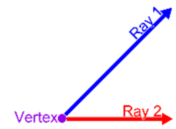
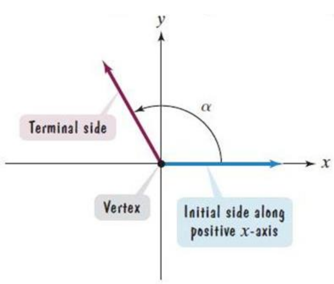
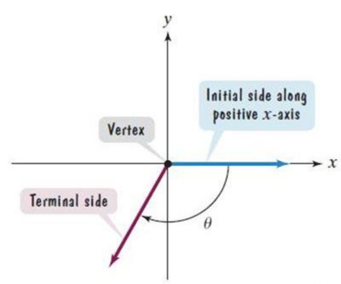
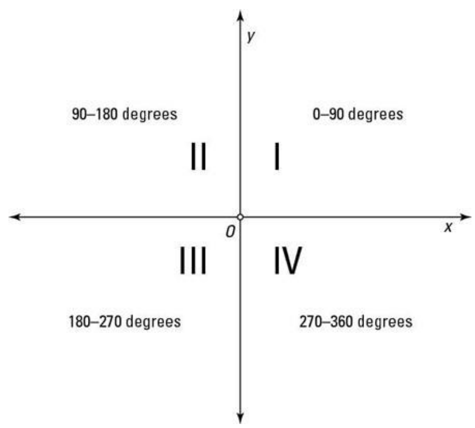
# 2.1 Angles in Standard Position and Special Triangles

- In **GEOMETRY**, an angle is formed by two rays having a common endpoint 


- In **TRIGONOMETRY** angles are often interpreted as rotations of a ray about a vertex.
    - These rotations are said to be in **STANDARD POSITION** when the vertex is located at the origin of the Cartesian plane.
    - In standard position the initial location of the ray is located on the positive x axis and is called the **INITIAL ARM**
    - Positive angles in standard position are created by rotating the initial arm counterclockwise about the vertex. The end location of the ray is called the **TERMINAL ARM.**
    - If the rotation of the terminal arm is clockwise, the angle has a negative measure



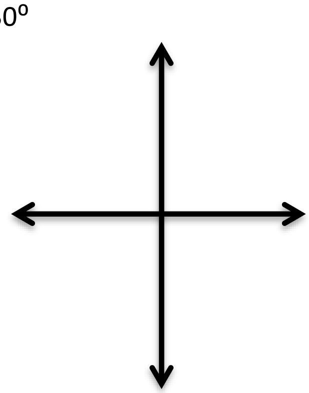




- We indicate the quadrant that an angle belongs in by the quadrant where the terminal arm is located
- If the terminal arm of an angle lies along an axis, then it is called a **QUADRANTAL ANGLE.**

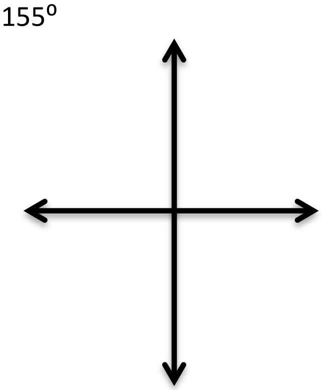
**READ PART A #1 and 2 on pg 75 of textbook**

