Pre- AP Pre-Calculus 20

## 5.1 Radicals

$$r\sqrt[n]{x}$$

"r" is the coefficient

"n" is the index

"x" is the radicand

**Perfect Squares:**

If a variable has an exponent that is a multiple of 2 it is a perfect square.

- 1, 4,
- $x^2$, $x^4$, ← because

$x \cdot x = x^2$ $\quad$ $x^2 \cdot x^2 = x^4$

**Perfect Cubes**

- 1, 8,
- $x^3$, $x^6$,

If a variable has and exponent that is a multiple of 3 it is a perfect cube

### Example #1- Changing Mixed Radicals to Entire Radicals (No Calc)

1) $7\sqrt{2}$

$= \sqrt{7^2 \cdot 2}$

$= \sqrt{49 \cdot 2}$

$= \sqrt{98}$

2) $a^4\sqrt{a}$

$= \sqrt{(a^4)^2 \cdot a}$

$= \sqrt{a^8 \cdot a^1}$

$= \sqrt{a^9}$

3) $-5b\sqrt[3]{3b^2}$

$= \sqrt{(-5b)^3 \cdot 3b^2}$

$= \sqrt{-125b^3 \cdot 3b^2}$

$= \sqrt{-375b^5}$ or $-\sqrt{375b^5}$

### Radicals in Simplest Form are Mixed Radicals such that:

- the radicand contains no factor that is a perfect square, cube, etc. ( Depending on the index)
- there is no radical in the denominator of a fraction.

*Example #2:* Write as a mixed radical in simplest form. (No Calc)

1) $\sqrt{52}$

$= \sqrt{4 \cdot 13}$

$= 2\sqrt{13}$

2) $\sqrt[3]{48c^4}$

$= \sqrt[3]{8 \cdot 6 \cdot c^3 \cdot c}$

$= 2c\sqrt[3]{6c}$

3) $5\sqrt{72}$

$= 5\sqrt{9 \cdot 8}$

$= 5 \cdot 3\sqrt{8}$

$= 15\sqrt{4 \cdot 2}$

$= 15 \cdot 2\sqrt{2}$ → $= 30\sqrt{2}$

4) $\sqrt{48y^5}$

$= \sqrt{16 \cdot 3y^4 \cdot y}$

$= 4y^2\sqrt{3y}$

5) $\sqrt{27x^4y^{12}}$

$= \sqrt{\boxed{9} \cdot 3 \, \boxed{\overset{2}{x} \cdot \overset{2}{x}} \, \boxed{\overset{6}{y} \cdot y^6}}$

$= 3x^2y^6\sqrt{3}$

6) $\sqrt[4]{m^7}$

$= \sqrt[4]{m^4 \cdot m^3}$

$= m\sqrt[4]{m^3}$

**Example #3:** Order the following five numbers in order from least to greatest without using a calculator

Change all to entire radicals

a) $4(13)^{\frac{1}{2}}$, $8\sqrt{3}$, $14$, $\sqrt{202}$, $10\sqrt{2}$

$4\sqrt{13}$

$\sqrt{4^2 \cdot 13}$

$= \sqrt{208}$

$\sqrt{64 \cdot 3}$

$\sqrt{192}$

$\sqrt{14^2}$

$= \sqrt{196}$

$\to \sqrt{200}$

$8\sqrt{3},\ 14,\ 10\sqrt{2},\ \sqrt{202},\ 4(13)^{\frac{1}{2}}$

**Adding/Subtracting Radicals** – simplify then combine like terms (terms that have the same index and the same radicand)

