7.1 Absolute Value

Absolute Value

For any real number, a, the absolute value is written as **|a|** and results in a positive number.

Absolute value can be used to represent the distance of a number from zero on a number line.

- This can be summarized as: $|a| = \begin{cases} a, \text{ if a} \geq 0 \\ -a, \text{ if a<0} \end{cases}$

Absolute value symbols should be treated in the same manner as brackets. First evaluate inside the absolute value symbol using order of operations, then take the absolute value of the result (make it a positive value).

Concept: **Evaluate numerical absolute value expressions and order a set of real numbers**

Example# 1: Evaluate the following.

1. |3|

$= 3$

2. |-8|

$= 8$

4. |4| - |-9|

$= 4 - 9$
$= -5$

5. |5 – 13|

$= |-8|$
$= 8$

6. 6 – 3 |-2 – 8|

$= 6 - 3|-10|$
$= 6 - 3(10)$
$= 6 - 30$
$= -24$

7. |-3(5 + 1)^2 – 4|

$= |-3(6)^2 - 4|$
$= |-3(36) - 4|$
$= |-108 - 4|$
$= |-112|$
$= 112$

Example#2: Write the real numbers in order from the least to greatest:

|3.5|, -2, |-5.75|, 1.05, $\left|-\frac{13}{4}\right|$, |-0.5|, -1.25, $\left|-3\frac{1}{3}\right|$

≈ 1.05

$\parallel$
$3.\overline{3}$

$= -2, -1.25, |-0.5|, \left|-\frac{13}{4}\right|, 1.05, \left|-3\frac{1}{3}\right|, |3.5|, |-5.75|$

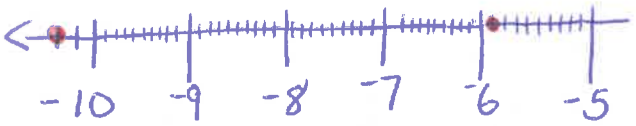
Example#3: **Evaluate** $|5x^2+3x-4|$ when $x=-3$

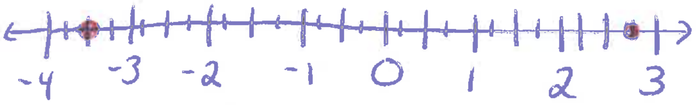
$$=|5(-3)^2+3(-3)-4|$$
$$=|5(9)-9-4|$$
$$=|45-9-4|$$
$$=|32| \qquad \boxed{=32}$$

Example#4: **Use absolute value to determine the distance between each pair of numbers on a number line. Sketch the numberline.**

a) -5.9 and -10.2

b) $2\frac{3}{4}$ and $-3\frac{1}{2}$





$$=|-5.9-(-10.2)| \text{ or } |-10.2-(-5.9)|$$
$$=|-5.9+10.2| \qquad =|-4.3|$$
$$=|4.3| \qquad = 4.3$$
$$=4.3$$

$$=|-3.5-2.75| \text{ or } |2.75-(-3.5)|$$
$$=|-6.25| \qquad =|6.25|$$
$$=6.25 \qquad =6.25$$

Example# 5: **When 7 is added to an integer, x, the absolute value of the sum is 12. Determine the value(s) for x.**

$$|x+7|=12$$

$$x+7\overset{-7}{=}12^{-7} \quad \text{or} \quad -(x+7)=12$$
$$x=5 \qquad -x-7\overset{+7}{=}12^{+7}$$
$$\frac{-x}{-1}=\frac{19}{-1}$$
$$x=-19$$

7.1 Assignment: Page 363 #1- 4, 5ac, 6, 7abd, 8, Ext. #18, 25

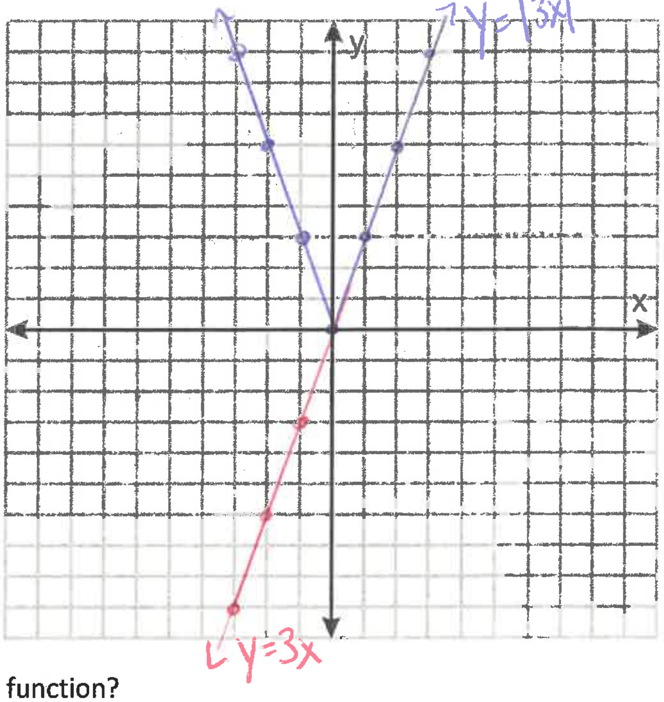
7.2 Absolute Value Functions (Day 1)

Concept: Sketch the graph of a linear absolute value function given its equation. Determine the intercepts, domain and range and piecewise function given its graph or equation.