**EX #1/** Sketch each angle in standard position and state what Quadrant they lie in:

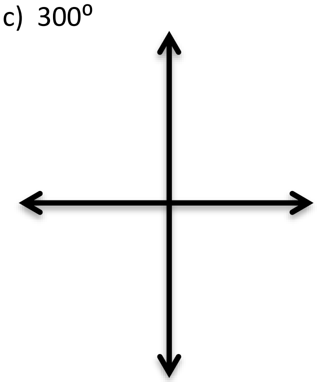
a) $30^{\circ}$



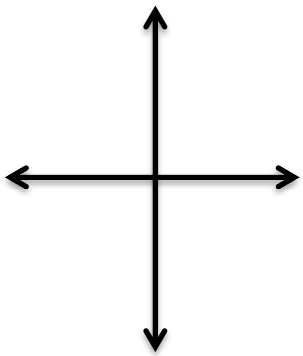
b) $155^{\circ}$



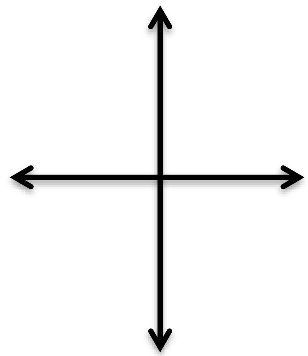
c) $300^{\circ}$



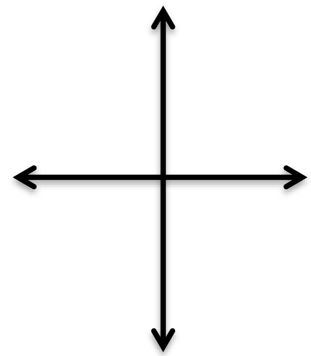
d) $-82^o$



e) $-210^o$



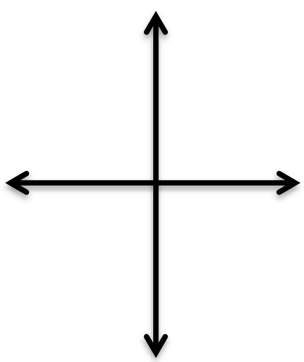
f) $425^o$



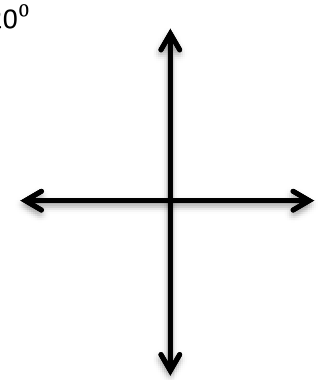
**<u>Reference Angle</u>** –

**Ex.#2/** Find the reference angle for each of the following:

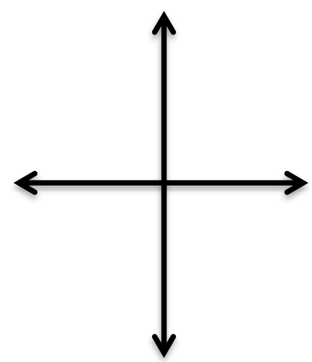
a) $70^0$



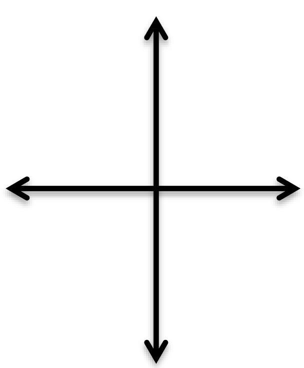
b) $220^0$



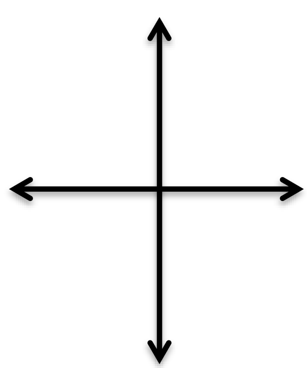
c) $272^0$



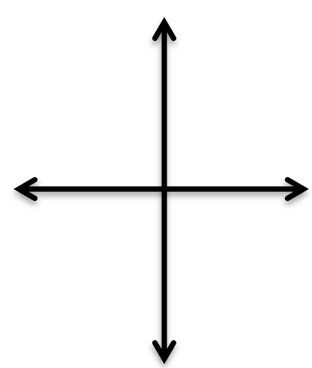
d) $-124^0$



e) $390^0$



f) $358^0$



**EX #3:** Determine the measure of 3 other angles in standard position 0°> Ө>360° that have a reference angle of 30°. Sketch.