*Example #4:* 1) $\sqrt{5} - 6\sqrt{5}$

$= 1\sqrt{5} - 6\sqrt{5}$

$= 1 - 6(\sqrt{5})$

$= -5\sqrt{5}$

2) $\boxed{3\sqrt{5}} + 1.2 + 2\sqrt{2} \boxed{-8\sqrt{5}} - 6\sqrt{2} + 5$

$= 3\sqrt{5} - 8\sqrt{5} + 2\sqrt{2} - 6\sqrt{2} + 1.2 + 5$

$= -5\sqrt{5} - 4\sqrt{2} + 6.2$

3) $-\sqrt{27} + 3\sqrt{5} - \sqrt{80} - 2\sqrt{12}$

① simplify each radical

$= -\sqrt{9 \cdot 3} + 3\sqrt{5} - \sqrt{16 \cdot 5} - 2\sqrt{4 \cdot 3}$

$= -3\sqrt{3} + 3\sqrt{5} - 4\sqrt{5} - 2 \cdot 2\sqrt{3}$

$= -3\sqrt{3} - 4\sqrt{3} + 3\sqrt{5} - 4\sqrt{5}$

$= -7\sqrt{3} - 1\sqrt{5}$

4) $\sqrt{4c} - 4\sqrt{9c}$ $(c \geq 0)$ ← restriction

$= 2\sqrt{c} - 4 \cdot 3\sqrt{c}$

$= 2\sqrt{c} - 12\sqrt{c}$

$= -10\sqrt{c}$

Restrictions – if a radical represents a real number and has an ***even index***, the radicand must be positive or 0.

Example #5: Identify the restrictions on the values for the variables.

a) $-5\sqrt{2a}$ b) $2a\sqrt{x-4}$ c) $\sqrt[3]{8r}$

restrictions $a \geq 0$

$\overset{+4}{x-4} \geq \overset{+4}{0}$

$\{x \mid x \geq 4, x \in \mathbb{R}\}$

$r = \{r \in \mathbb{R}\}$ because the index is odd.

$\frac{2a}{2} \geq \frac{0}{2}$

$\{a \mid a \geq 0, a \in \mathbb{R}\}$

Note: If the index is ***odd***, the radicand can be any real number.

Assignment:

Page 278 #1,2,3,4,6 (No calc), 8, 9, 10,12, 19(~~Hint: Pythagorean Theor~~em)

## 5.2 Multiplying and Dividing Radical Expressions (Day1 )

Concept : To multiply radical expressions with one or more terms and leave answers in simplest form. Also state restrictions.

Multiplying Radical Expressions

- multiply the coefficients and multiply the radicands (if they have the ***same index***)
- radicals should be simplified before multiplying
- answer in simplest form
- state restrictions for variables (if index is even, the radicand must be positive )

*Examples* – Multiply and simplify. State any restrictions on the values of the variables.

1) $(5\sqrt{2x})(3\sqrt{5x}) =$

Restrictions, $x \geq 0$

$= 5(3)(\sqrt{2x})(\sqrt{5x}$

$= 15\sqrt{10x^2}$

$\boxed{= 15x\sqrt{10}}$

2) $(3\sqrt{6})(-4\sqrt{2}) = (-4)(3)(\sqrt{6})(\sqrt{2}$

$= -12\sqrt{12}$

$= -12\sqrt{4\cdot 3}$

$= -12\cdot 2\sqrt{3} = \boxed{-24\sqrt{3}}$

← ★Distribute★ →

3) $7\sqrt{3}(2\sqrt{3} - 5\sqrt{7}) =$

$= 14\sqrt{9} - 35\sqrt{21}$

$= 14\cdot 3 - 35\sqrt{21}$

$\boxed{= 42 - 35\sqrt{21}}$

4) $5\sqrt[3]{9}(4\sqrt[3]{2} + 9\sqrt[3]{3}) =$

$= 5(4)(\sqrt[3]{9})(\sqrt[3]{2}) + 5(9)(\sqrt[3]{9})(\sqrt[3]{3})$

$= 20\sqrt[3]{18} + 45\sqrt[3]{27}$ Perfect cube

$= 20\sqrt[3]{18} + 45\cdot 3$

$\boxed{= 20\sqrt[3]{18} + 135}$

★FOIL★ or double distribute

5) $(4\sqrt{2} + 2\sqrt{3})(5\sqrt{2} - 6\sqrt{3}) =$

Like radicals

$= 20\sqrt{4} - 24\sqrt{6} + 10\sqrt{6} - 12\sqrt{9}$

$= 20\cdot 2 - 14\sqrt{6} - 12\cdot 3$

$= 40 - 36 - 14\sqrt{6}$

$= \boxed{4 - 14\sqrt{6}}$

★Distribute★

6) $9\sqrt[3]{2w}\left(\sqrt[3]{4w} + 7\sqrt[3]{28}\right) =$

$= 9\sqrt[3]{8w^2} + 63\sqrt[3]{56w}$

Perfect cube

$= 9\cdot 2\sqrt[3]{w^2} + 63\sqrt[3]{8\cdot 7w}$

$= 18\sqrt[3]{w^2} + 63\cdot 2\sqrt[3]{7w}$

$= \boxed{18\sqrt[3]{w^2} + 126\sqrt[3]{7w}}$ ; $w \in R$

Note: No restrictions because "w" can be negative since the index is odd.

