4.1 Graphical Solutions of Quadratic Equations

REVIEW:

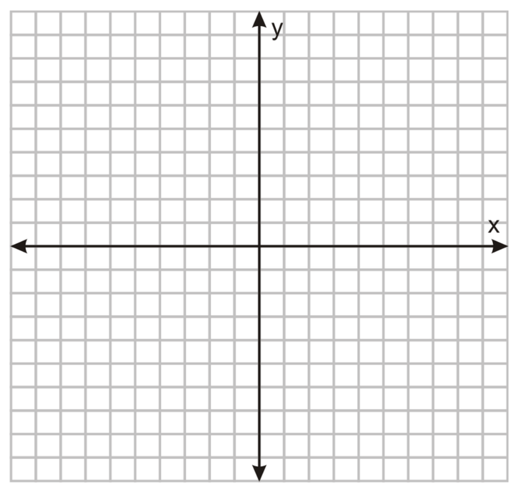
- A **QUADRATIC FUNCTION** is a function of degree two: $y = x^2$, $y = 2x^2 - 5x + 1$, $y = 2(x - 3)^2 - 3$, $y = (x + 1)^2$
- The place(s) where the quadratic function crosses the x axis are called the ________________
- A quadratic function may have ___________, _____________ or ____________ x intercepts
- There are four ways that a question may be asking you to find the x intercepts. They could ask you to:
  - Find the x intercepts
  - Find the _______________
  - Find the _______________
  - Find the _______________
- At this point, you may consider all four of the above terms to be exactly the same thing.

Concept #7: To solve a quadratic equation graphically (With and without technology)

EX #1:

Using a table of values, sketch $y = 2x^2 + 4x - 6$ and identify the roots. Verify your answer(s).

x	y



EX #2: Using technology, find the zeros to the quadratic equation $y - 9x = -\frac{4}{17}x^2 - 5$

- What is the difference between a QUADRATIC EQUATION, a QUADRATIC FUNCTION and a QUADRATIC EXPRESSION?

EX #3:

The manager of Jasmine's Fine Fashions is investigating the effect that raising or lowering dress prices has on the daily revenue from dress sales. The function $R(x) = 100+15x-x^2$ gives the store's revenue R, in dollars, from dress sales, where x is the price change, in dollars. Use technology to determine the price changes that will result in no revenue?

EX #4:

The product of two consecutive positive numbers is 110. Represent this as an algebraic equation and graph to solve the equation to find the numbers.

EX #5:

- Is the equation $\frac{x^2-3}{5}+2=\frac{4x+9}{3}$ a quadratic equation? _______________
- If it is a quadratic equation, rewrite it in standard form.

- Graph to solve the equation. (Use technology)

4.1 Assignment Pg 215 # 2,3bd(Use a table of values to graph #3bd),4ac,6,7,8,11,12(May use technology for 4,6,7,8,11,12)

4.2 Solving Quadratic Equations by Factoring (Day 1 – Review Factoring)

STEPS TO FOLLOW TO FACTOR AN EXPRESSION:

1. First, ALWAYS look for a GREATEST COMMON FACTOR (GCF)

EX #1: FACTOR OUT THE GCF OF THE FOLLOWING:

a) $2x + 4$

b) $22bc + 33ab^2c^5$

c) $-\frac{1}{2}x^2 + \frac{5}{4}$

2. IF YOU CAN'T FACTOR OUT A GCF AND YOU HAVE A QUADRATIC IN THE FORM $ax^2 + bx + c$ where $a \neq 0$, USE THE WINDOW (Box) METHOD, DECOMOPOSITION OR TRIAL AND ERROR TO FACTOR THE EXPRESSION

EX #2: FACTOR THE FOLLOWING USING THE METHOD OF YOUR CHOICE:

a) $x^2 + 6x - 16$

b) $x^2 - 4$

c) $3x^2 - 7x - 6$

d) $4m^2 - 36$

e) $4x^2 + 4x - 15$

f) $\frac{4}{49}x^2 - \frac{25}{81}y^2$

h) $16x^2 + 25y^2$

i) $x^2 + 10x + 25$

j) $36x^2 - 12x + 1$

3. IF YOU CAN FACTOR OUT A GCF YOU ALSO HAVE TO CHECK TO SEE IF THE EXPRESSION IN BRACKETS CAN CONTINUE TO BE FACTORED USING THE WINDOW METHOD/DECOMPOSTION/TRIAL AND ERROR.