## 2.1 DAY 2 Special Triangles:

Pg 76 Complete Part B: Create a $30^0$- $60^0$-$90^0$ triangle #6-8

Recall: SOH CAH TOA Pythagorean Theorem $a^2+b^2=c^2$

**$30^0$- $60^0$-$90^0$ triangle** **$45^0$-$45^0$-$90^0$ triangle**

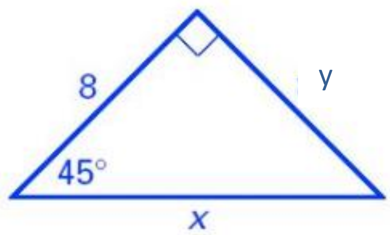
- We can determine the ***exact values*** of the trig ratios of these angles

Complete the chart using **exact values**

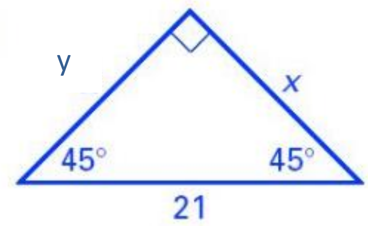
| $\theta$ | **Sin** $\theta$ | **Cos** $\theta$ | **Tan** $\theta$ |
|---|---|---|---|
| $30^0$ | | | |
| $60^0$ | | | |
| $45^0$ | | | |

**EX #4:** Use special triangles to determine the missing lengths.

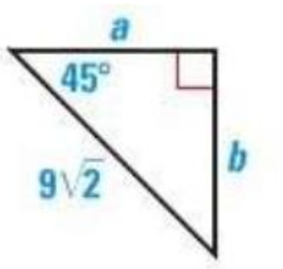
a)



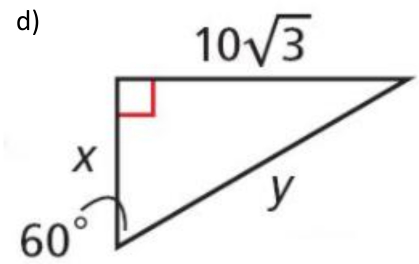

b)



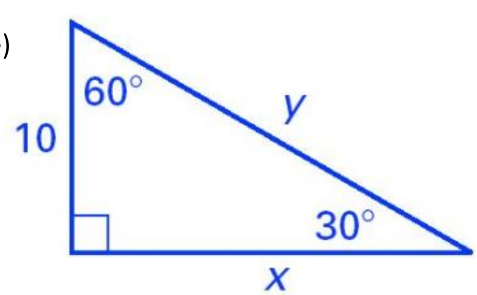

c)



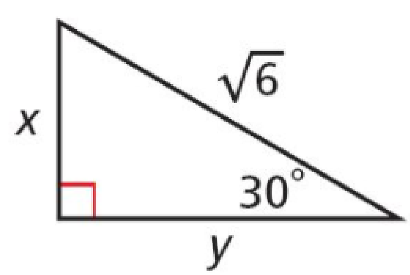

d)




e)

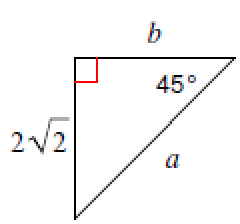



f)

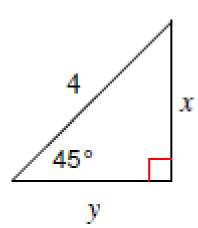



**<u>2.1 Assignment</u>:** Page 83 #1 – 7, 9 and questions below:

1.




2.

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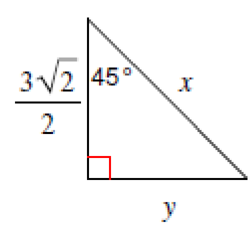


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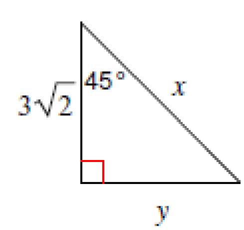
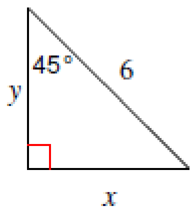


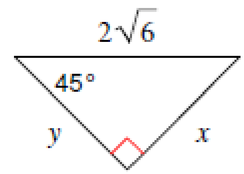
5.