Example #1/a) Graph $y = 3x$ and $y = |3x|$ on the same graph (using different colours) by using the following tables

x	y = 3x
-3	-9
-2	-6
-1	-3
0	0
1	3
2	6
3	9

x	$y = \|3x\|$
-3	9
-2	6
-1	3
0	0
1	3
2	6
3	9



b) Describe the shape of the graph of the absolute value of a linear function?

a "V"

c) What do you notice about the values of y for the absolute value function?

All positive values

d) What is the domain and range for each function:

$y = 3x$ $D=\{X \in \mathbb{R}\}$
$R=\{y \in \mathbb{R}\}$

$y = |3x|$ $D=\{x | x \in \mathbb{R}\}$ or $(-\infty, \infty)$
$R=\{y | y \geq 0, y \in \mathbb{R}\}$ or $[0, \infty)$

e) What are the invariant point(s)?

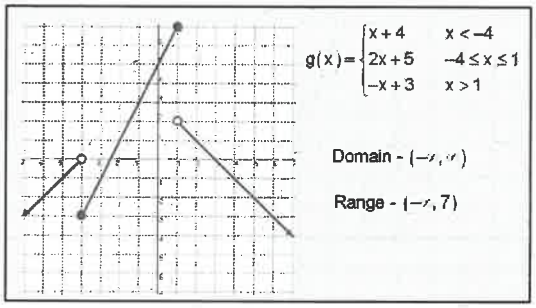
The x-intercept. and All points where $x \geq 0$

f) Is y=3x a function? Is $y = |3x|$ a function? Why or why not?

They are both functions because the domain does not repeat.

Piecewise Functions

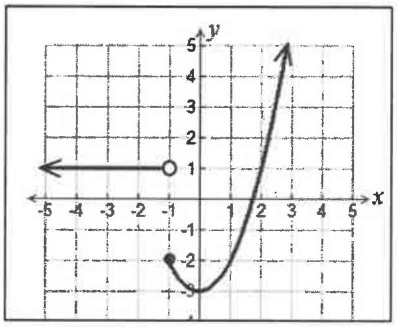
A piecewise function is a function that, when graphed, appears to have several distinct and different "pieces" of graphs within the same function. These "pieces" may be joined together or broken apart in separate pieces. The entire graph or function is found by, in turn, describing both the equation/function and domain of each individual piece and writing those descriptions collectivity as shown in the example below:



$$g(x)=\begin{cases} x+4 & x<-4 \\ 2x+5 & -4\le x\le 1 \\ -x+3 & x>1 \end{cases}$$

Domain - $(-\infty,\infty)$

Range - $(-\infty,7)$



In general, you can express an absolute value function, $y=|f(x)|$, as the piecewise function:

$$y=\begin{cases} f(x) \text{ if } f(x)\ge 0 \\ -f(x) \text{ if } f(x)<0 \end{cases}$$

Note: When sketching the graph $y=|f(x)|$ (line or parabola), sketch $y=f(x)$ first. Any part of the graph which lies ***below*** the x-axis must be reflected about the x-axis for the graph of $y=|f(x)|$

Example #2: Given $y=|2x-3|$

a) determine the y-intercept and the x-intercept

y-int
$y=|2(0)-3|$ $y=3$

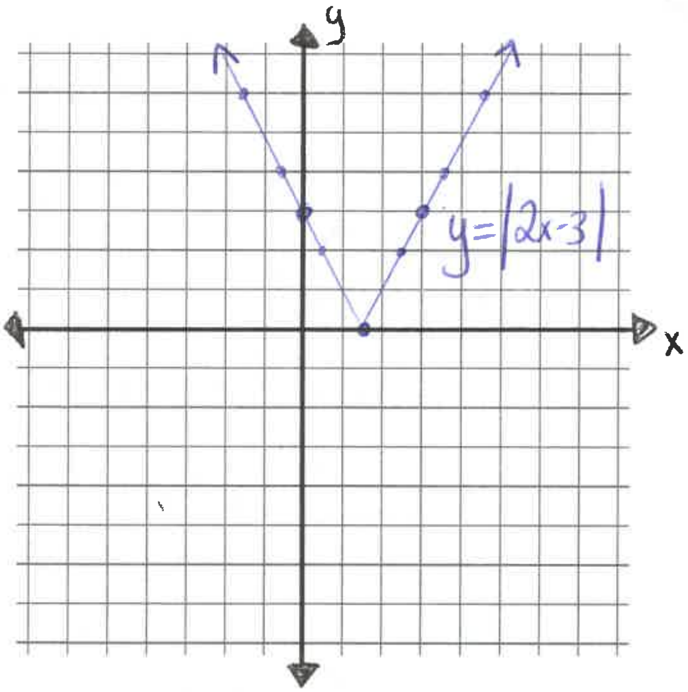
x-int
$0=|2x-3|$
$\frac{3}{2}=\frac{2x}{2}$
$x=1.5$

