

1. Simplify the following Radicals (8 marks – 2marks each)

$$\begin{aligned} \text{a) } \sqrt{175} \\ &= \sqrt{7 \cdot 25} \\ &= 5\sqrt{7} \end{aligned}$$

$$\begin{aligned} \text{b) } 7\sqrt[3]{32} \\ &= 7\sqrt[3]{8 \cdot 4} \\ &= 7 \cdot 2\sqrt[3]{4} \\ &= 14\sqrt[3]{4} \end{aligned}$$

$$\begin{aligned} \text{c) } 3\sqrt[3]{54x^7} \\ &= 3\sqrt[3]{27 \cdot 2 \cdot x^6 \cdot x} \\ &= 3 \cdot 3 \cdot x^2 \sqrt[3]{2x} \\ &= 9x^2\sqrt[3]{2x} \end{aligned}$$

$$\begin{aligned} \text{d) } \sqrt{108x^9y^4} \\ &= \sqrt{36 \cdot 3 \cdot x^8 \cdot x \cdot y^4} \\ &= 6x^4y^2\sqrt{3x} \end{aligned}$$

2. Change the mixed radicals to entire radicals (3 marks)

$$\begin{aligned} \text{a) } 2\sqrt{15} \\ &= \sqrt{2^2 \cdot 15} \\ &= \sqrt{4 \cdot 15} \\ &= \sqrt{60} \end{aligned}$$

$$\begin{aligned} \text{b) } 2xy^2\sqrt[3]{5x} \\ &= \sqrt[3]{2^3 x^3 y^6 5x} \\ &= \sqrt[3]{8 \cdot 5 x^4 y^6} \\ &= \sqrt[3]{40 x^4 y^6} \end{aligned}$$

3. Multiply and simplify the following radicals (6 marks – 3 marks each)

$$\begin{aligned} \text{a) } (-2\sqrt{10})(7\sqrt{2})(4\sqrt{30}) \\ &= -2 \cdot 7 \cdot 4 \sqrt{10 \cdot 2 \cdot 30} \\ &= -56\sqrt{600} \\ &= -56\sqrt{100 \cdot 6} \\ &= -56 \cdot 10\sqrt{6} \\ &= -560\sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{b) } (-2\sqrt{6xy})(3\sqrt{2x^2y})(5\sqrt{2xy^5}) \\ &= -2 \cdot 3 \cdot 5 \sqrt{24x^4y^7} \\ &= -30\sqrt{6 \cdot 4 \cdot x^4 \cdot y^6 \cdot y} \\ &= -30 \cdot 2x^2y^3\sqrt{6y} \\ &= -60x^2y^3\sqrt{6y} \end{aligned}$$

4. Add, subtract and then simplify the following radicals (4 marks – 2 marks each)

$$\begin{aligned} \text{a) } 5\sqrt{108} + 3\sqrt{75} \\ &= 5\sqrt{36 \cdot 3} + 3\sqrt{25 \cdot 3} \\ &= 5 \cdot 6\sqrt{3} + 3 \cdot 5\sqrt{3} \\ &= 30\sqrt{3} + 15\sqrt{3} \\ &= 45\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{b) } 5a\sqrt{28} - 3\sqrt{63a^2}, a \geq 0 \\ &= 5a\sqrt{4 \cdot 7} - 3a\sqrt{9 \cdot 7} \\ &= 5 \cdot 2a\sqrt{7} - 3a \cdot 3\sqrt{7} \\ &= 10a\sqrt{7} - 9a\sqrt{7} \\ &= a\sqrt{7}; a \geq 0 \end{aligned}$$

#4 Add, subtract and then simplify the following radicals (4 marks – 2 marks each)

c)  $-\frac{5}{2}\sqrt{20} - \frac{2}{5}\sqrt{125} + \frac{11}{3}\sqrt{45}$

$$= -\frac{5}{2}\sqrt{4 \cdot 5} - \frac{2}{5}\sqrt{25 \cdot 5} + \frac{11}{3}\sqrt{9 \cdot 5}$$

$$= -\frac{2 \cdot 5}{2}\sqrt{5} - \frac{2 \cdot 5}{5}\sqrt{5} + \frac{3 \cdot 11}{3}\sqrt{5}$$