7) $\left(2\sqrt{5}-3\sqrt{2}\right)^2 =$

*FOIL*

$= (2\sqrt{5}-3\sqrt{2})(2\sqrt{5}-3\sqrt{2})$

Like radicals

$= 4\sqrt{25} \; -6\sqrt{10} \; -6\sqrt{10} \; +9\sqrt{4}$

$= 4\cdot 5 - 12\sqrt{10} + 9\cdot 2$

$= 20+18-12\sqrt{10}$

$= \boxed{38-12\sqrt{10}}$

8) $\left(5\sqrt{2}-6\sqrt{3}\right)\left(5\sqrt{2}+6\sqrt{3}\right) =$

$= 25\sqrt{4}+30\sqrt{6}-30\sqrt{6}-36\sqrt{9}$

$= 25\cdot 2 + 0\sqrt{6} - 36\cdot 3$

$= 50 - 108$

$= \boxed{-58}$

* what do you notice about this question

**5.2 DAY 1 Assignment: Page 289 #1abcd, 2ac, 3ab, 4abc, 5**

## 5.2 (con't) Dividing Radical Expressions ( Day 2)

Concept: To divide radicals and rationalize a square root monomial and a square root binomial denominator using the conjugate

- Divide the coefficients and divide the radicands (if they have the ***same index***)
- A rational in simplest form does ***not*** have a radical in the denominator. If necessary, ***rationalize*** the denominator:

  > needs to have a rational denominat

  ***a) monomial denominator*** – multiply the numerator and denominator by an expression that produces a rational number in the denominator

  ***b) binomial denominator*** – multiply the numerator and denominator by the conjugate of the denominator. Conjugate: *(a + b)* and *(a – b)* are conjugates

*Examples* – Find the conjugate of the following

1) $5\sqrt{2}-\sqrt{3}$ → $(5\sqrt{2}+\sqrt{3})$

2) $-2\sqrt{6}+5\sqrt{7}$ → $(-2\sqrt{6}-5\sqrt{7})$

*Examples* – Divide and simplify

1) $\dfrac{\sqrt{24x^3}}{\sqrt{3x}}$; x ≥ 0

$= \sqrt{8x^2}$

$= \sqrt{4\cdot 2\, x\cdot x}$

$\boxed{= 2x\sqrt{2}}$

2) $\dfrac{12\sqrt{6}}{15\sqrt{3}}$

$= \dfrac{12^{\div 3}\sqrt{2}}{15^{\div 3}}$

$= \dfrac{4\sqrt{2}}{5}$ or $\dfrac{4}{5}\sqrt{2}$

3) $\frac{8\sqrt{5}}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

Note: 5 cannot be divided by 2. Therefore you must rationalize the denominator

$= \frac{2\sqrt{10}}{\sqrt{4}}$

$= \frac{2\sqrt{10}}{\cancel{2}}$

$\boxed{= \sqrt{10}}$

4) $\frac{7\sqrt{3}}{3\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$

$= \frac{7\sqrt{3x}}{3\sqrt{x^2}}$

$\boxed{= \frac{7\sqrt{3x}}{3x}}$

Restriction:

$x > 0$

↑ Can't be equal to as it would make the denominator zero which is undefined.