b) sketch the graph

c) state the domain and range

d) express as a piecewise function

e) identify the invariant points

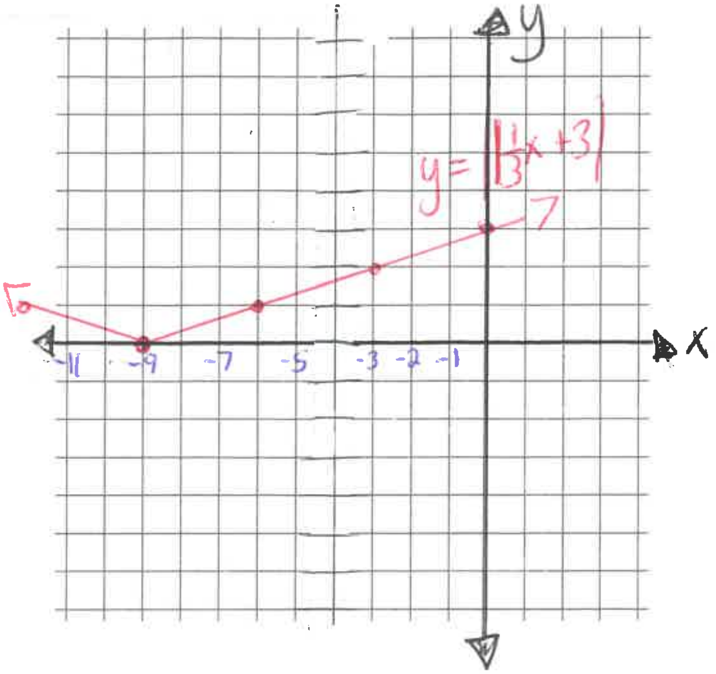


c) $D=(-\infty,\infty)$ $R=[0,\infty)$

d) $y=\begin{cases} 2x-3 \text{ if } x\ge 3/2 \\ -(2x-3) \text{ if } x<3/2 \end{cases}$

e) All points where $x\ge 3/2$

Example#3: Given $y=\left|\frac{1}{3}x+3\right|$



a) determine the y-intercept and the x-intercepts

y-int

=

b) sketch the graph

c) state the domain and range

d) express as a piecewise function

e) identify the invariant points

a) y-int

$y=\left|\frac{1}{3}(0)+3\right|$

$y=3$

x-int

$0\overset{-3}{=}\frac{1}{3}x+3^{-3}$

$-3^{(3)}=\frac{1}{3}x^{(3)}$

$-9=x$

c) $D=(-\infty,\infty)$ $R=[0,\infty)$

d) $y=\begin{cases}\frac{1}{3}x+3 & \text{if } x\geq -9\\ -(\frac{1}{3}x+3) & \text{if } x<-9\end{cases}$

e) All points where $x\geq-9$

7.2 Day 1 Assignment Pg375 #1a, 2, 3, 4, 5, 6bc, 9bc, 11ab, 16, 18 Ext.#20

7.2 Absolute Value Functions (Quadratic Functions) (Day 2)

Concept: Sketch the graph of a quadratic absolute value function given its equation. Determine the intercepts, domain and range and piecewise function given its graph or equation.

