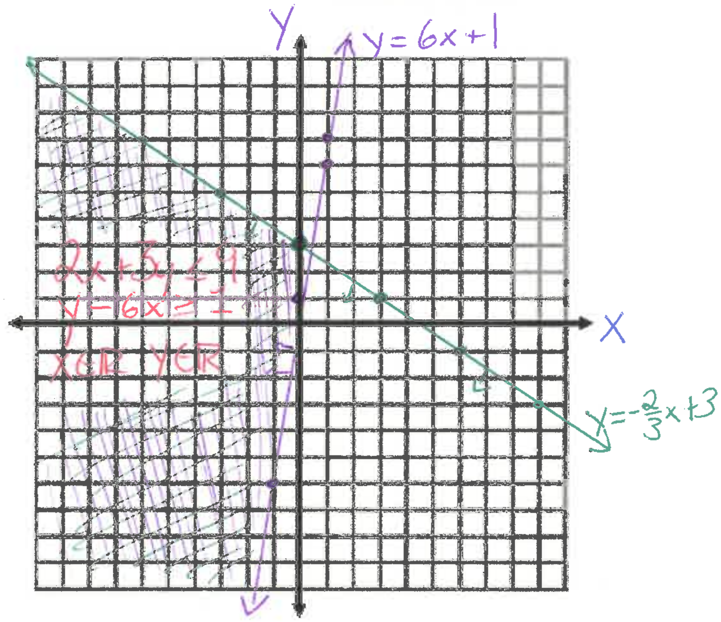
$y \geq 6x + 1$

slope y-int

→ boundary line is solid

→ shade above the line

Boundary line Equation ⇒ $y = 6x + 1$



EXAMPLE #2:

Graph the system of linear inequalities. Choose two possible solutions from the set. Assume $x \in I$, $y \in I$.

$-3x - 2y < 6$ and $y \leq 3$

$+3x$ $+3x$

$\frac{-2y}{-2} < \frac{3x}{-2} + \frac{6}{-2}$

☆Flip the inequality☆

$y > -\frac{3}{2}x - 6$

slope y-int.

→ Boundary line is dashed since >

→ Shade above the line

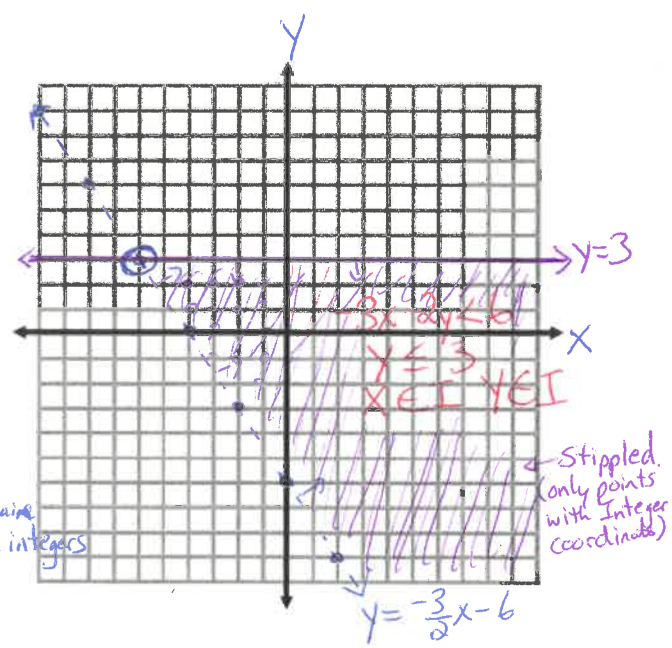
Boundary Line Equation ⇒ $y = -\frac{3}{2}x - 6$

$y = 3$ Horizontal line

→ Boundary line is solid

→ Shade Below the line

Note: Since the domain and range only include integers



Does the intersection point of the system have an open dot or a closed dot? Explain

It will have an open dot because the solutions must satisfy both inequalities and $-3x - 2y < 6$ does not include the points on the line, therefore the intersection point is not included in the solution region.

EXAMPLE #3:

The domain and range are within the set of whole numbers which can only be found in Quadrant I.

Graph the system of linear inequalities. Choose two possible solutions from the set. Assume $x \in W$, $y \in W$.

