Math Pre-Calc 20 Final Review (Solutions)

Chp1 Sequences and Series

#1. Write the first 4 terms of each sequence:

a) $t_1 = 3 \quad d = -2$

3, 1, -1, -3

b) $t_n = 3^n$

$3^1, 3^2, 3^3, 3^4$ OR 3, 9, 27, 81

#2. Find the value of the term indicated:

a) 1, 3, 9, … , t_7

$t_n = ar^{n-1}$

$t_7 = 1(3)^{7-1}$

$t_7 = 1(3^6)$

$t_7 = 729$

b) 17, 13, 9, … , t_{25}

$t_n = a + d(n-1)$

$t_{25} = 17 - 4(25-1)$

$t_{25} = 17 - 4(24)$

$t_{25} = 17 - 96$

$t_{25} = -79$

#3. Find the number of terms in each sequence:

a) $\frac{1}{2}, \frac{7}{8}, \frac{5}{4}, \ldots, \frac{31}{2}$

$\frac{4}{8}, \frac{7}{8}, \frac{10}{8}, \ldots, \frac{124}{8}$

$t_n = a + d(n-1)$

$\frac{124}{8} = \frac{4}{8} + \frac{3}{8}(n-1)$

$124 = 4 + 3(n - 1)$

$124 = 4 + 3n - 3$

$124 = 1 + 3n$

$123 = 3n$

$n = 41$

b) -5, -10, -20, … , -10240

$t_n = ar^{n-1}$ $\quad r = \frac{-10}{-5} = 2$

$-10240 = -5(2)^{n-1}$

$\frac{-10240}{-5} = \frac{-5(2)^{n-1}}{-5}$

$2048 = 2^{n-1}$

$2^{11} = 2^{n-1}$

$11 = n - 1$

$n = 12$

#4. Write the general term (t_n) for each sequence:

a) -8, 4, -2, …

$t_n = ar^{n-1}$

$t_n = -8(-½)^{n-1}$

b) -5, -10, -15, …

$t_n = a + d(n-1)$

$t_n = -5 + -5(n-1)$

$t_n = -5 - 5n + 5$

$t_n = -5n$

#5. The 20th term of an arithmetic sequence is 12 and the 32nd term is 48. Find the first term and the common difference.

Note: Between 20th and 32nd terms, there are 12 common differences.

$48 = 12 + 12d$

$36 = 12d$

$d = 3$

$t_n = a + d(n - 1)$

$48 = a + 3(32 - 1)$

$48 = a + 93$

$a = -45$

#6. Write out the first three terms of the geometric sequence whose fifth term is 48 and whose seventh term is 192.

Note: Between 5^{th} and 7^{th} terms, there are 2 common ratios.

$192 = 48r^2$

$4 = r^2$

$r = 2$ or -2

$t_n = ar^{n-1}$

$48 = a(2)^{5-1}$ or $48 = a(-2)^{5-1}$

$48 = a(16)$ $\quad$ $48 = a(16)$

$a = 3$ $\quad$ $a = 3$

1^{st} 3 terms==> **3, 6, 12** or **3, -6, 12**

#7. Find the sum of each series:

a) 100 + 90 + 80 + … + -200

Find n first.

$t_n = a + d(n-1)$

$-200 = 100 - 10(n-1)$

$-200 = 100 - 10n + 10$

$-200 = 110 - 10n$

$-310 = -10n$

$n = 31$

$$S_n = \frac{n}{2}(a + t_n)$$

$$S_{31} = \frac{31}{2}(100 - 200)$$

$S_{31} = 15.5(-100)$

$S_{31} = -1550$

b) $3 + 6 + 12 + \ldots + S_9$

n = 9 r = 2 (so use the r – 1 form)

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_9 = \frac{3(2^9 - 1)}{2 - 1} = \frac{3(511)}{1} = 1533$$

#8. Find the sum of the infinite geometric series:

a) $2 + 1 + \frac{1}{2} + \ldots$

r = ½ so the Sum is possible.

$$S_\infty = \frac{a}{1-r} = \frac{2}{1-\frac{1}{2}} = \frac{2}{\frac{1}{2}} = 2 \div \frac{1}{2} = 2 \times 2 = 4$$

b) $4 + \frac{20}{3} + \frac{100}{9} \ldots$

$$r = \frac{20}{3} \div 4 = \frac{20}{3} \times \frac{1}{4} = \frac{20}{12} = \frac{5}{3}$$

Since r > 1, the sum is not possible.

#9. Suppose that each year a tree grow 90% as much as it did the year before. If the tree was 2.35 m tall after the 1^{st} year, how tall would it eventually get?

This is an infinite sum. 2.35, .9x2.35, etc. So the ratio r = .9

$$S_\infty = \frac{a}{1-r} = \frac{2.35}{1-.9} = \frac{2.35}{.1} = 23.5$$

The tree would grow to 23.5 m in height.

#10. A man walks 5km in week1, 8 km in week2, 11 km in week3 and so forth. How many km would he walk in total over 10 weeks?

The series would be 5 + 8 + 11 + … with n=10

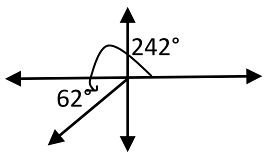
$$S_n = \frac{n}{2}[2a + d(n-1)]$$

$$S_{10} = \frac{10}{2}[2(5) + 3(10-1)] = 5[10 + 27] = 5(37) = 185$$

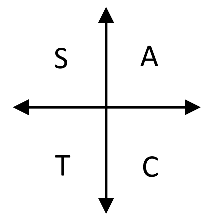
He would walk 185 km.

Chp2 Trig

#1. Sketch the angle and name its reference angle: 242°



The reference angle is 62°. (242-180)



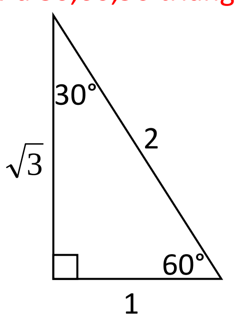
#2. Find the exact value of the following without using a calculator:

a) Cos 210°

Ref angle is 30° (210-180) in quadrant 3.
Cos is negative in quad 3.

Cos 30° = $\frac{\sqrt{3}}{2}$ So Cos 210° = $-\frac{\sqrt{3}}{2}$

Draw a 30,60,90 triangle to help find this.

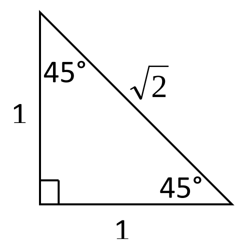


b) Sin 315°

Ref angle is 45° (360-315) in quadrant 4.
Sin is negative in quad 4.

Sin 45° = $\frac{1}{\sqrt{2}}$ So Sin 315° = $-\frac{1}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$

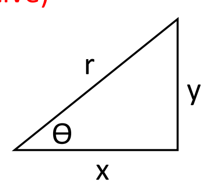
Draw a 45,45,90 triangle to help find this.



#3. A point P(4,-3) lies on the terminal arm of an angle ϴ in standard position. Determine the exact trigonometric ratios for Sin ϴ, Cos ϴ and Tan ϴ.

$x^2 + y^2 = r^2$ x=4 y=-3 $(4)^2 + (-3)^2 = r^2$ $25 = r^2$ r = 5   (r is always positive)

Sin ϴ= $\frac{y}{r}=-\frac{3}{5}$ Cos ϴ= $\frac{x}{r}=\frac{4}{5}$ Tan ϴ= $\frac{y}{x}=-\frac{3}{4}$



#4. If Sin ϴ = $\frac{5}{13}$, ϴ is in Q2, find the Cos ϴ and Tan ϴ.