$$= -5\sqrt{5} - 2\sqrt{5} + 11\sqrt{5}$$

$$= 4\sqrt{5}$$

d)  $5\sqrt[3]{80} + 3\sqrt[3]{270} - 6\sqrt[3]{10}$

$$= 5\sqrt[3]{8 \cdot 10} + 3\sqrt[3]{27 \cdot 10} - 6\sqrt[3]{10}$$

$$= 5 \cdot 2\sqrt[3]{10} + 3 \cdot 3\sqrt[3]{10} - 6\sqrt[3]{10}$$

$$= 10\sqrt[3]{10} + 9\sqrt[3]{10} - 6\sqrt[3]{10}$$

$$= 13\sqrt[3]{10}$$

5. Expand and simplify the following (5 marks)

a)  $5\sqrt{3}(4\sqrt{15} - 7\sqrt{21})$  (2 marks)

$$= 20\sqrt{45} - 35\sqrt{63}$$

$$= 20\sqrt{9 \cdot 5} - 35\sqrt{9 \cdot 7}$$

$$= 20 \cdot 3\sqrt{5} - 35 \cdot 3\sqrt{7}$$

$$= 60\sqrt{5} - 105\sqrt{7}$$

b)  $(5\sqrt{7} - 2\sqrt{5})(4\sqrt{7} + 5\sqrt{5})$  (3 marks)

$$= 20\sqrt{49} + 25\sqrt{35} - 8\sqrt{35} - 10\sqrt{25}$$

$$= 20(7) + 17\sqrt{35} - 10(5)$$

$$= 140 + 17\sqrt{35} - 50$$

$$= 90 + 17\sqrt{35}$$

6. STATE any restrictions. Divide and simplify the following radicals. Rationalize the denominator if needed. (8 marks – 2 marks each)

a)  $\frac{2\sqrt{50}}{6\sqrt{10}}$

$$= \frac{1\sqrt{5}}{3}$$

b)  $\frac{12\sqrt{27x^5}}{4\sqrt{9x}}$

~~x > 0~~  $x > 0$

$$= 3\sqrt{3x^4}$$

$$= 3x^2\sqrt{3}$$

6. Continued.....

$$c) \frac{2a}{\sqrt[3]{4a}} \cdot \frac{\sqrt[3]{4a}}{\sqrt[3]{4a}} \cdot \frac{\sqrt[3]{4a}}{\sqrt[3]{4a}} \in \mathbb{R}$$

$$= \frac{2a \sqrt[3]{16a^2}}{4a}$$

$$= \frac{2 \sqrt[3]{16a^2}}{2a} \quad a \neq 0$$

$$= \frac{\sqrt[3]{16a^2}}{a} = \frac{2 \sqrt[3]{2a^2}}{2} = \sqrt[3]{2a^2}$$

$$e) \frac{2\sqrt{10}-\sqrt{3}}{\sqrt{10}+\sqrt{3}} \quad (3 \text{ marks})$$

$$= \frac{(2\sqrt{10}-\sqrt{3})(\sqrt{10}-\sqrt{3})}{(\sqrt{10}+\sqrt{3})(\sqrt{10}-\sqrt{3})}$$

$$= \frac{2(10) - 2\sqrt{30} - \sqrt{30} + 3}{10 - 3}$$

$$= \frac{20 + 3 - 3\sqrt{30}}{7}$$

$$= \frac{23 - 3\sqrt{30}}{7}$$

$$d) \frac{6x\sqrt{3}}{\sqrt{20x^3}}, x \geq 0$$

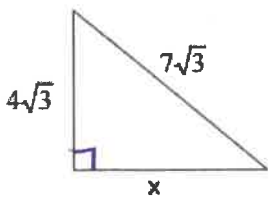
$$= \frac{6x\sqrt{3}}{\sqrt{20x^3}}$$

$$= \frac{6x\sqrt{3}}{2x\sqrt{5x}} \cdot \frac{\sqrt{5x}}{\sqrt{5x}}$$

$$= \frac{3\sqrt{3} \cdot \sqrt{5x}}{5x}$$

$$= \frac{3\sqrt{15x}}{5x} \quad ; x \geq 0$$

7. Solve for x in the right triangle below. Write your answer in EXACT form. (2 marks)



$$x^2 + (4\sqrt{3})^2 = (7\sqrt{3})^2$$

$$x^2 + 16(3) = 49(3)$$

$$x^2 + 48 = 147$$

$$\sqrt{x^2} = \sqrt{99}$$

$$x = \sqrt{99}$$

$$x = 3\sqrt{11}$$

8. For diamonds of comparable quality, the cost, "C", in dollars is related to the mass, "m", in carats, by the formula,

$$m = \sqrt{\frac{C}{700}}, C \geq 0.$$

What is the cost of a 3-carat diamond? (2 marks)

$$m = 3$$

$$(3)^2 = \left( \sqrt{\frac{C}{700}} \right)^2$$

$$(700) 9 = \frac{C}{700} (700)$$

$$\$6300 = C$$

The cost of a 3-carat diamond is \$6300.