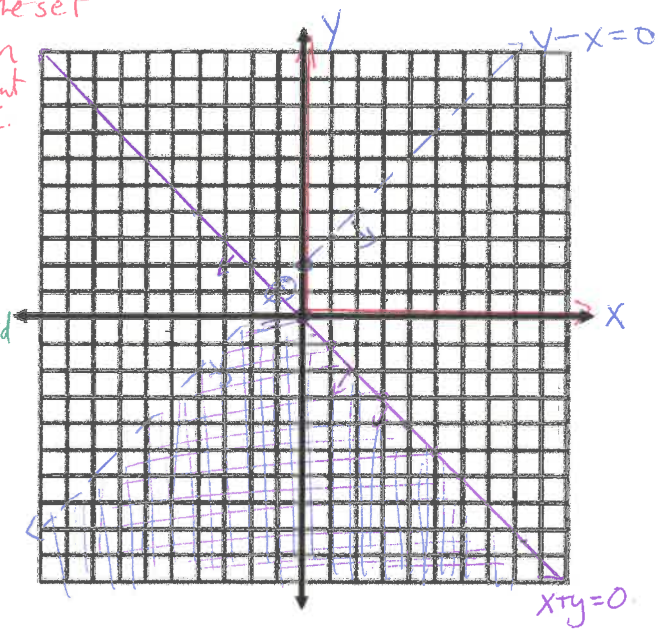
$y - x < 2$ and $x + y \le 0$

$y < x + 2$

- boundary line is dashed
- shade below the line

$y \le -x$

- boundary line is solid
- shade below



There is only one solution (0,0)

EXAMPLE #4:

Graph the system of linear inequalities. Choose two possible solutions from the set. Assume $x \in R$, $y \in R$.

$2x + 10 > 3y$ and $3x - y - 4 \le 0$ and $x \ge 0$

$10 > 3y - 2x$

$-3y + 10 > -2x$

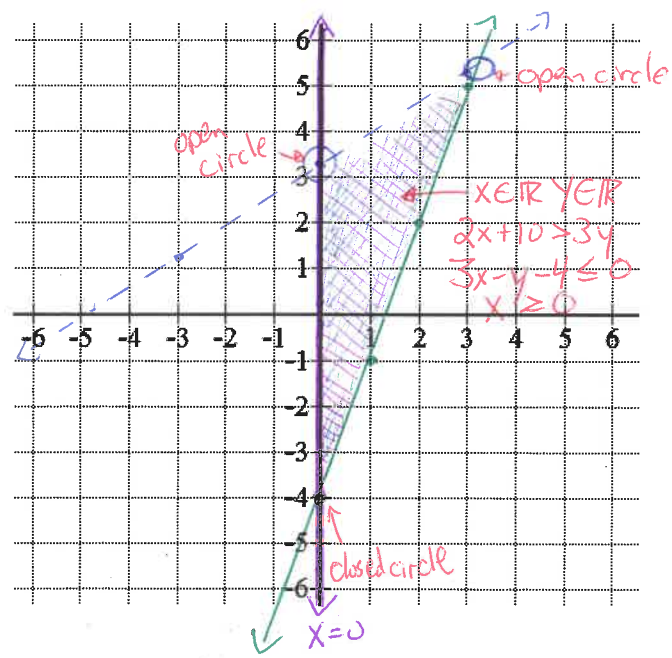
$-3y > -2x - 10$

$y < \frac{2}{3}x + \frac{10}{3}$

$-y \le -3x + 4$

$y \ge 3x - 4$

$x \ge 0$



Two possible solutions are $(1,1)$, $(\frac{1}{2}, \frac{3}{2})$

Assignment : pg 307 #1 Pg 317 #3(May not need to graph **) ,4,5**

6.2/3 Solving Situational Problems of Systems of Linear Inequalities – Day2 (Concept #3)

Example #1 A cupcake requires 35 grams of sugar and 50 grams of flour, and a muffin requires 30 grams of sugar and 65 grams of flour. Emily needs to use at least 460 grams of sugar to make cupcakes and muffins, and she wants to use at most 970 grams of flour. Use a graph to display all possible combinations of cupcakes and muffins to meet the inequalities.

Step 1: Define your variables let C = number of cupcakes
let M = number of muffins

Step 2: State the domain and range and any restrictions.

$X \in W,\ Y \in W$

Step 3: Write a system of inequalities. (Also known as constraints)

$35c + 30m \geq 460$ ← inequality represents grams of sugar

$50c + 65m \leq 970$ ← inequality representing grams of flour

Step 4: Graph the inequalities.
Be sure it is fully labelled.