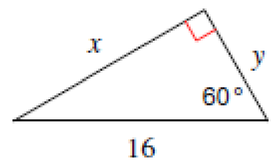
7.

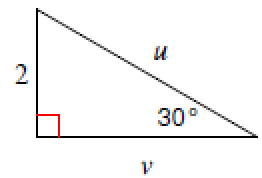



8.

6.

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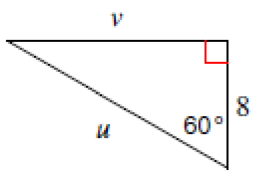


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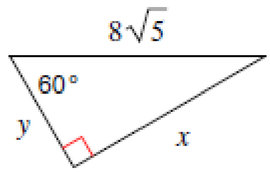


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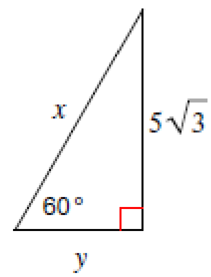


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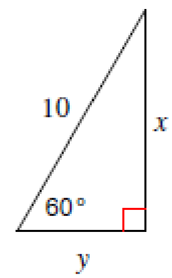


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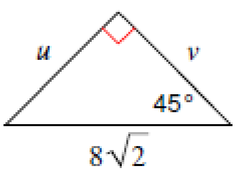


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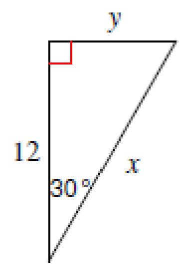


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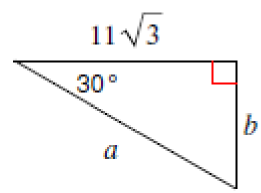


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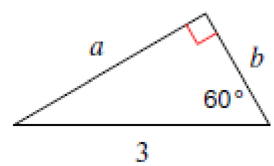


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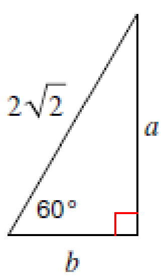
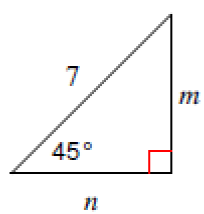


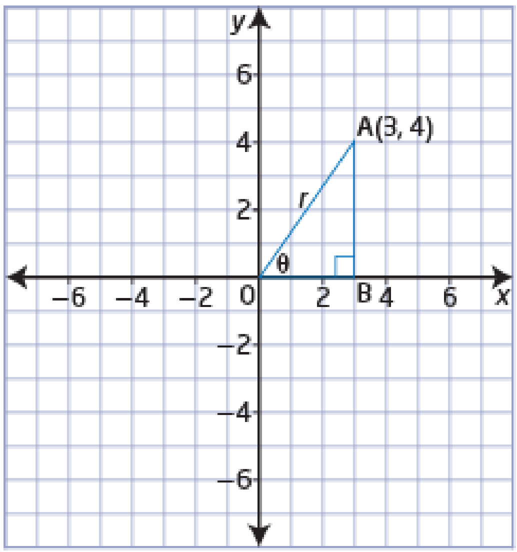

18.