5) $\frac{6}{3+\sqrt{2}} \cdot \frac{(3-\sqrt{2})}{(3-\sqrt{2})}$ ★ multiply by the conjugate to rationalize the denominator★

★FOIL★

$= \frac{18-6\sqrt{2}}{9-3\sqrt{2}+3\sqrt{2}\cancel{+}-\sqrt{4}}$

$= \frac{18-6\sqrt{2}}{9\cancel{\pm}2}$

$= \boxed{\frac{18-6\sqrt{2}}{\cancel{8}7}}$

6) $\frac{5\sqrt{2}}{3\sqrt{2}-\sqrt{3}} \cdot \frac{(3\sqrt{2}+\sqrt{3})}{(3\sqrt{2}+\sqrt{3})}$ ★FOIL★

$= \frac{15\sqrt{4}+5\sqrt{6}}{9\sqrt{4}+3\sqrt{6}-3\sqrt{6}-\sqrt{9}}$

$= \frac{15\cdot 2+5\sqrt{6}}{9\cdot 2-3}$

reduce $= \frac{30^{\div 5}+5^{\div 5}\sqrt{6}}{15^{\div 5}} = \boxed{\frac{6+\sqrt{6}}{3}}$

7) $\frac{4+\sqrt{2}}{\sqrt{3}+5\sqrt{2}} \cdot \frac{(\sqrt{3}-5\sqrt{2})}{(\sqrt{3}-5\sqrt{2})}$

$= \frac{4\sqrt{3}-20\sqrt{2}+\sqrt{6}-5\sqrt{4}}{\sqrt{9}-5\sqrt{6}+5\sqrt{6}-25\sqrt{4}}$

$= \frac{4\sqrt{3}-20\sqrt{2}+\sqrt{6}-10}{3-25\cdot 2}$

$= \frac{4\sqrt{3}-20\sqrt{2}+\sqrt{6}-10}{-47}$

★ To be in simplest form the negative sign must be in the numerator★

$= \frac{-(4\sqrt{3}-20\sqrt{2}+\sqrt{6}-10)}{47} \rightarrow = \boxed{\frac{-4\sqrt{3}+20\sqrt{2}-\sqrt{6}+10}{47}}$

8) $\frac{4\sqrt{11}}{y\sqrt[3]{6}} \cdot \frac{\sqrt[3]{6}}{\sqrt[3]{6}} \cdot \frac{\sqrt[3]{6}}{\sqrt[3]{6}}$

$= \frac{4\sqrt{11}(\sqrt[3]{36})}{y\sqrt[3]{6\cdot 6\cdot 6}}$

$= \frac{4^{\div 2}\sqrt{11}(\sqrt[3]{36})}{6^{\div 2}y}$

$= \boxed{\frac{2\sqrt{11}(\sqrt[3]{36})}{3y}}$

**5.2 DAY 2 Assignment: Page 290 #6abcd, 7b,8ab, 9ab, 10abd, 15,17,19,20**

## 5.3 Radical Equations (Day 1 and 2)

**Concept#23: To solve Radical equations (equations where the variable you are solving is in the radicand) and check for extraneous roots.**

**STEPS:**

1. State your restriction(s)
2. Isolate one radical (if there are two you should begin by getting the most complicated one by itself)
3. Square both sides ( if there is a binomial on one side you must square that binomial and get a trinomial!)
4. If there is still a radical in your equation, isolate it and repeat step 2 again
5. Solve for the variable
6. Verify your answers. very important!! Does it meet the restrictions? Is it extraneous?
7. Note: If none of your answers satisfy the check or restrictions there is "NO SOLUTION"

**Example#1:** a) State the restrictions on x. b ) Solve the radical

c) Verify your solution(s) and state any extraneous roots

a) Restrictions

$2x-\overset{+1}{1} \geq \overset{+1}{0}$

$\frac{2x}{2} \geq \frac{1}{2}$

$x \geq \frac{1}{2}$

b) $\overset{-5}{5}+\sqrt{2x-1}=12\ {-5}$

$(\sqrt{2x-1})^2=(7)^2$

$2x-\overset{+1}{1}=49^{+1}$

$\frac{2x}{2}=\frac{50}{2}$

$x=25$

c) Verify your solution $x=25$

$5+\sqrt{2(25)-1}=12$

$5+\sqrt{49}=12$

$5+7=12$

$12=12$ ✓

The solution is $x=25$ because it satisfies the restrictions and I verified my solution.

$x=\{25\}$

**Example #2** a) State the restrictions on m b) Solve the radical

c) Verify your solution(s) and state any extraneous roots

$m-\sqrt{2m+3}=6$

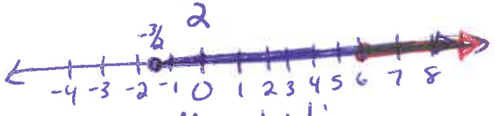
a) Restrictions

Whats under the radical must be positive, and what the radical equals also has to be positive.