① $35c + 30m \geq 460$

$30m \geq -35c + 460$

$\frac{30m}{30} \geq \frac{-35c}{30} + \frac{460}{30}$

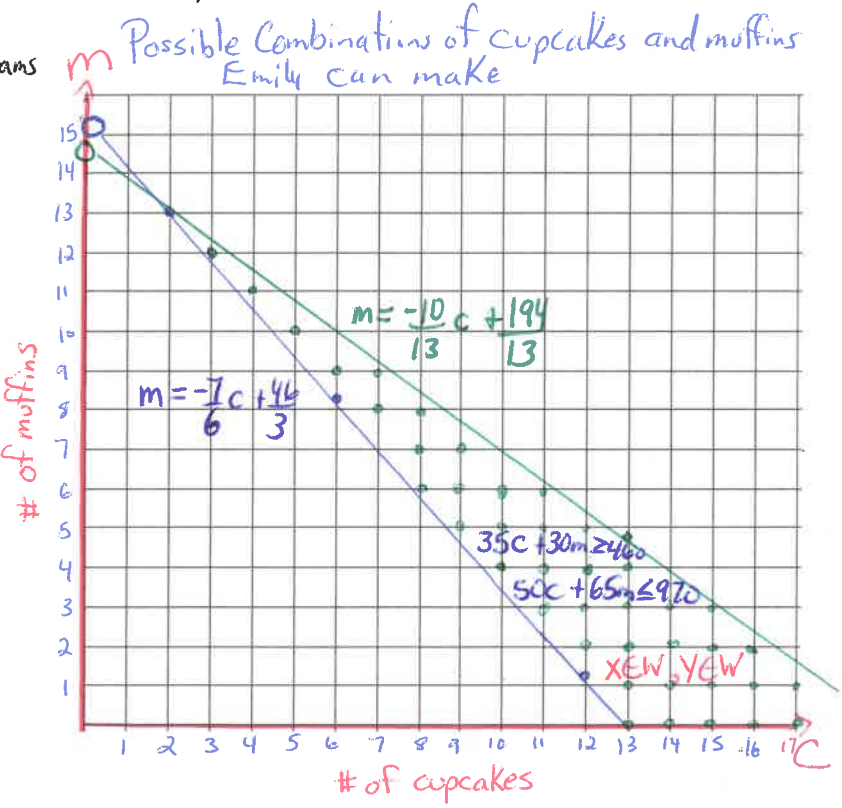
$m \geq -\frac{7}{6}c + \frac{46}{3}$ ← y-int ≈ 15.3

② $50c + 65m \leq 970$

$65m \leq -50c + 970$

$\frac{65m}{65} \leq \frac{-50c}{65} + \frac{970}{65}$

$m \leq -\frac{10}{13}c + \frac{194}{13}$ ← y-int ≈ 14.9



Step 5: Find two coordinates that satisfy both inequalities.

(12 cupcakes, 4 muffins) (9 cupcakes, 7 muffins)

Questions:

a) Verify one point and explain what the point means within the context of this situation.

$35(12) + 30(4) \geq 460$
$420 + 120 \geq 460$
$540 \geq 460$ ✓ True

$50(12) + 65(4) \leq 970$
$600 + 260 \leq 970$
$860 \leq 970$ ✓ True

This means that if Emily makes 12 cupcakes and 4 muffins she will use 825g of flour and 540g of sugar.

~~b) What is the minimum amount of cupcakes and muffins that she can bake that satisfy both inequalities?~~

~~cupcakes + muffins~~ Not a solution.

Example #2 A parkade can fit at most 100 cars and trucks on its lot. A car covers 100 ft^2 and a truck covers 200 ft^2. The lot has 12,000 ft^2 of space. Use a graph to display all possible combinations of trucks and cars that meet the constraints.

Step 1: Define the variables

Let x = # of cars

Let y = # of trucks

Step 2: State the domain and range and any restrictions

$x \in W$, $y \in W$

Step 3: Write the inequalities (Note: in the future these will be called the constraint inequalities)

$x + y \le 100$

$100x + 200y \le 12\,000$

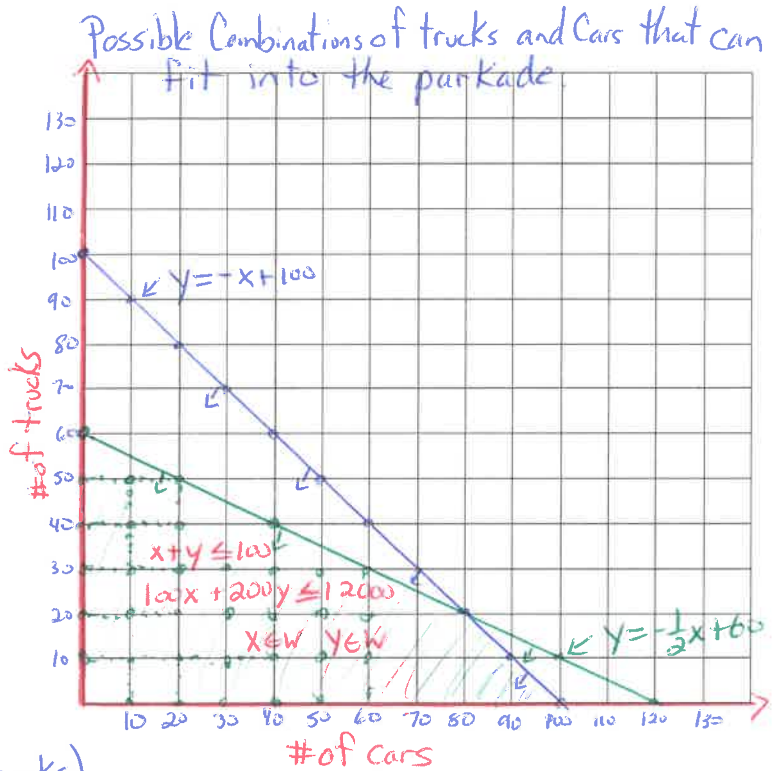
Step 4: Graph the system of inequalities within the restrictions.