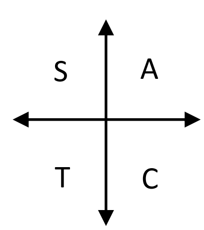
Sin ϴ= $\frac{y}{r}$ y = 5   r = 13 $x^2 + y^2 = r^2$ $x^2 + (5)^2 = (13)^2$ $x^2 + 25 = 169$ $x^2 = 144$ x = ±12

In quad 2, Cos ϴ and Tan ϴ are both neg, so Cos ϴ = $\frac{x}{r}=-\frac{12}{13}$ Tan ϴ= $\frac{y}{x}=-\frac{5}{12}$

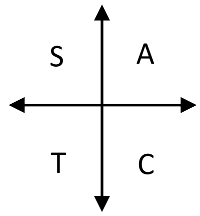
#5. Find the quadrant where Cos ϴ < 0 and Tan ϴ > 0.

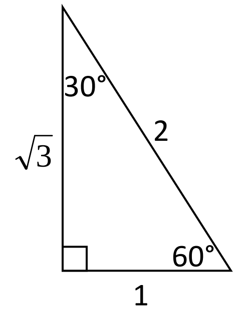
Cos neg in Q2 and Q3
Tan pos in Q1 and Q3

So **Q3** is where ϴ must be.



#6. Solve for Θ if 0° ≤ Θ ≤ 360°.





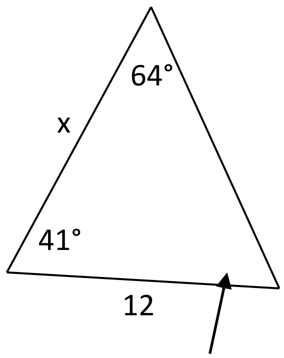
Sin Θ = $-\dfrac{\sqrt{3}}{2}$ $Θ_R$ = 60°   (See diagram)

Sin is neg in Q3 and Q4.

Θ = 180 + 60 = **240°**   or   Θ = 360 – 60 = **300°**.

#7. Find each measure indicated:

a)

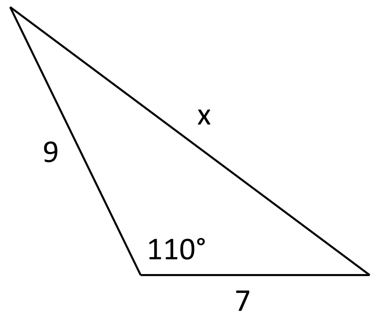


missing angle is 75°   (180-64-41)

$$\frac{12}{\text{Sin } 64} = \frac{x}{\text{Sin } 75}$$

x = $\dfrac{12(\text{Sin } 75)}{\text{Sin } 64} = 12.9$

b)



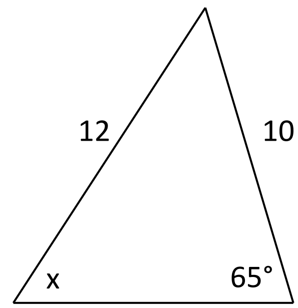
$c^2 = a^2 + b^2 – 2ab\text{Cos } C$

$x^2 = 9^2 + 7^2 – 2(9)(7)\text{Cos } 110°$

$x^2 = 173.09$

x = 13.16

c)



$$\frac{10}{\text{Sin x}} = \frac{12}{\text{Sin } 65}$$

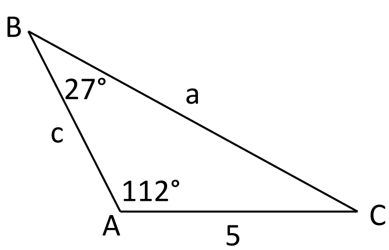
12 Sin x = 10(Sin 65)

$$\text{Sin x} = \frac{10(\text{Sin } 65)}{12}$$

x = 49°

#8. Solve each triangle ΔABC.

a) B = 27°, A = 112°, b = 5

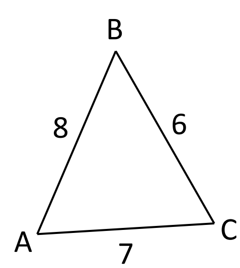


C = 180-27-112 = **41°**

$$\frac{a}{\text{Sin } 112} = \frac{5}{\text{Sin } 27} \qquad \frac{c}{\text{Sin } 41} = \frac{5}{\text{Sin } 27}$$

a = **10.2**   c = **7.2**

b) a = 6, b = 7, c = 8



$c^2 = a^2 + b^2 – 2ab\text{Cos } C$

$8^2 = 6^2 + 7^2 – 2(6)(7)\text{Cos } C$

64 = 85 – 84Cos C

-21 = -84Cos C

.25 = Cos C

C = **75.5°**

$$\frac{7}{\text{Sin B}} = \frac{8}{\text{Sin } 75.5}$$

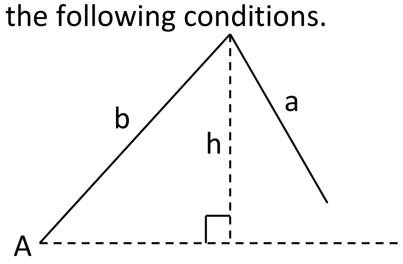
$$\text{Sin B} = \frac{7(\text{Sin } 75.5)}{8}$$

B = **57.9°**

A = 180-75.5-57.9 = **46.6°**

#9. Determine how many ABC triangles satisfy the following conditions.

a) $\angle A = 65°$, a = 9.1 cm, and b = 10.7 cm

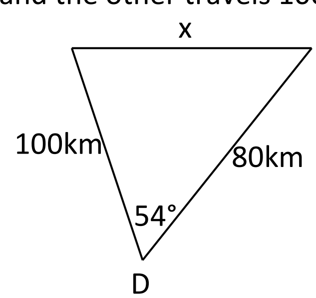


h = b Sin A
h = 10.7 Sin 65°
h = 9.7
Since "a" is the smallest in size, we can draw **"0" different triangles**.

b) $\angle A = 24°$, a = 5, and b = 7

h = b Sin A
h = 7 Sin 24°
h = 2.8
Since "h" is the smallest in size, we can draw **"2" different triangles**.

#10. Two boats leave a dock at the same time. Each travels in a different direction. The angle between their courses is 54°. If one boat travels 80 km and the other travels 100 km, how far apart are they?



$x^2 = 100^2 + 80^2 – 2(100)(80)\text{Cos } 54°$
$x^2 = 16400 – 9404.6$
$x^2 = 6995.4$
x = 83.6
They are 83.6km apart.