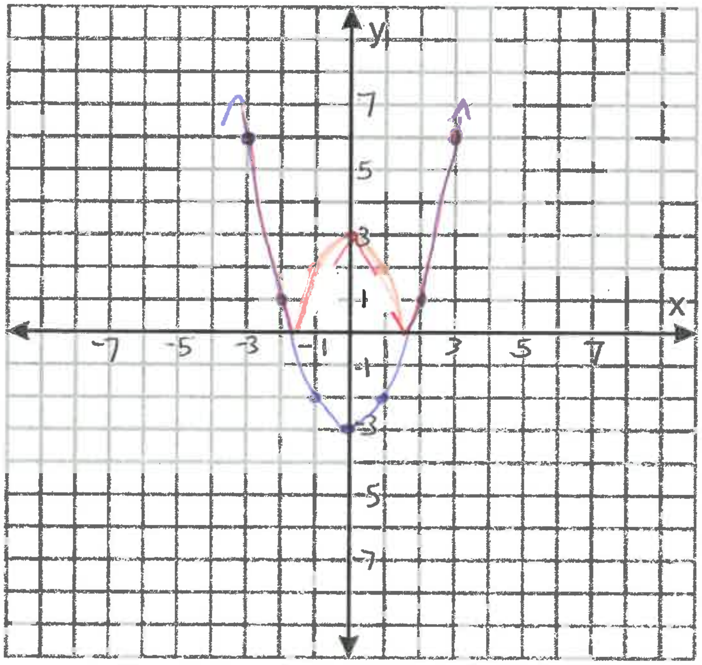
Example #1) Complete the table of values if $f(x) = x^2 - 3$ and $h(x) = | x^2 - 3 |$. Then graph.

$f(x) = x^2 - 3$

x	$f(x)$
-3	6
-2	1
-1	-2
0	-3
1	-2
2	1
3	6

$h(x) = |x^2 - 3|$

x	$h(x)$
-3	6
-2	1
-1	2
0	3
1	2
2	1
3	6



a)What domain of f(x)? Range of f(x)?

$D = (-\infty, \infty)$ $R = [-3, \infty)$

or or

$D = \{x | x \in \mathbb{R}\}$ $R = \{y | y \geq -3, y \in \mathbb{R}\}$

b)What is the domain of h(x)? Range of h(x)?

$D = (-\infty, \infty)$ $R = [0, \infty)$

or

$D = \{x | x \in \mathbb{R}\}$ $R = \{y | y \geq 0, y \in \mathbb{R}\}$

c)What is/are the invariant point(s)? Note: Need x-intercepts

All point where $x \leq -\sqrt{3}$ and $x \geq \sqrt{3}$

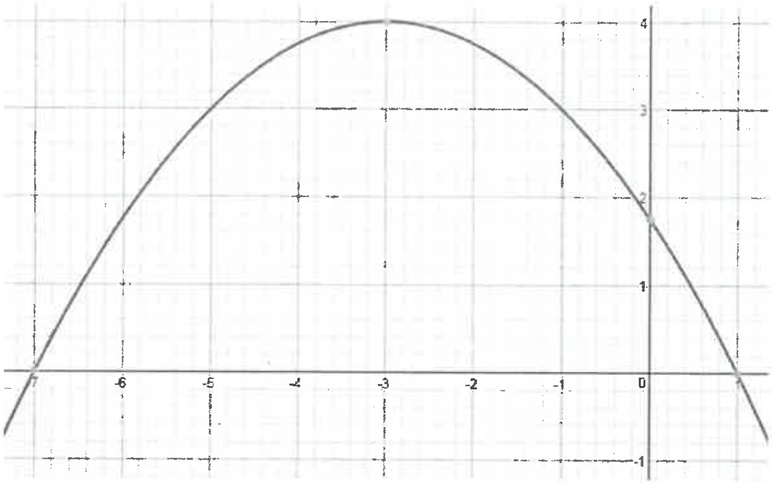
x-int

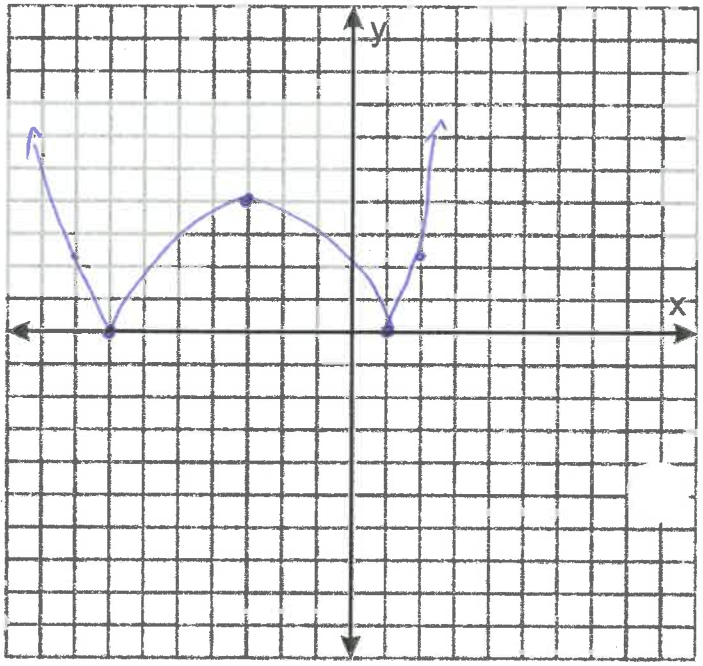
$0 = x^2 - 3$

$\sqrt{3} = \sqrt{x^2}$

$\pm\sqrt{3} = x$

Example #2) Given the graph $f(x) = -\frac{1}{4}(x+3)^2 + 4$ graph $h(x) = |f(x)|$