Fully label the graph!

$y \le -x + 100$ (slope, y-int)

$\frac{200y}{200} \le \frac{-100x}{200} + \frac{12000}{200}$

$y \le -\frac{1}{2}x + 60$ (slope, y-int)



Step 5: Determine two possible combinations of trucks and cars possible on the lot.

(25 cars, 10 trucks) (50 cars, 30 trucks)

A possible combination of vehicles the parkade could fit would be 25 cars and 10 trucks or 50 cars and 30 trucks

Question:

a) What do you think the points of intersection on the graph mean?

The amount of cars to maximize the area and/or the total # of vehicles allowed, which is 100 cars + trucks.

Assignment Pg 318 #6,8,12 Pg 323 #7

NOTE: The question below will be considered fully answered only by following all of the five steps in the examples in today's lesson.

1. Your company makes Ipods and MP3 players. Each one must be processed by 2 machines. An Ipod takes 1 hour at the moulding station and 1 hour at the wiring station. An MP3 player it takes 2 hours at the moulding station and 1 hour at the wiring station. The moulding station is available for 16 hours and the wiring for 10 hours. What combinations of each music item can be made to meet the constraints?

6.2/6.3 Solving Situational Problems of Systems of Linear Inequalities (Concept #3)

#1) Let $x=$ # of ipods
Let $y=$ # of MP3 players

Inequality to represent # of hours at the wiring station → $x+y \le 10$; $x+2y \le 16$ ← inequality to represent # of hours at the moulding station

$x \in W,\ y \in W.$

or

$\{(x,y) \mid x+y \le 10,\ x \in W,\ y \in W.\}$

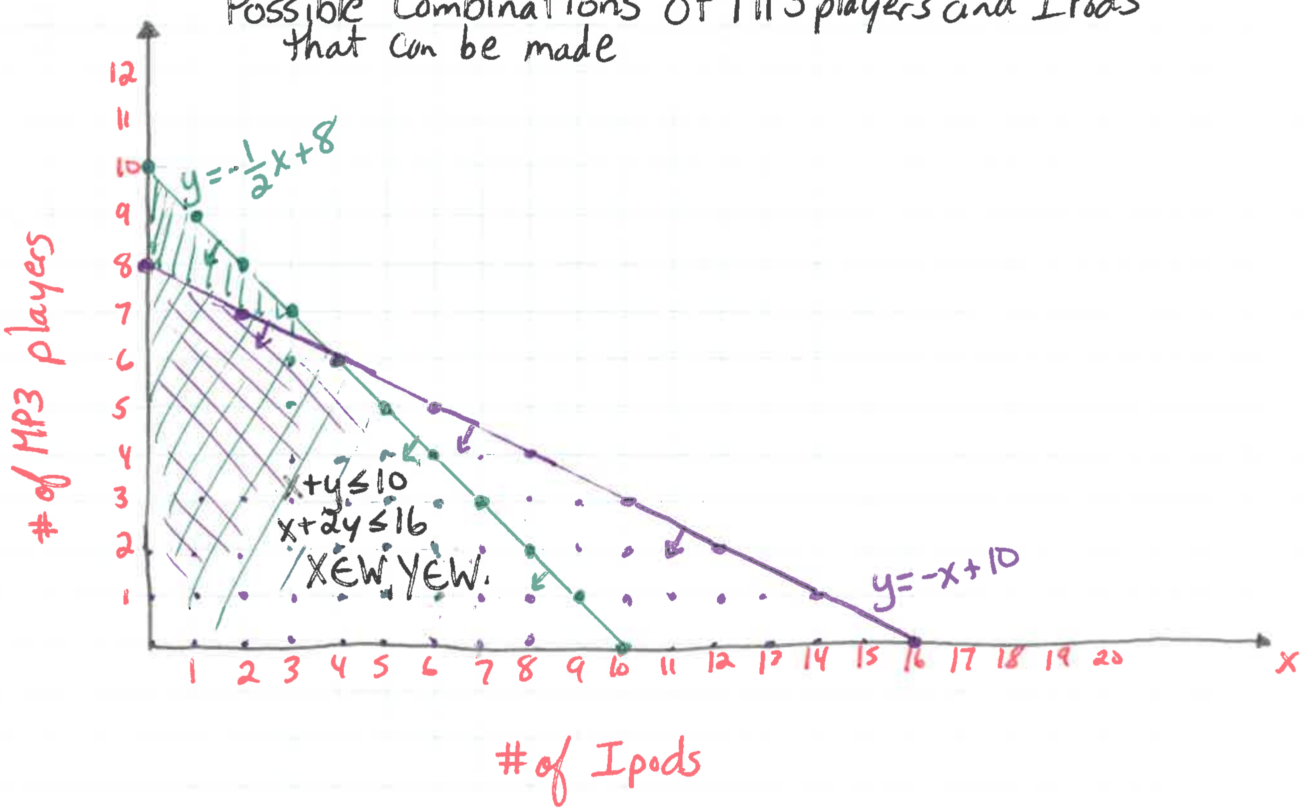
$\{(x,y) \mid x+2y \le 16,\ x \in W,\ y \in W\}$