<u>Chp 3 Quadratic Functions</u>

#1. Find the vertex of each quadratic:

a) $y = 3x^2$

vertex is (0, 0)

b) $y + 3 = -\frac{1}{2}x^2$

vertex is (0, -3)

c) $y = (x + 1)^2 + 2$

vertex is (-1, 2)

#2. Write each of the following in vertex-graphing form by completing the square:

a) $y = x^2 + 4x$

$y = x^2 + 4x + 4 – 4$
$y = (x + 2)^2 – 4$

b) $y = x^2 + x – 1$

$y + 1 = x^2 + 1x$
$y + 1 + ¼ = x^2 + x + ¼$
$y + 1 + ¼ = (x + ½)^2$
$y = (x + \frac{1}{2})^2 – \frac{5}{4}$

c) $y = -3x^2 + 12x – 2$

$y = -3x^2 + 12x$
$y + 2 = -3(x^2 – 4x)$
$y + 2 – 12 = -3(x^2 – 4x + 4)$
$y – 10 = -3(x^2 – 4x + 4)$
$y = -3(x – 2)^2 + 10$

#3. Answer the following questions for each quadratic function:
a) vertex b) equation of the axis of symmetry c) concavity (faces up or down)
d) maximum or minimum value e) domain and range f) x and y intercepts
g) sketch the graph

i) $y = -3(x + 2)^2 + 3$

Vertex is (-2, 3)
Eqn of A.O.S. is x = -2
Faces Down (a is neg)
Max Value of 3
Domain: $x \in R$
Range: $y \leq 3$

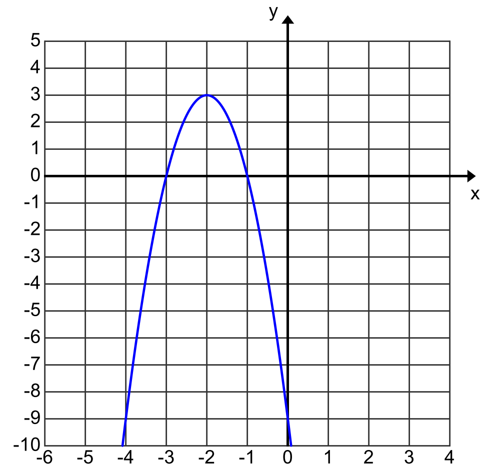
x-intercepts

$0 = -3(x + 2)^2 + 3$
$-3 = -3(x + 2)^2$
$1 = (x + 2)^2$
$\pm\sqrt{1} = x + 2$
$\pm 1 = x + 2$
$1 = x + 2$ $\quad -1 = x + 2$
$-1 = x$ $\quad -3 = x$
x ints are {-1, -3}

y-intercepts

$y = -3(0 + 2)^2 + 3$
$y = -3(4) + 3$
$y = -9$

y int is -9



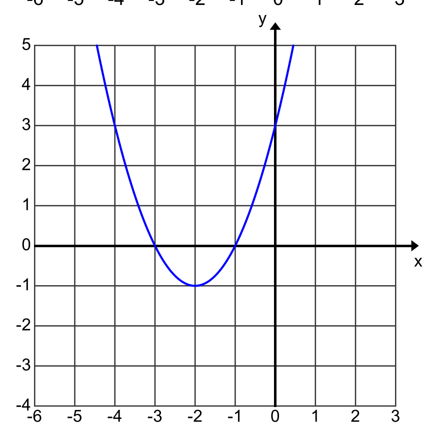
ii) $y = x^2 + 4x + 3$

Complete the square:
$y = x^2 + 4x + 4 - 4 + 3$
$y = (x + 2)^2 - 1$
(or use $p = -\frac{b}{2a}$)

Vertex is (-2, -1)
Eqn of A.O.S. is x = -2
Faces Up (a is pos)
Min Value of -1
Domain: $x \in R$
Range: $y \geq -1$

x-intercepts
$0 = x^2 + 4x + 3$
$0 = (x + 3)(x + 1)$
x ints are {-3, -1}

y-intercepts
$y = 0^2 + 4(0) + 3$
y int is 3



#4. Write a quadratic equation in vertex graphing form for each of the following:
a) a = 2 vertex is (-1, 2)

$y = a(x - p)^2 + q$
$y = 2(x + 1)^2 + 2$

b) vertex is (3, 2) and passes through the point (2, -1)

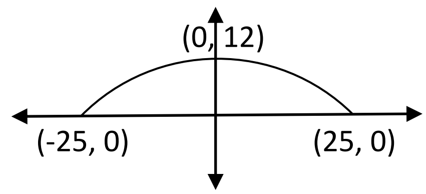
$y = a(x - p)^2 + q$ $\quad$ p=3, q=2, x=2, y=-1
$-1 = a(2 - 3)^2 + 2$
$-1 = a(1) + 2$
$-3 = a$ $\quad$ **$y = -3(x - 3)^2 + 2$**

#5. Write the new equation of the parabola $y = x^2$ after the following: (3 marks)
a) a horizontal translation 2 units to the left and a vertical translation 1 unit up
$y = a(x - p)^2 + q$ $\quad$ a=1, p=-2, q= 1 $\quad$ **$y = (x + 2)^2 + 1$**

b) a vertical translation 3 units down and a reflection across the x-axis
$y = a(x - p)^2 + q$ $\quad$ a=-1, p=0, q= -3 $\quad$ **$y = -1x^2 - 3$**

c) a multiplication of the y-values by -2 and then a horizontal translation 1 unit to the right
$y = a(x - p)^2 + q$ $\quad$ a=-2, p=1, q= 0 $\quad$ **$y = -2(x - 1)^2$**

#6. A bridge has the shape of a parabola. Its width is 50m and its height is 12m. Find the quadratic equation for this bridge.



$y = a(x - p)^2 + q$ $p=0,\ q=12,\ x=25,\ y=0$

$0 = a(25 - 0)^2 + 12$

$0 = a(625) + 12$

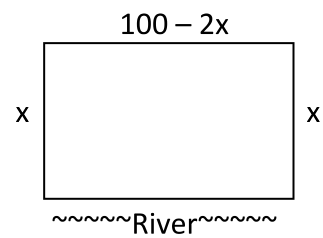
$-12 = 625a$ $a = -\frac{12}{625}$ $y = -\frac{12}{625}x^2 + 12$

#7. The height, "h", in metres, of a flare "t" seconds after it is fired into the air is given by the equation $h(t)=-4.9t^2 + 61.25t$. At what height is the flare at its maximum height? How many seconds after being shot does this occur?

$p = -\frac{b}{2a} = -\frac{61.25}{2(-4.9)} = 6.25$ $q = -4.9(6.25)^2 + 61.25(6.25) = 191.4$ Vertex is (6.25, 191.4)

Max height is at 191.4m. It happens 6.25 seconds after being shot.

#8. A farmer has 100m of fencing material to enclose a rectangular field adjacent to a river. No fencing is required along the river. Find the dimensions of the rectangle that will make its area a maximum. What is the maximum Area? (Hint: a diagram of the situation is given below)



$A = x(100 - 2x)$

$A = 100x - 2x^2$ or $A = -2x^2 + 100x$

$p = -\frac{b}{2a} = -\frac{100}{2(-2)} = 25$

$q = -2(25)^2 + 100(25) = 1250$ Vertex is (25, 1250)

100 – 2(25) = 50 So the rectangle is 25m by 50m. The maximum area is $1250m^2$.

Chp 4 Quadratic Equations

#1. Solve the quadratic equations by factoring:

a) $3x^2 - 36x = 0$

$3x(x - 12) = 0$

$x = 0$ $x = 12$

$x=\{0, 12\}$

b) $2x^2 - 7x - 15 = 0$

$(2x + 3)(x - 5) = 0$

$X=\left\{-\frac{3}{2}, 5\right\}$

c) $6x^2 - 11x + 3 = 24$

$6x^2 - 11x + 3 = 24$

$6x^2 - 11x - 21 = 0$

$(6x + 7)(x - 3) = 0$

$X=\left\{-\frac{7}{6}, 3\right\}$

#2. Solve the quadratic equations by completing the square: (Write answers in Exact Form)

a) $x^2 - 6x + 5 = 0$

$x^2 - 6x = -5$
$x^2 - 6x + 9 = -5 + 9$
$(x - 3)^2 = 4$
$x - 3 = \pm\sqrt{4}$
$x - 3 = \pm 2$
$x - 3 = 2$ or $x - 3 = -2$
$x = 5$ $x = 1$
$\{5, 1\}$

b) $x^2 + 4x + 1 = 0$

$x^2 + 4x = -1$
$x^2 + 4x + 4 = -1 + 4$
$(x + 2)^2 = 3$
$x + 2 = \pm\sqrt{3}$
$x = -2 \pm \sqrt{3}$
$\{-2 \pm \sqrt{3}\}$

c) $3x^2 - x - 2 = 0$

$3x^2 - x = 2$

$$x^2 - \frac{1}{3}x = \frac{2}{3}$$

$$x^2 - \frac{1}{3}x + \frac{1}{36} = \frac{2}{3} + \frac{1}{36}$$

$$\left(x - \frac{1}{6}\right)^2 = \frac{25}{36}$$

$$x - \frac{1}{6} = \pm\sqrt{\frac{25}{36}}$$

$$x = \frac{1}{6} \pm \frac{5}{6} \quad \left\{\frac{6}{6}, -\frac{4}{6}\right\} = \left\{1, -\frac{2}{3}\right\}$$