$2m+\overset{-3}{3} \geq \overset{-3}{0}$ $\qquad$ $-\overset{+6}{6}+m \geq 0^{+6}$

$\frac{2m}{2} \geq \frac{-3}{2}$ $\qquad$ $m \geq 6$

$m \geq -\frac{3}{2}$



So overall restriction is $m \geq 6$

b) $\overset{-m}{m}-\sqrt{2m+3}=6\ {-m}$

$\frac{-\sqrt{2m+3}}{-1}=\frac{6-m}{-1}$

$(\sqrt{2m+3})^2=(-6+m)^2$

$2m+3=(-6+m)(-6+m)$

$2\overset{-2m}{m}+\overset{-3}{3}=3\overset{-3}{6}-12\overset{-2m}{m}+m^2$

$0=m^2-14m+33$

$0=(m-11)(m-3)$

$m-\overset{+11}{11}=0^{+11}$ $\qquad$ $m-\overset{+3}{3}=0^{+3}$

$m=11$ $\qquad$ $m=3$

c) Verify your solutions

$m=11$

$11-\sqrt{2(11)+3}=6$

$11-\sqrt{22+3}=6$

$11-\sqrt{25}=6$

$11-5=6$

$6=6$ ✓

$m=3$

$3-\sqrt{2(3)+3}=6$

$3-\sqrt{9}=6$

$3-3=6$

$0 \neq 6$ "False"

$\therefore m=3$ is an extraneous root.

$m=\{11\}$

#6b) Pg 300 $\overset{+7}{-7} - 4\sqrt{2x-1} = \overset{+7}{17}; x \ge \frac{1}{2}$

$$\frac{-4\sqrt{2x-1}}{-4} = \frac{24}{-4}$$

$$\sqrt{2x-1} = -6$$

"No solution"

A radical cannot equal a negative #.

**Example #3:** Solve $\sqrt{x+2} \overset{-\sqrt{x+2}}{-} \sqrt{3x-5} = -1 \overset{-\sqrt{x+2}}{}$

$\boxed{\text{Step1}}$ Isolate the more complicated radical

$\boxed{\text{Step2}}$ Square both sides of the equal sign

$\boxed{\text{Step3}}$ Isolate the 2nd radical

$\boxed{\text{Step4}}$ Square both sides of the equal sign

$$\frac{-\sqrt{3x-5}}{-1} = \frac{-1-\sqrt{x+2}}{-1}$$

$$(\sqrt{3x-5})^2 = (1+\sqrt{x+2})^2$$

$$3x-5 = (1+\sqrt{x+2})(1+\sqrt{x+2})$$

$$\overset{-3x}{3x} \overset{+5}{-5} = (1 + 2\sqrt{x+2} + x+2) -3x+5$$

$$\overset{-8\ +2x}{0} = \overset{-8}{8} + 2\sqrt{x+2} - 2x \overset{+2x}{}$$

$$\frac{2x-8}{2} = \frac{2\sqrt{x+2}}{2}$$

$$(x-4)^2 = (\sqrt{x+2})^2$$

$$(x-4)(x-4) = x+2$$

$$x^2 - 8x \overset{-x}{+} \overset{-2}{16} = \overset{-x}{x} + 2 \overset{-2}{}$$

$$x^2 - 9x + 14 = 0$$

$$(x-7)(x-2) = 0$$

$$x-\overset{+7}{7} = \overset{+7}{0} \qquad x-\overset{+2}{2} = \overset{+2}{0}$$