**SOLUTIONS TO EXTRA QUESTIONS**

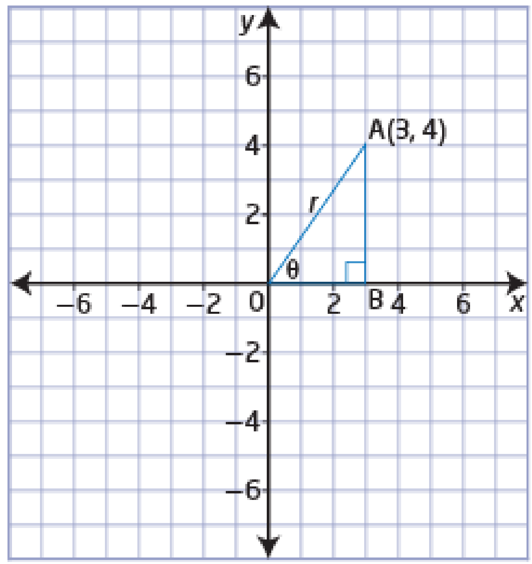
1. $a = 4; b = 2\sqrt{2}$
2. $x = 2\sqrt{2}; y = 2\sqrt{2}$
3. $x = 3; y = \frac{3\sqrt{2}}{2}$
4. $x = 6; y = 3\sqrt{2}$
5. $x = 3\sqrt{2}; y = 3\sqrt{2}$
6. $x = 2\sqrt{3}; y = 2\sqrt{3}$
7. $x = 8\sqrt{3}; y = 8$
8. $u = 4; v = 2\sqrt{3}$
9. $u = 16; v = 8\sqrt{3}$
10. $x = 4\sqrt{15}; y = 4\sqrt{5}$
11. $x = 10; y = 5$
12. $x = 5\sqrt{3}; y = 5$
13. $u = 8; v = 8$
14. $x = 8\sqrt{3}; y = 4\sqrt{3}$
15. $a = \dfrac{3\sqrt{3}}{2}; b = \dfrac{3}{2}$
16. $a = 22; b = 11$
17. $a = \sqrt{6}; b = \sqrt{2}$
18. $m = \frac{7\sqrt{2}}{2}; n = \frac{7\sqrt{2}}{2}$