9. Solve the following radical equations. State any restrictions and verify your solutions.

a)  $\sqrt{3x+2} + 5 = 2$  (2 marks)

$$\sqrt{3x+2} = -3$$

No solution.

b)  $\sqrt{x+4} - \sqrt{2x+3} = 0$  (4 marks)

$$(\sqrt{x+4})^2 = (\sqrt{2x+3})^2$$

$$x+4 = 2x+3$$

$$4 = x+3-3$$

$$1 = x$$

check

$$\sqrt{1+4} - \sqrt{2+3} = 0$$

$$\sqrt{5} - \sqrt{5} = 0$$

$$0 = 0$$

$$x = \{1\}$$

Restrictions

$$x \geq -4$$

$$2x+3 \geq 0$$

$$\frac{2x}{2} \geq \frac{-3}{2}$$

$$x \geq -\frac{3}{2}$$

Restriction

$$3x+3 \geq 0$$

$$\frac{3x}{3} \geq \frac{-3}{3}$$

$$c) \sqrt{3x+3} - x = 1 \quad (5 \text{ marks})$$

$$(\sqrt{3x+3})^2 = (x+1)^2$$

$$3x+3 = x^2 + 2x + 1$$

$$0 = x^2 - x - 2$$

$$0 = (x-2)(x+1)$$

$$x = 2 \quad x = -1$$

Check x=2

$$\sqrt{3(2)+3} - 2 = 1$$

$$\sqrt{6+3} - 2 = 1$$

$$3 - 2 = 1$$

$$1 = 1$$

✓

$$x = \{2, -1\}$$

check x=-1

$$\sqrt{3(-1)+3} - (-1) = 1$$

$$\sqrt{-3+3} + 1 = 1$$

$$0 + 1 = 1$$

$$1 = 1$$

✓

$$x+1 \geq 0$$
  
$$\boxed{x \geq -1}$$

$$d) \sqrt{4x+1} - \sqrt{3x-5} = 2 \quad (5 \text{ marks})$$

$$(\sqrt{4x+1} - 2)^2 = (\sqrt{3x-5})^2$$

$$(4x+1) - 4\sqrt{4x+1} + 4 = 3x-5$$

$$-4\sqrt{4x+1} = 3x - 4x - 5 - 5$$

$$\frac{-4\sqrt{4x+1}}{-4} = \frac{-1x - 10}{-4}$$

$$(\sqrt{4x+1})^2 = \left(\frac{x+10}{4}\right)^2$$

$$4x+1 = \left(\frac{1}{4}x + \frac{10}{4}\right)^2$$

$$4x+1 = \frac{1}{16}x^2 + \frac{10}{16}x + \frac{10}{16}x + \frac{100}{16}$$

$$4x+1 = \frac{1}{16}x^2 + \frac{20}{16}x + \frac{100}{16}$$

$$64x+16 = x^2 + 20x + 100$$

$$0 = x^2 - 44x + 84$$

$$0 = (x-42)(x-2)$$

$$x = 42 \quad x = 2$$

$$x = \{42, 2\}$$

$$4x+1 \geq 0 \quad 3x-5 \geq 0$$

$$\frac{4x}{4} \geq \frac{-1}{4}$$

$$x \geq \frac{-1}{4}$$

$$\frac{3x}{3} \geq \frac{5}{3}$$

$$\boxed{x \geq \frac{5}{3}}$$

$$(\sqrt{4x+1})^2 = (2 + \sqrt{3x-5})^2$$

$$4x+1 = 4 + 4\sqrt{3x-5} + 3x-5$$

$$1x + 6 = 4\sqrt{3x-5}$$

$$(x+2)^2 = (4\sqrt{3x-5})^2$$

$$x^2 + 4x + 4 = 16(3x-5)$$

$$x^2 + 4x + 4 = 48x - 80$$

$$x^2 - 44x + 84 = 0$$

$$(x-42)(x-2) = 0$$

$$x=42 \quad x=2$$