$$x = 7 \qquad x = 2$$

Check

$x = 7$

$$\sqrt{7+2} - \sqrt{3(7)+5} = -1$$

$$\sqrt{9} - \sqrt{16} = -1$$

$$3 - 4 = -1$$

$$-1 = -1$$ ✓ True

$x = 2$

$$\sqrt{2+2} - \sqrt{3(2)-5} = -1$$

$$\sqrt{4} - \sqrt{1} = -1$$

$$2 - 1 = -1$$

$$1 \neq -1$$ × False

∴ $x=2$ is an extraneous root

$$x = \{7\}$$

**Example#4:** Solve $\overset{-5}{7}+\sqrt{3x}=\sqrt{5x+4}+5\;-5$

$(2+\sqrt{3x})^2=(\sqrt{5x+4})^2$

$(2+\sqrt{3x})(2+\sqrt{3x}) = 5x+4$

$\overset{-4}{4}+4\sqrt{3x}+\overset{-3x}{3x} = \overset{-3x}{5x}+4\;{-4}$

$\frac{4\sqrt{3x}}{4} = \frac{2x}{4}$

$(\sqrt{3x})^2=(\frac{1}{2}x)^2$

$\overset{-3x}{3x} = \frac{1}{4}x^2-3x$

$0=\frac{1}{4}x^2-3x$ ✪ Factor ✪

$0=x(\frac{1}{4}x-3)$

$x=0$ $\quad \frac{1}{4}x-\overset{+3}{3}=0^{+3}$

$(4)\frac{1}{4}x=3(4)$

$x=12$

Check $x=0$

$7+\sqrt{3(0)}=\sqrt{5(0)+4}+5$

$7=2+5$

$7=7$ ✓ True

Check $x=12$

$7+\sqrt{3(12)}=\sqrt{5(12)+4}+5$

$7+\sqrt{36}=\sqrt{64}+5$

$7+6=8+5$

$13=13$ ✓ True

The solutions are

$x=\{0,12\}$

**5.3 Day 2**

**Example #5** What is the speed, in metres per second , of a 0.4kg football that has 28.8 J of Kinetic energy? Use the kinetic energy formula, $E_k=\frac{1}{2}mv^2$ , where $E_k$ represents the kinetic energy, in joules; $m$ represents mass, in kilograms; and $v$ represents speed in metres per second.

$m=0.4Kg$

$E_K=28.2J$

$\boxed{\text{Step1}}$ Isolate the "v" variable

$(2)E_K=(2)\frac{1}{2}mv^2$

$\frac{2E_K}{m}=\frac{mv^2}{m}$

$\sqrt{\frac{2E_K}{m}}=\sqrt{v^2}$

$V=\sqrt{\frac{2E_K}{m}}$

$\boxed{\text{Step2}}$ Substitute values in

$V=\sqrt{\frac{2(28.2)}{0.4}}$

$V=\sqrt{144}$

$V=12\,m/s$

The speed of the football was 12 m/s

**Example #6/**State restrictions and solve $\sqrt{3x^2-12}=x+5$ *(Video approx. 5 mins)* *https://www.youtube.com/watch?v=rbJjp9KrA6s&t=1s&list=WL&index=2 )*

Restrictions

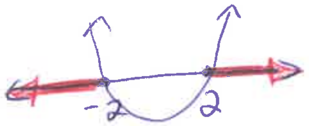
$3x^2-12 \ge 0$

$3(x^2-4) \ge 0$

$3(x-2)(x+2) \ge 0$

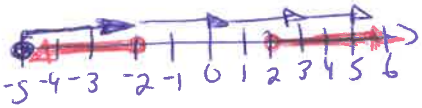
$x+5 \ge 0$ $\quad -5 \quad -5$

$x \ge -5$



$x<-2$ or $x>2$

overall restrictions



$-5 \le x < -2$ and $x \ge 2$

Solve

$$\left(\sqrt{3x^2-12}\right)^2=(x+5)^2$$

$$3x^2-12=(x+5)(x+5)$$

$$\overset{-x^2-10x-25}{3x^2-12}=\overset{-x^2\ \ -10x\ \ -25}{x^2+10x+25}$$

$$2x^2-10x-37=0$$

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(2)(-37)}}{2(2)}$$

$$x=\frac{10\pm\sqrt{100+296}}{4}$$

$$x=\frac{10\pm\sqrt{396}}{4}$$

$$x=\frac{10\pm\sqrt{4\cdot9\cdot11}}{4}$$

$$x=\frac{\overset{\div2}{10}\pm\overset{\div2}{6}\sqrt{11}}{4\div2}$$

$$x=\frac{5\pm3\sqrt{11}}{2}$$

$\boxed{\text{Check}}$ use calculator

**5.3 Assignment: Page 300 #3ab, 4ab, 5, 6a,7bc, 8d,9bc,10, 13, 14, 18**