$x = -8$

$y = -\frac{1}{4}(-8+3)^2 + 4$

$y = -\frac{1}{4}(-5)^2 + 4$

$y = -\frac{1}{4}(25) + 4$

$y =$

$x = 2$

$y = |-\frac{1}{4}(2+3)^2 + 4|$

$y = -\frac{1}{4}(5)^2 + 4$

$y = -\frac{25}{4} + 4$

$y = |-2.25|$ $y = 2.25$

$-2>x>4$

Example #3) : Given $f(x)=|-x^2+2x+8|$

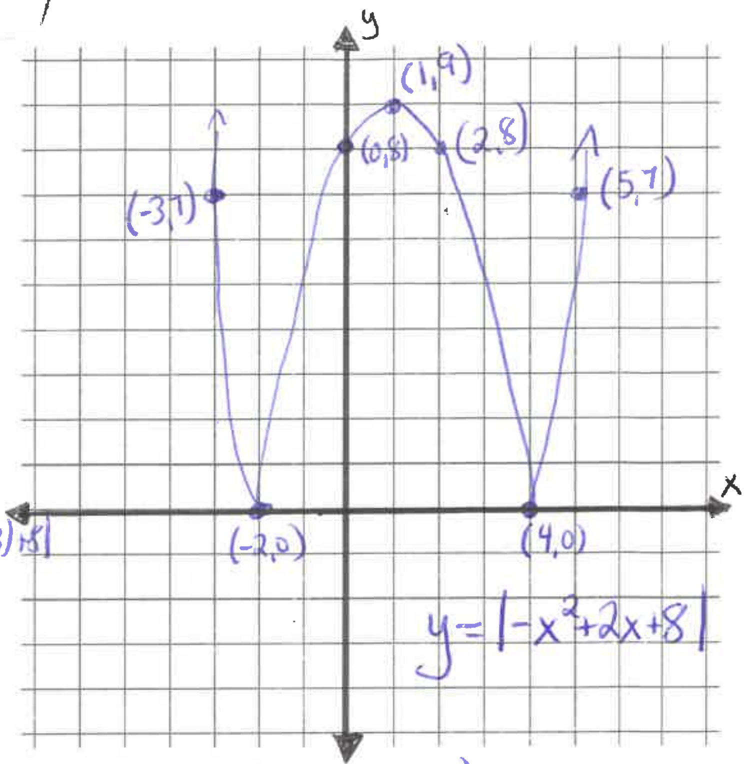
a) Sketch f(x) (Determine the y-intercept, x-intercept(s), vertex) Note: you may need points in each section of graph

~~b) Sketch the graph (find the vertex)~~

b c) State the domain and range

C d) express as a piecewise function

d e) identify the invariant points



a) $\boxed{y\text{-int} = 8}$

x-int

$0=-x^2+2x+8$

$0=-1(x^2-2x-8)$

$0=-1(x-4)(x+2)$

$\boxed{x=4 \quad x=-2}$

when $x=-3$

$y=|-(-3)^2+2(-3)+8|$

$y=|-9-6+8|$

$y=|-15+8|$

$y=|-7|$

$y=7$

$(-3,7)$

Vertex:

$p=\frac{-b}{2a}$

$p=\frac{-2}{2(-1)}$

$p=1$

$q=-(1)^2+2(1)+8$

$q=-1+2+8$

$q=-1+10$

$q=9$

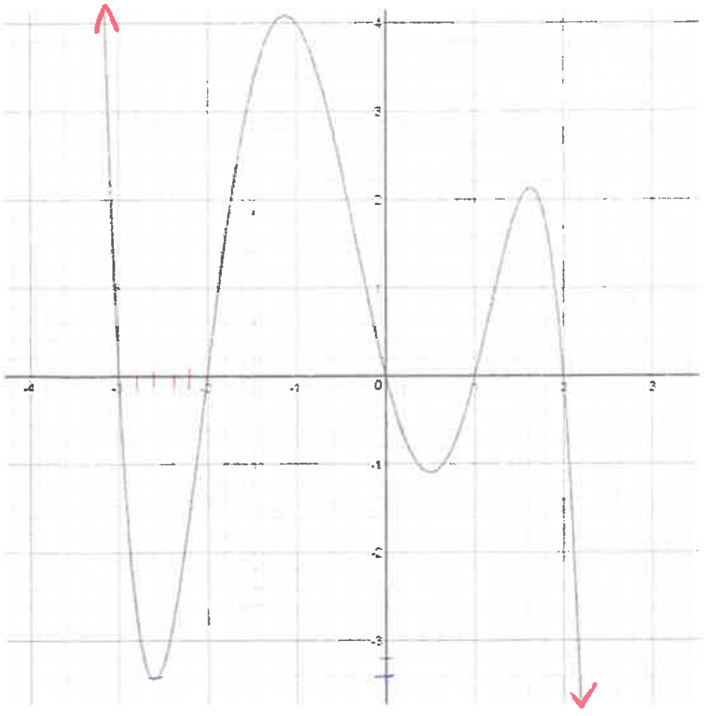
$\boxed{\text{Vertex }(1,9)}$

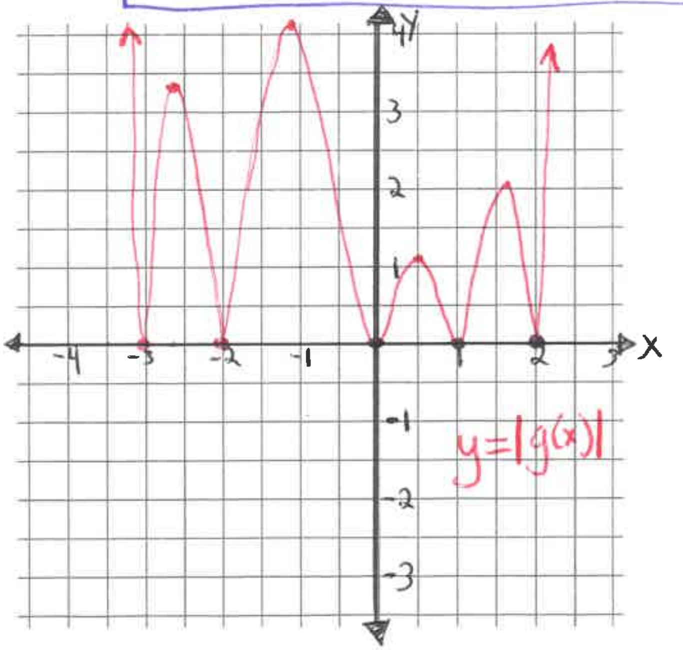
b) $D=(-\infty,\infty)$ $R=[0,\infty)$

c) $y=\begin{cases}-x^2+2x+8, & -2\le x\le 4\\ -(-x^2+2x+8), & x<-2 \text{ or } x>4\end{cases}$

d) All ordered pairs when $-2\le x\le 4$

Extensions Question: Given the following graph of g(x), sketch $y=|g(x)|$





7.2 Day 2 Assignment: **Page 375 #8 ace, 10, 13, 19 and Ext. 21(AP STYLE QUESTION)**

7.3 Absolute Value Equations

Concept: Solve Absolute value linear and quadratic equations.

Steps for solving absolute value equations

1. Isolate the absolute value expression first
2. The expression inside the absolute value symbols could be ***positive*** or it could be ***negative.*** Write 2 equations. Solve both.
3. Check your answers – there may be extraneous roots.
4. Write your answer in solution brackets.

Example #1): Solve algebraically: $|x-3|=7$

① Absolute value expression is already isolated

② Write both scenarios

$x-3\overset{+3}{=}7^{+3}$ or $\dfrac{-(x-3)}{-1}=\dfrac{7}{-1}$

$x=10$

$x-3\overset{+3}{=}-7^{+3}$

$x=-4$

③ Check for extraneous roots (Note: always substitute into original s function)

When $x=10$

$|10-3|=7$

$|7|=7$

$7=7$ ✓

when $x=4$

$|-4-3|=7$

$|-7|=7$

$7=7$ ✓

④ Solution

$x=\{10,-4\}$

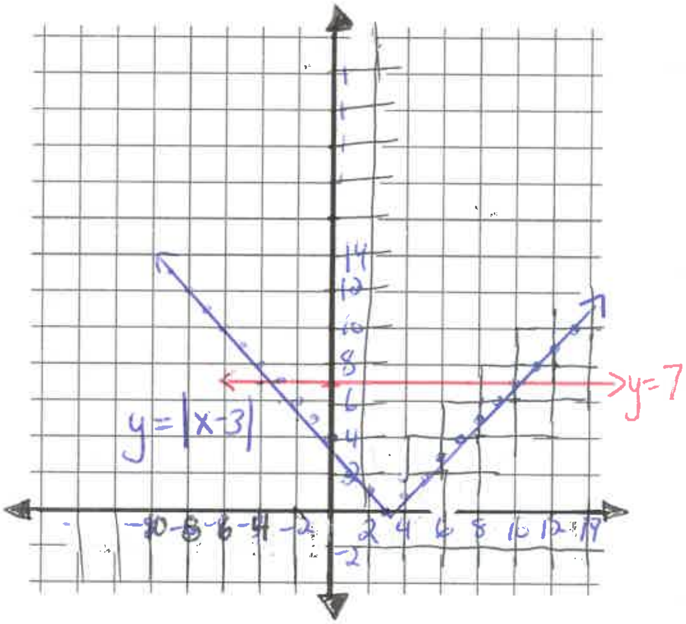
Solve $|x-3|=7$ by graphing.