#3. Solve the quadratic equations using the quadratic formula: (Write answers in Exact Form)

a) $x^2 + 4x - 96 = 0$ a =1, b=4, c=-96

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-96)}}{2(1)} = \frac{-4 \pm \sqrt{400}}{2} = \frac{-4 \pm 20}{2} = -2 \pm 10 \quad \{-12, 8\}$$

b) $3x^2 = 4$ (Hint: Same as $3x^2 - 0x - 4 = 0$) a=3, b=0, c=-4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0 \pm \sqrt{(0)^2 - 4(3)(-4)}}{2(3)} = \frac{\pm\sqrt{48}}{6} = \frac{\pm 4\sqrt{3}}{6} = \frac{\pm 2\sqrt{3}}{3} \quad \left\{\frac{\pm 2\sqrt{3}}{3}\right\}$$

#4. Find the zeros of the function $f(x) = x^2 - 10x + 16$.

$0 = x^2 - 10x + 16$
$0 = (x - 8)(x - 2)$
$x = 8$ $x = 2$ The zero's are 8 and 2. {Note: The zeros are the same as x-intercepts!}

#5. Find the quadratic equation with the roots of $\left\{\frac{1}{2}, -\frac{2}{3}\right\}$

$(2x - 1)(3x + 2) = 0$
$6x^2 + x - 2 = 0$

#6. Find the discriminant and state the nature of the roots:

a) $x^2 - 4x - 5 = 0$

a=1 b=-4 c=-5
Discr = $b^2 - 4ac$
Discr = $(-4)^2 - 4(1)(-5)$
Discr = 16 + 20 = 36

So there are **2** roots.

b) $x^2 = -9$

$x^2 + 9 = 0$ a=1 b=0 c=9
Discr = $b^2 - 4ac$
Discr = $(0)^2 - 4(1)(9)$
Discr = 0 – 36 = -36

So there are **0** roots.

c) $x^2 + 2x + 1 = 0$

a=1 b=2 c=1
Discr = $b^2 - 4ac$
Discr = $(2)^2 - 4(1)(1)$
Discr = 4 – 4 = 0

So there is **1** root.

#7. The hypotenuse of a right triangle is 13. If the sum of the legs is 17, find the legs. (Hint: Let one leg be x and the other is therefore 17-x…since the sum is 17.)

$a^2 + b^2 = c^2$

$x^2 + (17 - x)^2 = 13^2$ $(17 - x)(17 - x) = 289 - 17x - 17x + x^2$

$x^2 + 289 - 34x + x^2 = 169$

$2x^2 - 34x + 120 = 0$

$2(x^2 - 17x + 60) = 0$

$(x - 12)(x - 5) = 0$

$x = 12$ $x = 5$ The legs are 5 and 12.

#8. If $h(t) = 5t^2 - 30t + 45$, find t when h = 20. (Hint: $20 = 5t^2 - 30t + 45$)

$20 = 5t^2 - 30t + 45$

$0 = 5t^2 - 30t + 25$

$0 = 5(t^2 - 6t + 5)$

$0 = (t - 5)(t - 1)$

$t= 5$ $t = 1$ $\{5, 1\}$

Chp 5 Radicals

#1. Simplify:

Squares	Cubes	Fourths
4	8	16
9	27	64
16	64	81
25	125	625
36	216	x^4
49	x^3	x^8
64	x^6	
81		
100		
x^2		
x^4		

a) $\sqrt{150}$

$=\sqrt{25}\sqrt{6}$
$=5\sqrt{6}$

b) $\sqrt[3]{32x^5}$

$=\sqrt[3]{8x^3}\ \sqrt[3]{4x^2}$
$=2x\ \sqrt[3]{4x^2}$

c) $\sqrt[4]{32x^9y^6}$

$=\sqrt[4]{16x^8y^4}\ \sqrt[4]{2xy^2}$
$=2x^2y\ \sqrt[4]{2xy^2}$

#2. Change each mixed radical into an entire radical:

a) $4\sqrt{3}$

$=\sqrt{16}\sqrt{3}$
$=\sqrt{48}$

b) $2x\ \sqrt[3]{3x^2}$

$=\sqrt[3]{8x^3}\ \sqrt[3]{3x^2}$
$=\sqrt[3]{24x^5}$

#3. Simplify:

a) $5\sqrt{2}-6\sqrt{3}+7\sqrt{2}-\sqrt{3}$

$=12\sqrt{2}-7\sqrt{3}$

b) $\sqrt{108}-2\sqrt{27}-\sqrt{40}-5\sqrt{160}$

$=\sqrt{36}\sqrt{3}-2\sqrt{9}\sqrt{3}-\sqrt{4}\sqrt{10}-5\sqrt{16}\sqrt{10}$
$=6\sqrt{3}-6\sqrt{3}-2\sqrt{10}-20\sqrt{10}$
$=-22\sqrt{10}$

c) $3\sqrt[3]{54}+2\sqrt[3]{128}$

$=3\sqrt[3]{27}\sqrt[3]{2}+2\sqrt[3]{64}\sqrt[3]{2}$
$=9\sqrt[3]{2}+8\sqrt[3]{2}$
$=17\sqrt[3]{2}$

#4. Multiply (Expand) the following and simplify:

a) $\left(\sqrt{6}\right)\left(\sqrt{2}\right)$

$=\sqrt{12}$
$=2\sqrt{3}$

b) $\left(3\sqrt{2x}\right)^2$

$=9\sqrt{4x^2}$
$=9(2x)$
$=18x$

c) $\left(\sqrt[3]{4x^2}\right)^2$

$=\sqrt[3]{16x^4}$
$=\sqrt[3]{8x^3}\ \sqrt[3]{2x}$
$=2x\ \sqrt[3]{2x}$

Recall:

$\sqrt{\text{num}}\sqrt{\text{num}}=\text{num}$
$\sqrt{5}\sqrt{5}=5$

d) $\left(2x\sqrt{3y}\right)\left(3x\sqrt{6y^3}\right)$

$=6x^2\sqrt{18y^4}$
$=6x^2\sqrt{9y^4}\sqrt{2}$
$=6x^2 3y^2\sqrt{2}$
$=18x^2y^2\sqrt{2}$

e) $3\sqrt{2}\left(\sqrt{2}+\sqrt{3}\right)$

$=3(2)+3\sqrt{6}$
$=6+3\sqrt{6}$

f) $\left(3\sqrt{2}-2\sqrt{5}\right)^2$

$=\left(3\sqrt{2}-2\sqrt{5}\right)\left(3\sqrt{2}-2\sqrt{5}\right)$
$=9(2)-6\sqrt{10}-6\sqrt{10}+4(5)$
$=38-12\sqrt{10}$

g) $(2+\sqrt{x})(3-\sqrt{x})$

$= 6 - 2\sqrt{x} + 3\sqrt{x} - x$

$= 6 - x + \sqrt{x}$

#5. Divide the following and be sure to rationalize all denominators:

a) $\dfrac{3\sqrt{6}}{6\sqrt{2}}$

$= \dfrac{\sqrt{3}}{2}$

b) $\dfrac{\sqrt{2}}{\sqrt{10}}$

$= \dfrac{\sqrt{1}}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

c) $\dfrac{3\sqrt{2}}{2\sqrt{3}}$

$\dfrac{3\sqrt{2}}{2\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{3\sqrt{6}}{2(3)} = \dfrac{\sqrt{6}}{2}$

d) $\dfrac{3x}{\sqrt{2x}}$

$\dfrac{3x}{\sqrt{2x}} \cdot \dfrac{\sqrt{2x}}{\sqrt{2x}} = \dfrac{3x\sqrt{2x}}{2x} = \dfrac{3\sqrt{2x}}{2}$

e) $\dfrac{3\sqrt{3}-\sqrt{2}}{2\sqrt{2}}$

$= \dfrac{3\sqrt{3}-\sqrt{2}}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{3\sqrt{6}-2}{4}$

f) $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$= \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \cdot \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \dfrac{5+\sqrt{15}+\sqrt{15}+3}{5-3} = \dfrac{8+2\sqrt{15}}{2} = 4+\sqrt{15}$

g) $\dfrac{2}{\sqrt[3]{9}}$

$= \dfrac{2}{\sqrt[3]{9}} \cdot \dfrac{\sqrt[3]{3}}{\sqrt[3]{3}} = \dfrac{2\sqrt[3]{3}}{3}$

#6. Solve the radical equations:

a) $\sqrt{3x-2} = 7$

$3x - 2 = 49$

$3x = 51$

$x = 17$

X={17}

b) $6 - 2\sqrt{x+7} = -2$

$-2\sqrt{x+7} = -8$

$\sqrt{x+7} = 4$

$x + 7 = 16$

$x = 9$

X= {9}

c) $\sqrt{2x+5} = x-5$

$2x+5 = (x-5)^2$

$2x+5 = x^2 - 10x + 25$

$0 = x^2 - 12x + 20$

$0 = (x-10)(x-2)$

X = {10, 2}

d) $\sqrt{x^2+4} = 3$

$x^2 + 4 = (3)^2$

$x^2 = 9 - 4$

$x^2 = 5$

$x = \pm\sqrt{5}$

X= $\{\pm\sqrt{5}\}$

e) $\sqrt{y-5} + \sqrt{y} = 5$

$\sqrt{y-5} = 5 - \sqrt{y}$

$y - 5 = (5-\sqrt{y})^2$ ← $(5-\sqrt{y})(5-\sqrt{y})$

$y - 5 = 25 - 10\sqrt{y} + y$

$-30 = -10\sqrt{y}$

$3 = \sqrt{y}$

$9 = y$ y= {9}

Chp 6 Rationals

#1. Simplify:

a) $\dfrac{12x^2y^2}{15xy^3}$

$= \dfrac{4x}{5y}$

b) $\dfrac{16x^2-25}{12x-15}$

$= \dfrac{(4x+5)(4x-5)}{3(4x-5)} = \dfrac{4x+5}{3}$

c) $\dfrac{3x-6}{2x^2+x-10}$

$= \dfrac{3(x-2)}{(2x+5)(x-2)} = \dfrac{3}{2x+5}$

#2. Multiply/Divide the following and simplify:

a) $\dfrac{12m^2f}{5cf} \cdot \dfrac{15c}{4m}$

$= \dfrac{9m}{1} = 9m$

b) $\dfrac{a^2-16}{16a-4a^2} \cdot \dfrac{2a^3+6a^2}{a^2+7a+12}$

$= \dfrac{\overset{-1}{\cancel{(a-4)}}(a+4)}{4a\cancel{(4-a)}} \cdot \dfrac{2a^2(a+3)}{(a+4)(a+3)} = -\dfrac{a}{2}$

c) $\dfrac{8y^2-2y-3}{y^2-1} \div \dfrac{2y^2-3y-2}{2y-2} \div \dfrac{3-4y}{y+1}$

$= \dfrac{\overset{-1}{\cancel{(4y-3)}}(2y+1)}{(y-1)(y+1)} \cdot \dfrac{2(y-1)}{(2y+1)(y-2)} \cdot \dfrac{y+1}{\cancel{3-4y}} = \dfrac{-2}{y-2}$

#3. Add/Subtract the following and simplify:

a) $\dfrac{3}{m} + \dfrac{2}{n} - \dfrac{3}{c}$

$= \dfrac{3nc+2mc-3mn}{mnc}$

b) $\dfrac{a-5}{2} - \dfrac{a-2}{3}$

$= \dfrac{3(a-5)}{6} - \dfrac{2(a-2)}{6}$

$= \dfrac{3a-15-2a+4}{6} = \dfrac{a-11}{6}$

c) $\dfrac{y^2-20}{y^2-4} - \dfrac{y-2}{y+2}$

$= \dfrac{y^2-20}{(y+2)(y-2)} - \dfrac{(y-2)(y-2)}{(y+2)(y-2)}$

$= \dfrac{y^2-20}{(y+2)(y-2)} - \dfrac{y^2-4y+4}{(y+2)(y-2)}$

$= \dfrac{y^2-20-y^2+4y-4}{(y+2)(y-2)}$

$= \dfrac{4y-24}{(y+2)(y-2)}$

d) $\dfrac{5}{x^2-5x+6} - \dfrac{4}{x^2-x-6}$

$= \dfrac{5}{(x-3)(x-2)} - \dfrac{4}{(x-3)(x+2)}$

$= \dfrac{5(x+2)}{(x-3)(x-2)(x+2)} - \dfrac{4(x-2)}{(x-3)(x-2)(x+2)}$

$= \dfrac{5x+10-4x+8}{(x-3)(x-2)(x+2)}$

$= \dfrac{x+18}{(x-3)(x-2)(x+2)}$

e) $\dfrac{1+\frac{1}{x}}{x-\frac{1}{x}}$

$= \left(1+\dfrac{1}{x}\right) \div \left(x-\dfrac{1}{x}\right)$

$= \dfrac{x+1}{x} \div \dfrac{x^2-1}{x}$

$= \dfrac{x+1}{x} \cdot \dfrac{x}{(x+1)(x-1)} = \dfrac{1}{x-1}$

#4. Solve each rational equation and list all the restrictions:

a) $\frac{x-2}{2}=\frac{2x+4}{5}-1$

$(10)\left[\frac{x-2}{2}=\frac{2x+4}{5}-1\right]$

$5(x - 2) = 2(2x + 4) - 10(1)$

$5x - 10 = 4x + 8 - 10$

$5x - 10 = 4x - 2$

$x = 8$ {8} no restrictions

b) $\frac{12}{x}-1=\frac{9}{x}$

$(x)\left[\frac{12}{x}-1=\frac{9}{x}\right]$

$12 - 1x = 9$

$-1x = -3$

$x = 3$ {3}

$x \neq 0$

c) $\frac{x}{x-2}=\frac{x-6}{x-4}$

$(x-2)(x-4)\left[\frac{x}{x-2}=\frac{x-6}{x-4}\right]$

$x(x - 4) = (x - 6)(x - 2)$

$x^2 - 4x = x^2 - 8x + 12$

$4x = 12$

$x = 3$ {3} $x \neq 2$ $x \neq 4$

d) $\frac{d}{d+4}=\frac{2-d}{d^2+3d-4}+\frac{1}{d-1}$

$(d+4)(d-1)\left[\frac{d}{d+4}=\frac{2-d}{(d+4)(d-1)}+\frac{1}{d-1}\right]$

$d(d - 1) = (2 - d) + 1(d + 4)$

$d^2 - d = 2 - d + d + 4$

$d^2 - d - 6 = 0$

$(d - 3)(d + 2) = 0$

$d = 3$ $d = -2$ {3, -2} $d \neq -4$ $d \neq 1$

#5. The sum of two numbers is 12. The sum of their reciprocals is $\frac{4}{9}$. Find the numbers.