Graph boundary lines

$y \le -x+10$

$\frac{2y}{2} \le -\frac{x}{2} + \frac{16}{2}$

$y \le -\frac{1}{2}x + 8$



Two possible combinations would be 4 ipods and 1 MP3 player or 6 ipods and 3 MP3 players.

6.4 Creating the model for Optimization Problems (Concept #4)

KEY NEW IDEAS FOR THIS LESSON

You will be able to develop algebraic and graphical reasoning by solving optimization problems using linear programming

OPTIMIZATION PROBLEM: A problem where a quantity must be maximized or minimized following a set of guidelines or conditions.

CONSTRAINT: A limiting condition of the optimization problem being modelled, represented by a linear inequity.

OBJECTIVE FUNCTION: In an optimization problem, the equation that represents the relationship between the two variables in the system of linear inequalities and the quantity to be optimized

FEASIBLE REGION: The solution region for a system of linear inequalities that is modelling an optimization problem.

OPTIMAL SOLUTION: A point in the solution set that represents the maximum or minimum value of the objective function. If a vertex isn't included in the feasible region the optimal solution will be a point, within the feasible region that is close to the vertex

LINEAR PROGRAMMING: A mathematical technique used to determine which solutions in the feasible region result in the optimal solutions of the objective function.

Steps to Solving an Optimization Problem:

- The solution to an optimization problem is usually found at one of the vertices of the feasible region.
- To determine the optimal solution to an optimization problem using linear programming, follow these steps:

Step 1. Create an algebraic model that includes:
- a defining statement of the variables used in your model
- the restrictions on the variables
- a system of linear inequalities that describes the constraints
- an objective function that shows how the variables are related to the quantity to be optimized

Step 2. Graph the system of inequalities to determine the coordinates of the vertices of its feasible region.

Step 3. Evaluate the objective function by substituting the values of the coordinates of each vertex.

Step 4. Compare the results and choose the desired solution.

Step 5. Verify that the solution(s) satisfies the constraints of the problem situation.

Note: In optimization problems, any restrictions on the variables are considered constraints. For example, if you are working with positive real numbers, $x \geq 0$ and $y \geq 0$ are constraints and should be included in the system of linear inequalities.

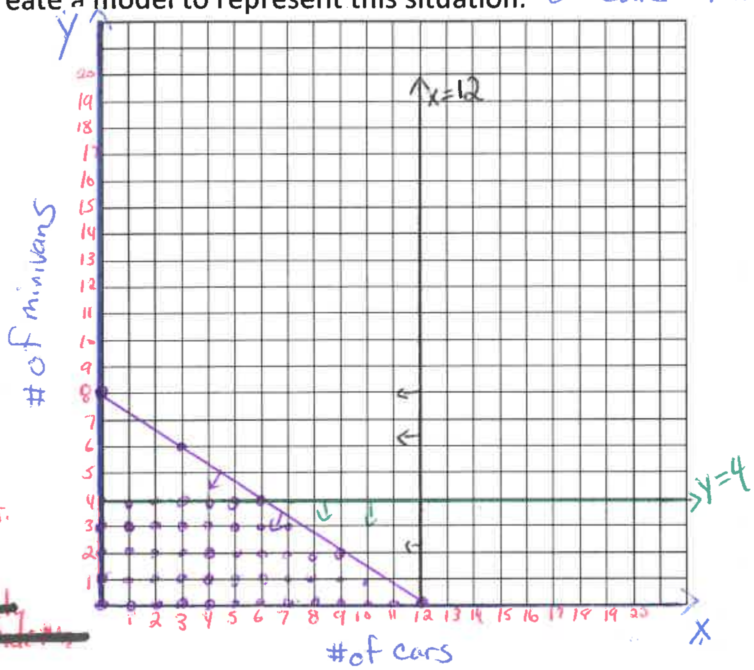
EXAMPLE #1:

Three teams are travelling to a basketball tournament in cars and minivans.