EX #3: FACTOR THE FOLLOWING

a) $2x^2 + 10x - 28$

b) $5x^2 - 20$

c) $-3x^2 + 42x - 147$

d) $-6x^2 - 13x + 5$

e) $-\frac{3}{10}x^2 + \frac{11}{10}x + 2$

f) $0.4x^2 - 1.8x - 1 = 0$ (Equation)

g) $3x^2 = \frac{29}{2}x - 14$

h) $-x^2 + \frac{625}{121}$

NEW: HOW TO FACTOR QUESTIONS THAT AREN'T QUADRATIC BUT IN THE FORM OF A QUADRATIC EQUATION

EX #4: Factor the following polynomials in quadratic form using substitution

a) $-2(x + 3)^2 + 12(x + 3) + 14$

b) $4(x - 2)^2 - 0.25(y - 4)^2$

4.2 Day 1 Assignment Pg 229 #1-6 and 1-4 below

1. Factor.

a) $2x^2 - 50y^2$ b) $0.1x^2 - 0.001$ c) $20x^2 - 125y^2$ d) $\frac{1}{100}x^2 - \frac{1}{25}y^2$

2. Factor.

a) $2x^2 + 16x + 24$ b) $3x^2 - 9x - 30$ c) $x^2 + \frac{5}{2}x - 6$ d) $x^2 + 2.5x - 1.5$

3. Factor each polynomial.

a) $\frac{x^2}{9} - \frac{4}{25}$ b) $6 + 5x - x^2$ c) $-x^2 + \frac{121}{64}$ d) $7 - \frac{5}{3}x - 2x^2$

4. Factor each polynomial expression.

a) i) $9x^2 - 4y^2$ ii) $9(x - 3)^2 - 4(2y + 1)^2$

b) i) $50x^2 - 162y^2$ ii) $50(2x - 5)^2 - 162(3y - 2)^2$

SOLUTIONS TO EXTRA QUESTIONS IN 4.2 DAY 1

a) $2(x - 5y)(x + 5y)$ b) $0.001(10x - 1)(10x + 1)$ c) $5(2x - 5y)(2x + 5y)$ d) $\frac{1}{100}(x - 2y)(x + 2y)$

a) $2(x + 6)(x + 2)$ b) $3(x - 5)(x + 2)$ c) $\frac{1}{2}(2x - 3)(x + 4)$ d) $0.5(2x - 1)(x + 3)$

a) $\left(\frac{x}{3} - \frac{2}{5}\right)\left(\frac{x}{3} + \frac{2}{5}\right)$ b) $(1 + x)(6 - x)$ c) $\left(\frac{11}{8} - x\right)\left(\frac{11}{8} + x\right)$ d) $\frac{1}{3}(7 + 3x)(3 - 2x)$

a) i) $(3x - 2y)(3x + 2y)$ ii) $(3x - 4y - 11)(3x + 4y - 7)$ b) i) $2(5x - 9y)(5x + 9y)$

4.2 Solving Quadratics by Factoring (Day 2)

Zero Product Property: If $a \times b = 0$ then a = 0 or b= 0

To solve equations:

1. Set equation = 0 (write in descending order)
2. Factor fully
3. Set each factor = 0 and solve.
4. Write a solution set x ={#,# } or X= #,#

Concept #8: To solve quadratic equations by factoring

Example 1 : Solve the following.