Let f(x) = $|x-3|$ and g(x) = 7. Where do they intersect?

Graph f(x) and g(x)

The two functions intersect at

$x=\{10,-4\}$



Have students cross off. Assignment is on the last page.

~~Note: Example~~ #2,3,4,5 Complete on looseleaf.

7.3 ~~Assignment: Page 389 #2a, 4ac, 5ace, 6abe 14,~~ 15 and 20, 23, 24

Example #2): Solve $|x+5| = 4x-1$

$\overset{-x}{x}+5=4\overset{-x}{x}-1$ or $-(x+5)=4x-1$

$\overset{+1}{5}=3x-1^{+1}$

$\frac{6}{3}=\frac{3x}{3}$

$2=x$

$-\overset{+x}{x}-5=4\overset{+x}{x}-1$

$-\overset{+1}{5}=5x-1^{+1}$

$\frac{-4}{5}=\frac{5x}{5}$

$-\frac{4}{5}=x$

check when $x=2$

$|2+5|=4(2)-1$

$|7|=8-1$

$7=7$ ✓

check when $x=-4/5$

$\left|-4/5+\frac{5}{1}\frac{\cdot 5}{\cdot 5}\right|=4(-4/5)-1$

$\left|\frac{-4}{5}+\frac{25}{5}\right|=\frac{-16}{5}-\frac{1\cdot 5}{1\cdot 5}$

$\left|\frac{24}{5}\right|=\frac{-16}{5}-\frac{5}{5}$

$\frac{24}{5}\neq\frac{-21}{5}$ Not true $\therefore x=-\frac{4}{5}$ is an extraneous root

☆Show student graphs on DESMOS and how the solution is when they cross.

$f(x)=|x+5|$

$g(x)=4x-1$

$x=\{2\}$

Example #3): Solve a) $|2x+5| = x+1$

$2\overset{-x}{x}+5=\overset{-x}{x}+1$ or $-(2x+5)=x+1$

$1x+\overset{-5}{5}=1^{-5}$

$x=-4$

$-2\overset{-x}{x}-5=\overset{-x}{x}+1$

$-3x-\overset{+5}{5}=1^{+5}$

$\frac{-3x}{-3}=\frac{6}{-3}$

$x=-2$

Check: $x=-4$

$|2(-4)+5|=-4+1$

$|-8+5|=-3$

$|-3|=-3$

$3\neq-3$

when $x=-2$

$|2(-2)+5|=-2+1$

$|-4+5|=-2+1$

$|1|=-1$

$1\neq-1$

$\therefore$ No solutions

$x=\{\ \}$ or $x=\emptyset$

b) $|4x-5|+\overset{-9}{9}=2^{-9}$

$\boxed{\text{Step 1}}$ → Isolate the absolute value expression

$|4x-5|=-7$

An absolute value expression cannot equal a negative # $\therefore$ No solution because it is the distance from zero which is always positive

$x=\{\ \}$

"Show graphically solving on Desmos" ← To help explain why there are extraneous roots

Example #4) Solve $3x+18=2|x^2+6x|$

$\boxed{\text{Step 1}}$ → Isolate the absolute value expression

$$\frac{3x+18}{2}=\frac{2|x^2+6x|}{2}$$

$$\frac{3}{2}x+9=|x^2+6x|$$

$\boxed{\text{Step 2}}$ → The expression inside the absolute value symbols could be positive or negative write both equations and solve.