* Each team has no more than 2 coaches and 14 athletes = 16 team members x 3 = 48 people
* Each car can take 4 team members, and each minivan can take 6 team members.
* No more than 4 minivans and 12 cars are available.

The school wants to know the combination of cars and minivans that will require the minimum and maximum number of vehicles. Create a model to represent this situation.

Possible Combinations of Cars + Minivans



Step 1

1. Variable Statement:

Let x = # of cars

y = # of minivans

2. Domain, Range and Restrictions:

$x \in W$ $y \in W$

Since x + y are whole #'s our graph is restricted to Quadrant I.

3. Constraint Inequalities:

$4x + 6y \leq 48$ purple → $\frac{6y}{6} \leq \frac{-4x}{6} + \frac{48}{6}$

$y \leq -\frac{2}{3}x + 8$ (slope, y-int.)

$x \leq 12$ Black

$y \leq 4$ green

~~$x \geq 0$ $y \geq 0$~~

Step 2

4. Graph the above Constraint Inequalities within the restrictions:

5. Objective Function to be Maximized/Minimized:

Step 3

Let V = Total # of vehicles $x + y = V$

~~Leave Blank for now:~~ Identify 3 possible solutions. (Coordinates to the vertices)

Evaluate the objective function by substituting the values of the coordinates of each vertex

Step 4

Vertex 1 = (0 cars, 4 minivans) Vertex 2 = (6 cars, 4 minivans) Vertex 3 = (12 cars, 0 minivans)

$0 + 4 = \boxed{4 \text{ vehicles}}$ minimum $6 + 4 = 10$ vehicles $12 + 0 = \boxed{12 \text{ vehicles}}$ maximum

Step 5

Check

$4(0) + 6(4) \leq 48$

$24 \leq 48$ ✓ True

$0 \leq 12$ ✓ True

$4 \leq 4$ True

Check

$4(12) + 6(0) \leq 48$

$48 \leq 48$ ✓ True

$12 \leq 12$ ✓ True

$0 \leq 4$ ✓ True

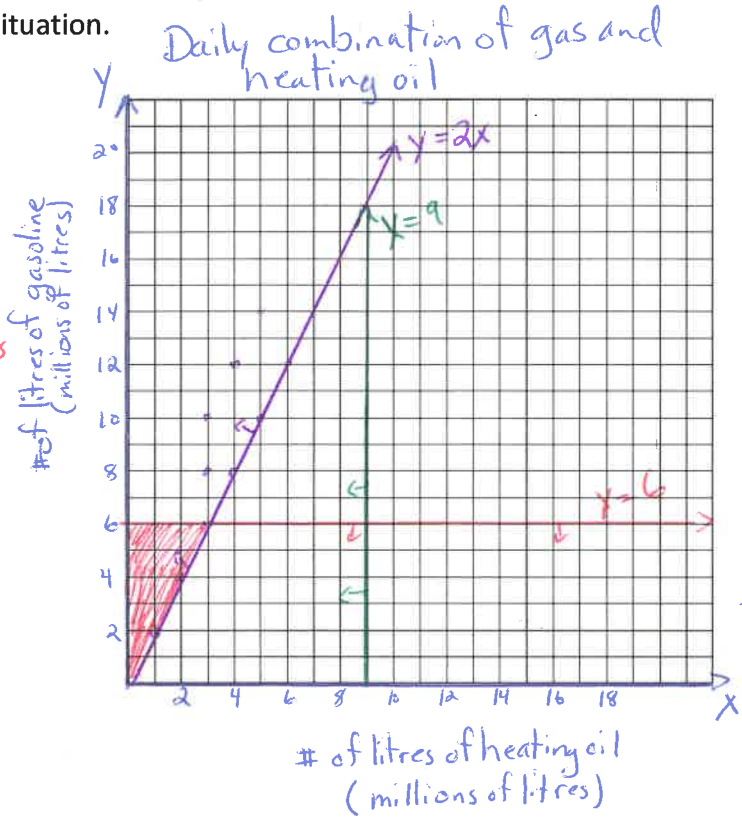
The minimum # of vehicles is 4 and the maximum is 12

EXAMPLE #2:

A refinery produces oil and gas.

* At least 2 L of gasoline is produced for each litre of heating oil
* The refinery can produce up to 9 million litres of heating oil and 6 million litres of gasoline each day.
* Gasoline is projected to sell for $1.10 per litre.
* Heating oil is projected to sell for $1.75 per litre.

The company needs to determine the daily combination of gas and heating oil that must be produced to maximize revenue. Create a model to represent this situation.