A) $x^2 + 6x + 9 = 0$ (Verify your solution)

B) $2x^2 + 8x = 42$ (Verify your solution)

C) $2x(x – 6) + 3x = 2x – 9$

D) $2a^2+a + 2= 0$

e) $x^2 + 6x = 0$

f) $\frac{x^2}{2} + \frac{7}{6}x = 1$

Example #2 Without factoring, determine if d – 5 is one of the factors of $-\frac{3}{10}d^2 + \frac{11}{10}d + 2 = 0$

Example #3 - Page 231 #13

A flare is launched from a boat. The height , h, in metres, of the flare above the water is approximately modelled by the function h(t) = 150t – 5t^2, where t is the number of seconds after the flare is launched.

a) What equation could you use to determine the time it takes for the flare to return to the water?
b) How many seconds will it take for the flare to return to the water?

4.2 Day2 Assignment: Page 230 #7acde, 8ace, 9ac, 10ace, 12 Challenge : #11, 17

4.3 Solving Quadratic Equations by Completing the Square and Square Root Property

To solve equations that are ***non-factorable (yet may have x-intercepts)***, complete the square (if necessary) and then:

1. Isolate the squared term , if there is no term with just x(Degree1)
2. Take the $\pm$ square root of both sides.
3. Solve. If necessary, write 2 equations.
4. Check. Watch for ***extraneous*** roots (an answer not satisfying the restrictions on the variable)
5. Write a solution set x={#,#} or x=#,#

REVIEW SIMPLIFYING RADICALS

Simplify the following:

a) $\sqrt{75}$ b) $-\sqrt{98}$ $\sqrt{48}$

Concept #9: To solve a quadratic equation by completing the square and/or the square root property

EX #1: Solve each equation using the square root method. Leave your answers in exact form.

a) $(x-4)^2 = 16$ b) $2x^2 - 1 = 5$ Verify your solution

c) $x^2 + 6x + 16 = 0$ d) $x^2 - 10x = 3$ (approx. to the nearest tenth)

g) $9(x-2)^2 = 27$

h) $2x^2 = 12x - 3$

i) $-2x^2 - 3x + 7 = 0$

Ex.#3 (Ex. 1 on pg 236) A wide-screen television has a diagonal measure of 42 in. The width of the screen is 16 in. more than the height. Determine the dimensions of the screen, to the nearest tenth of an inch.

4.3 Assignment Pg 240 #4bcd, 5bcef, 6bcde,7abde, 8,9

4.4 The Quadratic Formula and the Discriminant

Show how the quadratic formula is derived by taking standard form and solve by completing the square and square root property.

STEPS to solving equations using the QUADRATIC FORMULA.

1. Make sure your quadratic equation is in standard form **$ax^2 + bx + c = 0$**

2. To find the roots use the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3.. Write the solution by either listing each root using x =# or by listing all roots in a solution set {#,…. }

Concept #10: To solve quadratic equations by using the quadratic formula

EX #1: Solve the following using the quadratic formula. Write your answer in exact form.

a) $x^2 + 4x - 1 = 0$

b) $x^2 + 6x + 9 = 0$

c) $2x = 3(x - 1)(x + 1)$

d) $\dfrac{2}{3}x^2 + 1 = \dfrac{5}{6}x$

EX #2: The surface area of a cylinder is 250 cm^2. The height of the cylinder is 7 cm. What is the radius of the cylinder to the nearest thousandth of a centimetre?

Concept #11 – To determine how many roots/solutions/x-intercepts a quadratic equation will have by using the discriminant b^2-4ac (a.k.a nature of the roots)

Discriminant – the expression b^2-4ac from the quadratic formula.

The discriminant tells us how many roots (or solutions, or zeroes, or x-intercepts) there are.

- If $b^2-4ac<0$

- If $b^2-4ac=0$

- If $b^2-4ac>0$

Ex 3: How many roots/solutions/x-intercepts do the following have? (Determine the nature of the roots)

1. $x^2-5x+4=0$
2. $3x^2+4x+\frac{4}{3}=0$
3. $2x^2-8x=-9$

4.4 Assignment Pg. 254 #1cef, 2bcde,3, 4ac,5bce,9,10