Let x be one number Let 12 – x be the other {Sum of the numbers is 12}

$\frac{1}{x}+\frac{1}{12-x}=\frac{4}{9}$

$(9)(x)(12-x)\left(\frac{1}{x}+\frac{1}{12-x}=\frac{4}{9}\right)$

$(1)(9)(12 - x) + (1)(9)(x) = (4)(x)(12 - x)$

$108 - 9x + 9x = 48x - 4x^2$

$4x^2 - 48x + 108 = 0$

$4(x^2 - 12x + 27) = 0$

$4(x - 9)(x - 3) = 0$

$x = 9$ $x = 3$ The numbers are 9 and 3.

#6. Two hoses are used to fill up a pool. If one hose fills the pool in 6 hrs and the other fills the pool in 12 hrs, how much time would it take the fill the pool using both hoses?

$\frac{x}{6}+\frac{x}{12}=1$

$(12)\left(\frac{x}{6}+\frac{x}{12}=1\right)$

$2x + x = 12$

$3x = 12$

$x = 4$

It will take 4 hrs to fill the pool.

Chp 7 Absolute Value and Reciprocal Functions

#1. Evaluate:

a) $|-3|$

=3

b) $-2|-6|$

=-2(6)

=-12

c) $3|-2|-4|-2|$

= 3(2) – 4(2)

=6 – 8

=-2

d) $|2-6-3|-|5-4+3(2)|$

=|-7| – |7|

= 7 – 7

= 0

#2. Solve each equation:

a) $|3x| = 9$

Pos Case
3x = 9
x = 3

Neg Case
3x = -9
x = -3

Soln: { 3, -3}

b) $5|4x| + 10 = 5$

5|4x|= -5
|4x| = -1
Not possible, abs value is never neg

Soln: { }

c) $|4x+3| = 7$

Pos Case
4x + 3 = 7
4x = 4
x = 1

Neg Case
4x + 3 = -7
4x = -10
x = -2.5

Soln: { 1, -2.5}

d) $|3x+3| = 2x-5$

Pos Case
3x + 3 = 2x – 5
x = -8
(reject, it doesn't check)

Neg Case
3x + 3 = -2x + 5
5x = 2
x = .4
(reject, it doesn't check

Solution: { } no soln

e) $|x^2-2x+2| = 3x-4$

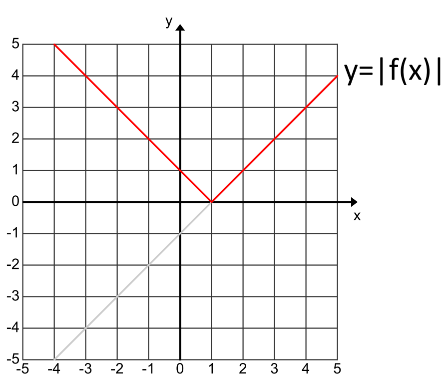
Pos Case
$x^2 - 2x + 2 = 3x - 4$
$x^2 - 5x + 6 = 0$
(x – 3)(x – 2) = 0
x = 3 x = 2

Soln: { 3, 2}

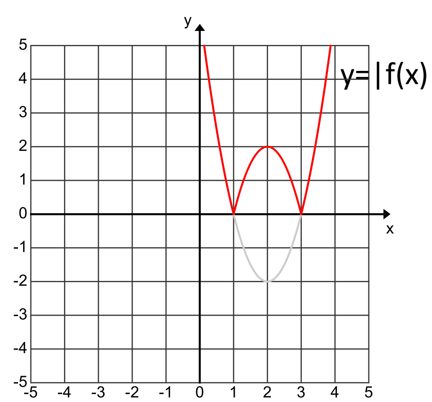
Neg Case
$x^2 - 2x + 2 = -3x + 4$
$x^2 + x - 2 = 0$
(x + 2)(x – 1) = 0
x = -2 x = 1
reject both since neither check

#3. Use the graph of y=f(x) to sketch the graph of y=|f(x)|

a)

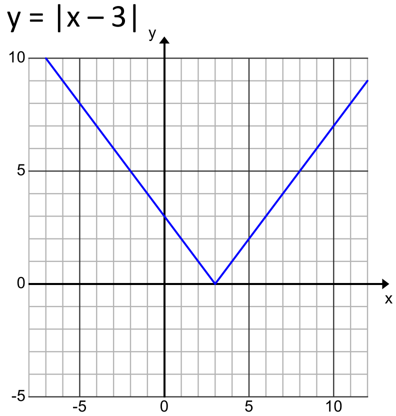


b)

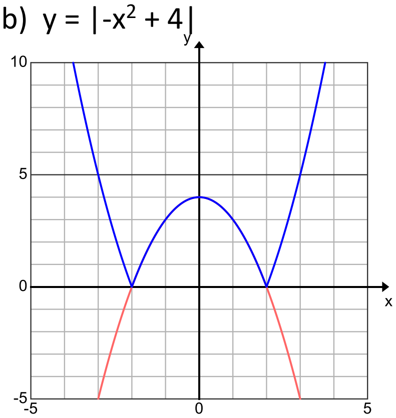


#4. Sketch the graph of:

a) y = |x – 3|



b) $y = |-x^2 + 4|$



$$p = -\frac{b}{2a} = -\frac{0}{2(-1)} = 0$$

$q = -(0)^2 + 4 = 4$

vertex: (0, 4)

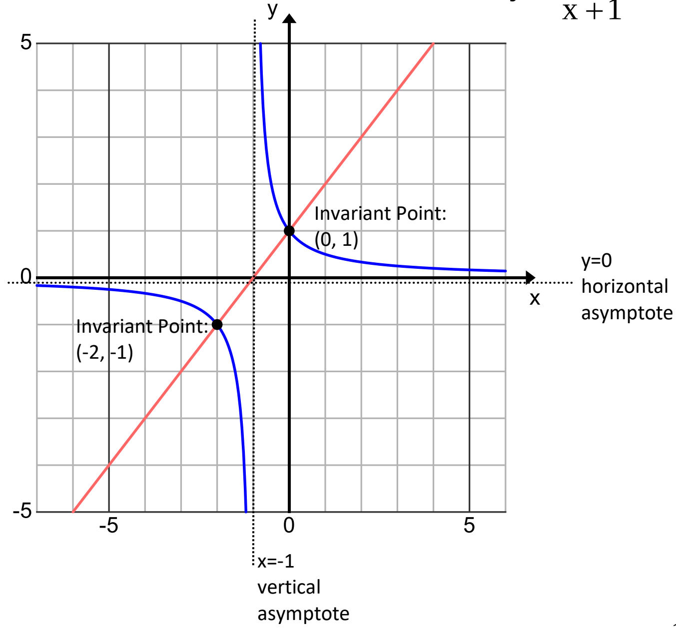
Reflect neg values across the x-axis.

#5. Express y = |x – 3| as a piecewise function.

0 = x – 3   x int is 3

$$y = \begin{cases} x-3 & \text{if } x \geq 3 \\ -x+3 & \text{if } x < 3 \end{cases}$$

#6. Sketch the graph of y = x + 1 and $y = \frac{1}{x+1}$. State the invariant points.