$$\frac{3}{2}x^{-\frac{3}{2}x}+9^{-9}=x^2+6x^{-\frac{3}{2}x\;-9}$$

$$0=x^2+\frac{6\cdot 2}{1\cdot 2}x-\frac{3}{2}x-9$$

$$0=x^2+\frac{12}{2}x-\frac{3}{2}x-9$$

$$^{(2)}0={}^{(2)}x^2+{}^{(2)}\frac{9}{2}x-9^{(2)}$$

$$0=2x^2+9x-18$$

$$0=(2x-3)(x+6)$$

$$2x-3^{+3}=0^{+3}\qquad x+6^{-6}=0^{-6}$$

$$\frac{2x}{2}=\frac{3}{2}\qquad x=-6$$

$$x=\frac{3}{2}$$

or

$$\frac{3}{2}x+9=\overset{\curvearrowright}{-(x^2+6x)}$$

$$\frac{3}{2}x^{-\frac{3}{2}x}+9^{-9}=-x^2-6x^{-\frac{3}{2}x\;-9}$$

$$0=-x^2-\frac{6\cdot 2}{1\cdot 2}x-\frac{3}{2}x-9$$

$$0=-x^2-\frac{12x}{2}-\frac{3}{2}x-9$$

$$^{(2)}0={}^{(2)}-x^2-{}^{(2)}\frac{15}{2}x-9^{(2)}$$

$$0=-2x^2-15x-18$$

$$0=-1(2x^2+15x+18)$$

$$0=-1(2x+3)(x+6)$$

$$2x+3^{-3}=0^{-3}\qquad x+6^{-6}=0^{-6}$$

$$\frac{2x}{2}=-\frac{3}{2}\qquad x=-6$$

$$x=-\frac{3}{2}$$

$\boxed{\text{Step 3}}$ - Check your answers there may be extraneous roots.

$\boxed{\text{Option 1}}$

An extraneous root will occur in this case if the x-value you substitute in makes the left side of the expression negative, because absolute value can never equal a negati
#

$$\underbrace{\frac{3}{2}x+9}=|x^2+6x|$$

If this becomes a negative value the solution is extraneous

See next page. →

Example #4 cont.....

Option 2 Substitute x-value into the original equation see if the left side of the equal sign equals the right

check when $x = \frac{3}{2}$

$$3\left(\frac{3}{2}\right)+18 = 2\left|\left(\frac{3}{2}\right)^2 + \overset{3}{\cancel{6}}\left(\frac{3}{\cancel{2}_1}\right)\right|$$

$$\frac{9}{2} + \frac{18\cdot 2}{1\cdot 2} = 2\left|\frac{9}{4} + \frac{9\cdot 4}{1\cdot 4}\right|$$

$$\frac{9}{2} + \frac{36}{2} = 2\left|\frac{9}{4} + \frac{36}{4}\right|$$

$$\frac{45}{2} = \overset{1}{\cancel{2}}\left(\frac{45}{\cancel{4}_2}\right)$$

$$\frac{45}{2} = \frac{45}{2} \checkmark$$

∴ $x = \frac{3}{2}$ is a solution

check when $x = -6$

$$3(-6)+18 = 2\left|(-6)^2+6(-6)\right|$$

$$-18+18 = 2|36-36|$$

$$0 = 2|0|$$

$$0 = 0 \checkmark$$

∴ $x = -6$ is a solution

Check when $x = -\frac{3}{2}$

$$3\left(-\frac{3}{2}\right)+18 = 2\left|\left(-\frac{3}{2}\right)^2 + \overset{3}{\cancel{6}}\left(\frac{-3}{\cancel{2}_1}\right)\right|$$

$$-\frac{9}{2} + \frac{18\cdot 2}{1\cdot 2} = 2\left|\frac{9}{4} - \frac{9\cdot 4}{1\cdot 4}\right|$$

$$-\frac{9}{2} + \frac{36}{2} = 2\left|\frac{9}{4} - \frac{36}{4}\right|$$

$$\frac{27}{2} = 2\left|-\frac{27}{4}\right|$$

$$\frac{27}{2} = \overset{1}{\cancel{2}}\left(\frac{27}{\cancel{4}_2}\right)$$

$$\frac{27}{2} = \frac{27}{2} \checkmark$$

The solutions are

$x = -6, \frac{3}{2}$ and $-\frac{3}{2}$

Example #5) Solve $|x^2-2x|=1$

Step1 : Isolate the absolute value expression

DONE

Step2 : The expression inside the absolute value symbols could be positive or negative write both equations and solve.

$x^2-2x\overset{-1}{=}1^{-1}$ or $\overset{\curvearrowright}{-(x^2-2x)}=1$

$x^2-2x-1=0$

$(x\quad)(x\quad)=0$

Not factorable use quadratic formula

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$$

$$x=\frac{2\pm\sqrt{4+4}}{2}$$

$$x=\frac{2\pm\sqrt{8}}{2}$$

★leave answer in exact simplest form★

$$x=\frac{2\pm\sqrt{\textcircled{4}\cdot 2}}{2}$$

$$x=\frac{2^{\div 2}\pm 2^{\div 2}\sqrt{2}}{2^{\div 2}}$$

$$x=1\pm\sqrt{2}$$

$-x^2+2x\overset{-1}{=}1^{-1}$

$-x^2+2x-1=0$

$-1(x^2-2x+1)=0$

$-1(x-1)(x-1)=0$

$x-1^{+1}=0^{+1}$

$x=1$

The solutions are

$$x=\{1\pm\sqrt{2},1\}$$

Step3 : Check your answers.

For pre-ap students Option: Show students how to check using graphing calculator or Desmos App

Graph $y=|x^2-2x|$

and

$y=1$

Where the cross are your solutions.

7.3 Assignment Pg 389 #2a, 4ac, 5ace, 6abe, 14, 15 and 20