1. Variable Statement:

Let x = # of litres of heating oil
y = # of litres of gasoline

2. Domain and range and restrictions:

$x \in \mathbb{R}$ $y \in \mathbb{R}$, $x \geq 0$, $y \geq 0$ ← these restrictions become a part of your constraints

3. Constraint Inequalities:

① $x \geq 0$ ② $y \geq 0$

③ $2x \leq y$

④ $x \leq 9\,000\,000$

⑤ $y \leq 6\,000\,000$

4. Graph the above Constraint Inequalities within the restrictions:

5. Objective Function to be Maximized/Minimized:

Let R = Total ~~rev~~ daily revenue $1.75x + 1.10y = R$

Leave Blank for now: Identify 3 possible solutions. (Coordinates of the vertices in the feasible region)

vertex 1 = (0, 6) ← millions vertex2 = (3, 6) vertex3 = (0, 0)

Evaluate the objective function by substituting values of the coordinates of each vertex

$1.75x + 1.10y = R$

$1.75(0) + 1.10(6\,000\,000) = \boxed{6\,600\,000 \text{ Revenue}}$

$1.75(3\,000\,000) + 1.10(6\,000\,000) = \boxed{\$11\,85\,0000 \text{ Revenue}}$ Maximum

$1.75(0) + 1.10(0) = \boxed{\$0 \text{ Revenue}}$ Minimum

Check (make sure the maximum's coordinates meet all the constraints)

① $3 \geq 0$ ✓ True ② $6 \geq 0$ ✓ True ③ $2(3\,000\,000) \leq 6\,000\,000$, $6\,000\,000 \leq 6\,000\,000$ ✓ True ④ $3 \leq 9\,000\,000$ ✓ True ⑤ $6\,000\,000 \leq 6\,000\,000$ ✓ True.

The maximum revenue is $11850000 daily if 3000000 L of heating oil and 6 000 000 L of gasoline are produced.

ASSIGNMENT:

Note: Please do each question on one side of a page. Please make sure that there are at least 10 lines that are blank on the bottom of each page as we will be doing a bit more work on these questions tomorrow!

TEXTBOOK P 330 # 1-7

6.5/6.6 Optimization Problems and Exploring Solutions

WHAT IS OPTIMIZATION?

Optimization in real life is a method of making the best of anything. In mathematics it is a mathematical technique for finding a maximum or minimum value of a function of several variables subject to a set of constraints. We can also describe it as is the selection of a best element (with regard to some criterion) from some set of available alternatives. Often this is used to find the minimum cost to produce an item or the maximum profit that can be obtained. The method we use to find the maximum or minimum is called Linear Programming.

Example #1 Find the maximum and minimum to each objective function below.:

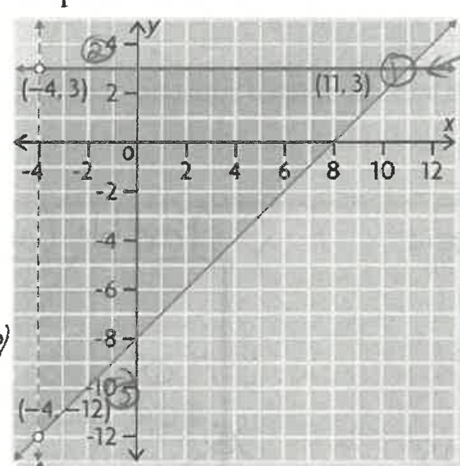
a) Model A

Restrictions:
$x \in R, y \in R$

Constraints:
$x > -4$
$x - y \leq 8$
$y \leq 3$

Objective function:
$T = 2x + 5y$

Graph of Model A:



Max. occurs at (11,3)

Min occurs near (-4,-12) but not equal to

① $2(11)+5(3)$
$22+15$
Max $= 37$

② $2(-4)+5(3)$
$-8+15$
$=7$

③ $2(-4)+5(-12)$
$-8+-60$
Min $=-68$

$12 \times 5 = 60$

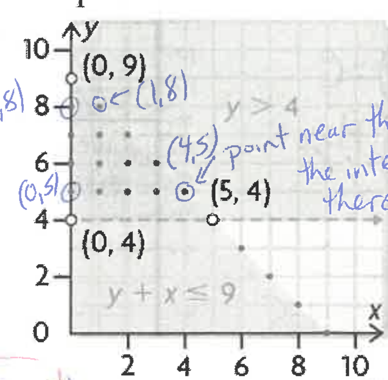
b) Model B

Restrictions:
$x \in W, y \in W$

Constraints:
$x \geq 0$
$y \geq 0$
$y > 4$
$y + x \leq 9$

Objective function:
$N = 3x - 2y$

Graph of Model B:



(4,5) point near the intersection point because the intersection point is an open circle therefore not included in the solution region.

Max
Vertex (4,5)
$N=3(4)-2(5)$
$N=12-10$
$N=2$

Vertex (0,5)
$N=3(0)-2(5)$
$N=-10$

Min
Vertex (0,8)
$N=3(0)-2(8)$
$N=-16$

Vertex (1,8)
$N=3(1)-2(8)$
$N=3-16$
$N=-13$

Example #2 L&G Construction is competing for a contract to build a fence.