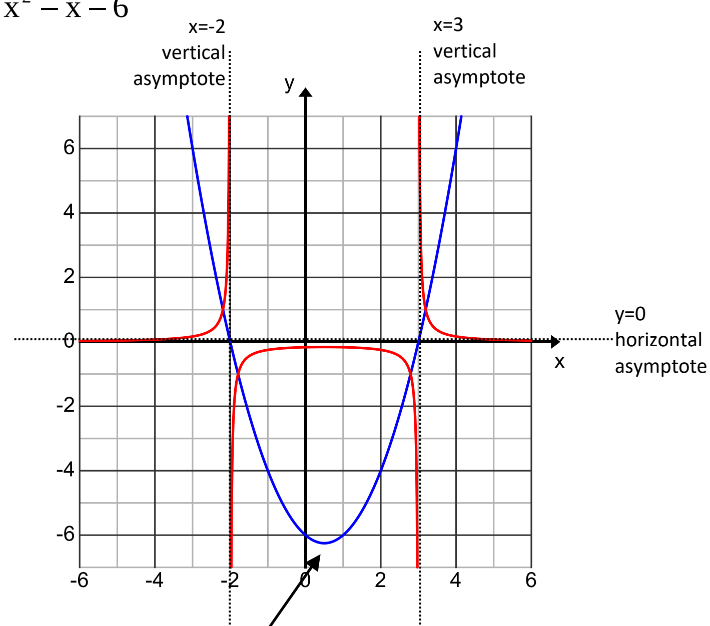


#7. Sketch the graph of $y = x^2 - x - 6$ and $y = \frac{1}{x^2 - x - 6}$. State the invariant points.

Vertical Asymptotes at: (N.P.V)

$0 = x^2 - x - 6$

0 = (x – 3)(x + 2)   x = 3   x = -2



Invariant Points:
y =1

$1 = x^2 - x - 6$

$0 = x^2 - x - 7$

Use quad formula to find the invariant pts:
x = 3.2   x = -2.2

(3.2, 1)   (-2.2, 1)

Invariant Points:
y =-1

$-1 = x^2 - x - 6$

$0 = x^2 - x - 5$

Use quad formula to find the invariant pts:
x = 2.8   x = -1.8

(2.8, 1)   (-1.8, 1)

To find the vertex of $y = x^2 - x - 6$:

$$p = -\frac{b}{2a} = -\frac{-1}{2(1)} = \frac{1}{2} = .5$$

$q = (.5)^2 - (.5) - 6$

vertex: (.5, -6.25)

Chp 8 Systems

#1. Solve by graphing. Give approximate solutions if needed. Verify your solutions.

$y = \frac{1}{2}x + 2$
$y + x^2 + 2x = 8$

$y = -x^2 - 2x + 8$

$$p = -\frac{b}{2a} = \frac{-(-2)}{2(-1)} = -1$$

$q = -(-1)^2 - 2(-1) + 8 = 9$

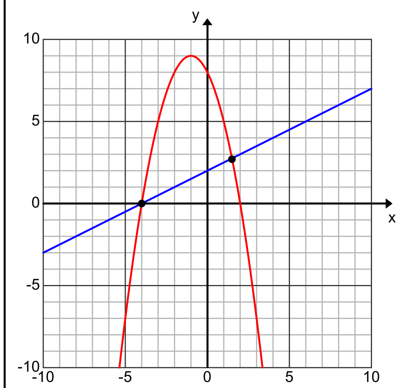
Vertex is at (-1, 9)

Check (-4,0):
$y = \frac{1}{2}x+2$
$0 = \frac{1}{2}(-4)+2$
$0 = -2+2$
$0 = 0$ yes

$y + x^2 + 2x = 8$
$0 + (-4)^2 + 2(-4) = 8$
$0 + 16 - 8 = 8$
$8 = 8$ yes

Check (1.5,2.7):
$y = \frac{1}{2}x+2$
$2.7 = \frac{1}{2}(1.5) + 2$
$2.7 = .75 + 2$
$2.7 = 2.75$ yes, close

$y + x^2 + 2x = 8$
$2.7 + (1.5)^2 + 2(1.5) = 8$
$2.7 + 2.25 + 3 = 8$
$7.95 = 8$ yes, close



Solutions: {(-4,0) (1.5,2.7)} approximately

#2. Solve algebraically. Verify your solutions.

$y = 3x + 1$
$y = 6x^2 + 10x - 4$

Substitute 3x + 1 in for y in the 2nd equation:

$3x + 1 = 6x^2 + 10x - 4$
$0 = 6x^2 + 7x - 5$
$0 = (2x - 1)(3x + 5)$

$x = \frac{1}{2}$ $\quad$ $x = -\frac{5}{3}$

substitute x to find y values

$y = 3x + 1$ $\quad$ $y = 3x + 1$

$y = 3\left(\frac{1}{2}\right) + 1$ $\quad$ $y = 3\left(-\frac{5}{3}\right) + 1$

$y = \frac{5}{2}$ $\quad$ $y = -4$

Solutions: $\left\{\left(\frac{1}{2}, \frac{5}{2}\right), \left(-\frac{5}{3}, -4\right)\right\}$

Check: $\left(\frac{1}{2}, \frac{5}{2}\right)$

$y = 3x + 1$

$$\frac{5}{2} = 3\left(\frac{1}{2}\right) + 1$$
$$\frac{5}{2} = \frac{3}{2} + \frac{2}{2}$$
$$\frac{5}{2} = \frac{5}{2}$$

$y = 6x^2 + 10x - 4$

$$\frac{5}{2} = 6\left(\frac{1}{2}\right)^2 + 10\left(\frac{1}{2}\right) - 4$$
$$\frac{5}{2} = 6\left(\frac{1}{4}\right) + 5 - 4$$
$$\frac{5}{2} = \frac{3}{2} + 1$$
$$\frac{5}{2} = \frac{5}{2}$$

Check: $\left(-\frac{5}{3}, -4\right)$

$y = 3x + 1$

$$-4 = 3\left(-\frac{5}{3}\right) + 1$$
$$-4 = -5 + 1$$
$$-4 = -4$$

$y = 6x^2 + 10x - 4$

$$-4 = 6\left(-\frac{5}{3}\right)^2 + 10\left(-\frac{5}{3}\right) - 4$$
$$-4 = 6\left(\frac{25}{9}\right) - \frac{50}{3} - 4$$
$$-4 = \frac{50}{3} - \frac{50}{3} - 4$$
$$-4 = -4$$

#3. Solve algebraically. Verify your solutions.

$x^2 + y - 3 = 0$
$x^2 - y + 1 = 0$ Add both together to eliminate the y terms

$2x^2 - 2 = 0$
$2x^2 = 2$
$x^2 = 1$
$x = \pm 1$

You could also use substitution to solve this problem!

substitute x to find y values

x = 1	x = -1
$x^2 + y - 3 = 0$	$x^2 + y - 3 = 0$
$(1)^2 + y - 3 = 0$	$(-1)^2 + y - 3 = 0$
$1 + y - 3 = 0$	$1 + y - 3 = 0$
$y = 2$	$y = 2$

Solutions: {(1,2) , (-1,2)}

Check: (1, 2)

$x^2 + y - 3 = 0$
$(1)^2 + 2 - 3 = 0$
$1 + 2 - 3 = 0$
$0 = 0$

$x^2 - y + 1 = 0$
$(1)^2 - 2 + 1 = 0$
$1 - 2 + 1 = 0$
$0 = 0$

Check: (-1, 2)

$x^2 + y - 3 = 0$
$(-1)^2 + 2 - 3 = 0$
$1 + 2 - 3 = 0$
$0 = 0$

$x^2 - y + 1 = 0$
$(-1)^2 - 2 + 1 = 0$
$1 - 2 + 1 = 0$
$0 = 0$

#4. Solve algebraically. Verify your solutions.

$y = x^2 - 4x + 1$
$2y = -x^2 + 4x + 2$

substitute $x^2 - 4x + 1$ in for y in the 2nd equation:

$2(x^2 - 4x + 1) = -x^2 + 4x + 2$
$2x^2 - 8x + 2 = -x^2 + 4x + 2$
$3x^2 - 12x = 0$
$3x(x - 4) = 0$
$x = 0 \quad x = 4$

substitute x to find y values

x = 0	x = 4
$y = x^2 - 4x + 1$	$2y = -x^2 + 4x + 2$
$y = (0)^2 - 4(0) + 1$	$2y = -(4)^2 + 4(4) + 2$
$y = 1$	$2y = 2$
	$y = 1$

Solutions: {(0,1) , (4,1)}

Check: (0, 1)

$y = x^2 - 4x + 1$
$1 = (0)^2 - 4(0) + 1$
$1 = 1$

$2y = -x^2 + 4x + 2$
$2(1) = -(0)^2 + 4(0) + 2$
$2 = 2$

Check: (4, 1)

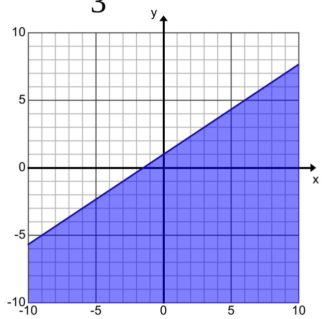
$y = x^2 - 4x + 1$
$1 = (4)^2 - 4(4) + 1$
$1 = 16 - 16 + 1$
$1 = 1$

$2y = -x^2 + 4x + 2$
$2(1) = -(4)^2 + 4(4) + 2$
$2 = -16 + 16 + 2$
$2 = 2$

Chp 9 Quadratic Inequalities

#1. Solve by graphing:

a) $y < \frac{2}{3}x + 1$



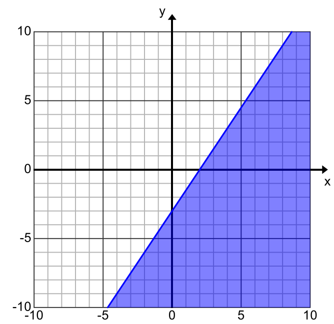
Slope is $\frac{2}{3}$ y-int: 1

Test Point: (0,0)

$0 < \frac{2}{3}(0) + 1$

0 < 1 true, so shade towards the pt (0,0)

b) $3x - 2y \geq 6$



$3x - 2y \geq 6$

$-2y \geq -3x + 6$

$\frac{-2y}{-2} \leq \frac{-3x}{-2} + \frac{6}{-2}$

$y \leq \frac{3}{2}x - 3$

Test Point: (0,0)

$0 \leq \frac{3}{2}(0) - 3$

0 ≤ -3 false, so shade away from (0,0)

#2. Solve:

a) $x^2 + x - 12 < 0$

$(x + 4)(x - 3) < 0$ **zeros at -4 and 3**

Interval	x < -4	-4 < x < 3	x > 3
Test Point	-5	0	4
Substitution (Work Area)	$(-5)^2 + (-5) - 12$ 25-5-12 20-12 8	$0^2 + 0 - 12$ -12	$4^2 + 4 - 12$ 16 + 4 - 12 20 - 12 8
Result: + or –	+	-	+

Solution is $x = \{x | -4 < x < 3, x\varepsilon\mathbb{R}\}$

b) $x^2 > 5x$

$x^2 - 5x > 0$ $x(x - 5) > 0$ **zeros at 0 and 5**

Interval	x < 0	0 < x < 5	x > 5
Test Point	-1	1	6
Substitution (Work Area)	$(-1)^2 - 5(-1)$ 1 + 5 6	$(1)^2 - 5(1)$ 1 - 5 -4	$(6)^2 - 5(6)$ 36 - 30 6
Result: + or –	+	-	+

Solution is x < 0 and x > 5 $x = \{x | 5 > x > 0, x\varepsilon\mathbb{R}\}$

c) $x^2 - 3x + 6 < 2x$

$x^2 - 5x + 6 < 0$ $(x - 3)(x - 2) < 0$

zeros at 2 and 3

Interval	x < 2	2 < x < 3	x > 3
Test Point	-3	2.5	4
Substitution (Work Area)	$(-3)^2 - 5(-3) + 6$ 9+15+6 30	$(2.5)^2 - 5(2.5) + 6$ 6.25-12.5+6 -.25	$(4)^2 - 5(4) + 6$ 16-20+6 2
Result: + or –	+	-	+

Solution is 2 < x < 3 $x = \{x | 2 < x < 3, x\varepsilon\mathbb{R}\}$

d) $2x^2 < 3 - 5x$

$2x^2 + 5x - 3 < 0$ $(2x - 1)(x + 3) < 0$

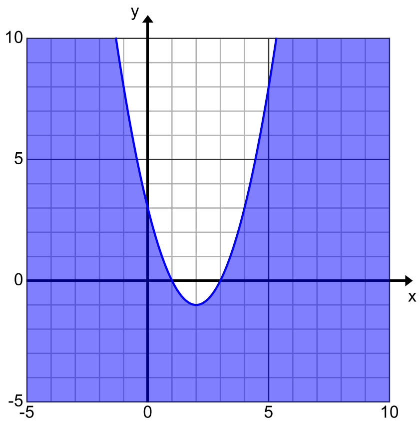
zeros at -3 and ½

Interval	x < -3	-3 < x < ½	x > ½
Test Point	-4	0	1
Substitution (Work Area)	$2(-4)^2 + 5(-4) - 3$ 32-20-3 9	$2(0)^2 + 5(0) - 3$ -3	$2(1)^2 + 5(1) - 3$ 2+5-3 4
Result: + or –	+	-	+

Solution is -3 < x < ½ $x = \{x | -3 < x < 0.5, x\varepsilon\mathbb{R}\}$

#3. Solve by graphing:

a) $y < (x - 2)^2 - 1$

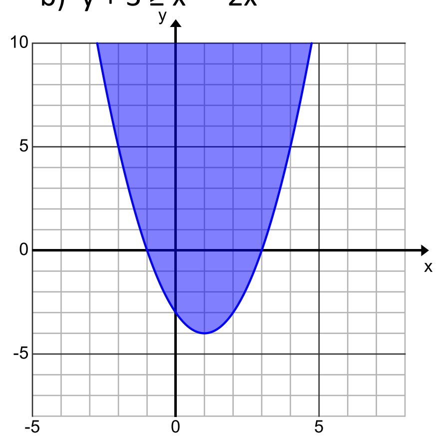


Vertex: (2, -1) Use 1a/3a/5a to graph

Test Point: (0,0)

$y < (x - 2)^2 - 1$
$0 < (0 - 2)^2 - 1$
$0 < 4 - 1$
$0 < 3$ True, so shade towards pt (0,0)

b) $y + 3 \geq x^2 - 2x$



$y \geq x^2 - 2x - 3$

$$p = -\frac{b}{2a} = -\frac{-2}{2(1)} = \frac{2}{2} = 1$$

$q = (1)^2 - 2(1) - 3 = -4$

Vertex: (1, -4) Use 1a/3a/5a to graph
or use x-intercepts: $(x - 3)(x + 1)$
x-intercepts: 3 and -1

Test Point: (0,0)

$y \geq x^2 - 2x - 3$
$0 \geq (0)^2 - 2(0) - 3$
$0 \geq -3$ True, so shade towards pt (0,0)