- The fence will be no longer than 50 yd and will consist of narrow boards that are 6 in. wide and wide boards that are 8 in. wide.
- There must be no fewer than 100 wide boards and no more than 80 narrow boards.
- The narrow boards cost \$3.56 each, and the wide boards cost \$4.36 each.

Determine the maximum and minimum costs for the lumber to build the fence.

Note: In questions like these, you won't always be asked for a series of steps. In order to completely answer the question you will be expected to know to do the following:

- Define the variables and the restrictions
- Write a system of linear inequalities (constraint inequalities) to model the situation.
- Sketch the solution and label your graph.
- Decide if the boundaries are part of the solution
- Find the verticies of the feasible region
- Find the optimal equation (An equation that requires finding a maximum or minium – often a PROFIT equation or a COST equation)
- Substitute all ordered pairs from the verticies into the optimal equation, the largest answer is your MAXIMUM and the smallest answer is your MINIMUM.

Let x = # of narrow boards   50yds = 1800in
y = # of wide boards $x \in W$ $y \in W$.

$6x + 8y \leq 1800 \rightarrow \frac{8y}{8} \leq \frac{-6x}{8} + \frac{1800}{8}$

$y \geq 100$ $y \leq -\frac{3}{4}x + 225$

$x \leq 80$

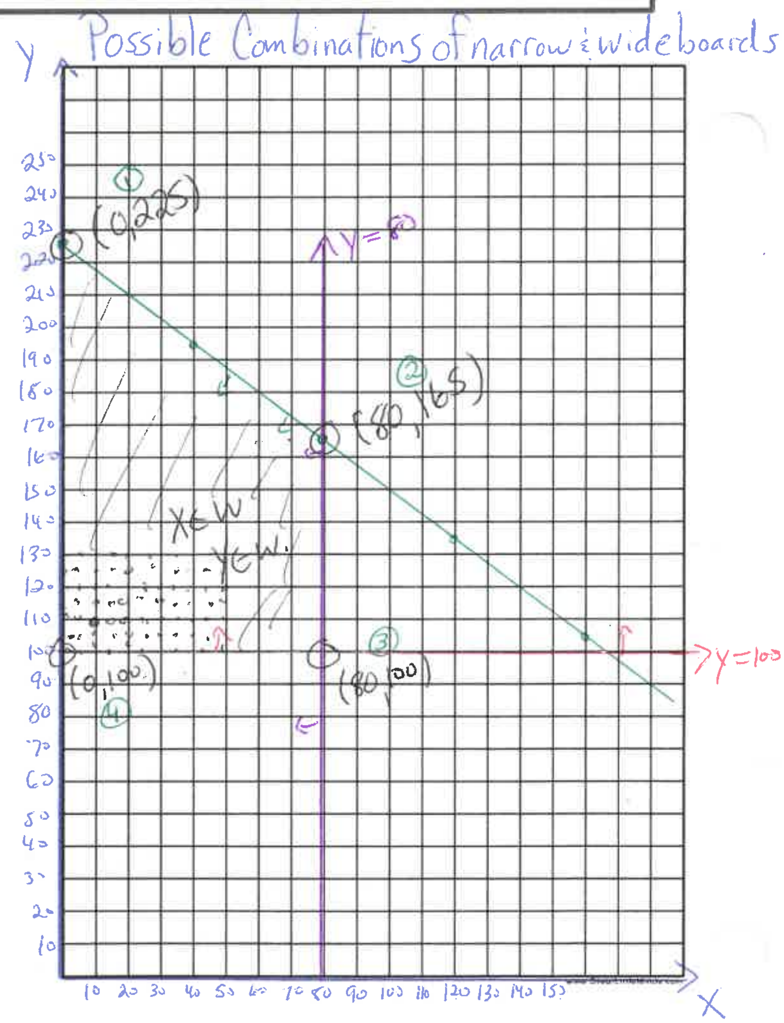
Objective Function: $3.56x + 4.36y = C$
Let C = Cost

Vertex① (0,225) Substitute into Objective Function.
$3.56(0) + 4.36(225) = C$
\$981 = C

vertex② (80,165)
$3.56(80) + 4.36(165) = C$
$\boxed{\$1004.20 = C}$ Max

Vertex③ (80,100)
$3.56(80) + 4.36(100) = C$
\$720.80 = C

vertex④ (0,100)
$3.56(0) + 4.36(100) = C$
Min $\boxed{\$436 = C}$



Assignment Pg 345 Pick Two Questions #11-15

The maximum cost of lumber to build the fence would be \$1004.20 and minimum cost of lumber would be \$436.00