# 2.2 Trigonometric Ratios of any Angle

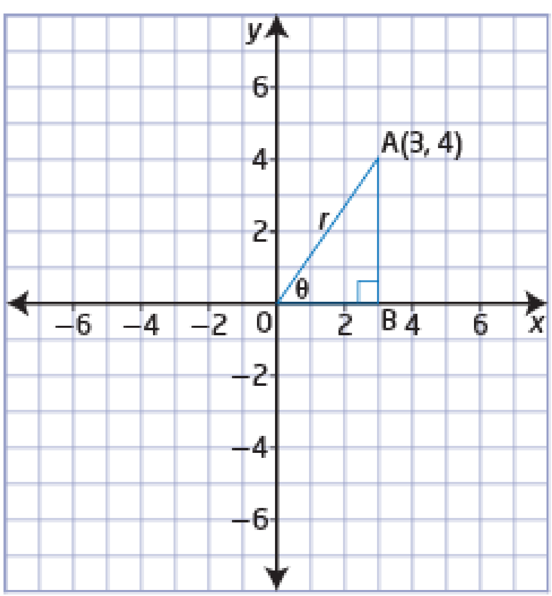
1. Find the 3 trig values for θ



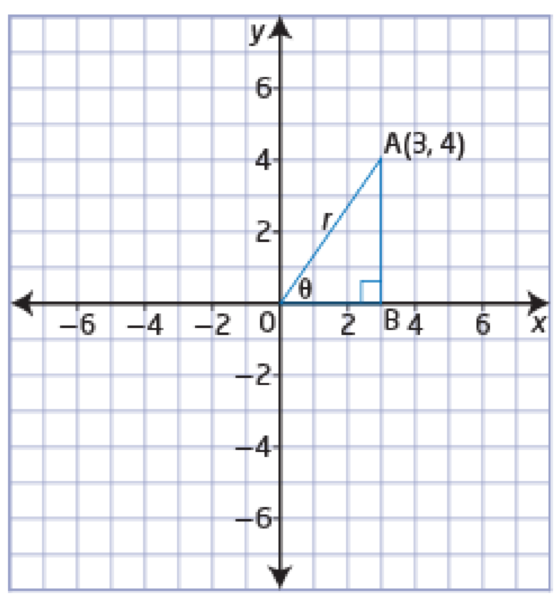

2. Reflect this triangle about the y-axis. Find the 3 trig values for this new angle




3. Reflect this triangle about the y-axis and then the x-axis. Find the 3 trig values for this new angle.




4. Reflect this triangle about the x-axis. Find the 3 trig values for this new angle

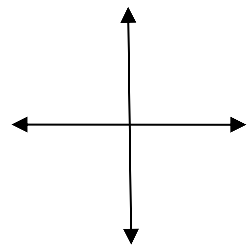


**1. The Trig ratios for $\theta$ can be written as follows:**

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

**2. The ASTC (All Soup Turns Cold) Rule**



- In the 1st quadrant, **ALL** trig functions have positive values.
- In the 2nd quadrant, the **SINE** function has positive values.
- In the 3rd quadrant, the **TANGENT** function has positive values.
- In the 4th quadrant, the **COSINE** function has positive values.

**Ex#1/ Given the following description, in which quadrant does the terminal arm of the angle lie?**

a) $\cos\theta > 0\ and\ \tan\theta < 0$

b) $\sin\theta > 0 and\ \tan\theta > 0$

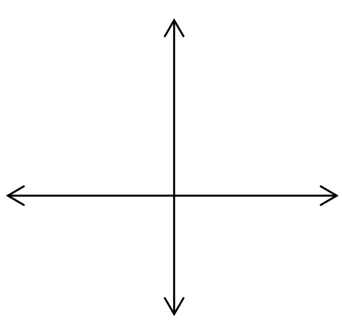
Ex#2/ **Determining the exact value of a trig ratio given its angle which has a reference angle of 30, 45 or 60**

a) Determine the exact value of $\cos 135^0$

b) Determine the exact value of $\sin 330^0$

Ex #3/ **Determine the exact trig ratios of an angle given a point on the terminal arm**

The point (-8, 15) is on the terminal arm of an angle θ in standard position. Draw the angle. Create a right triangle. Determine the exact trig ratios for $\sin\theta, \cos\theta, \tan\theta$. Determine θ to the nearest thousandth of a degree.



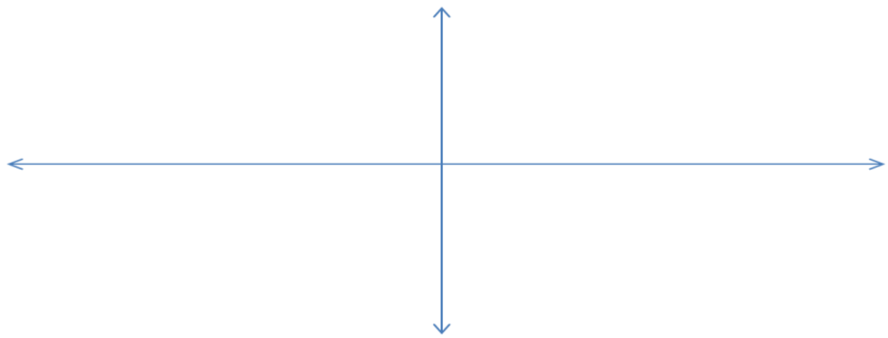
**Determining Trig Ratios of Quadrantal Angles**

| | 0˚ | 90˚ | 180˚ | 270˚ |
|---|---|---|---|---|
| Sin Θ | | | | |
| Cos Θ | | | | |
| Tan Θ | | | | |

Recall: $\sin\theta = \dfrac{y}{r}$

$\cos\theta = \dfrac{x}{r}$

$\tan\theta = \dfrac{y}{x}$



Ex#4/ Suppose θ is an angle in standard position with point ( 0, -4) on the terminal arm . What are the exact values of cos θ , $\sin\theta$ and $\tan\theta$? What is θ ?

# 2.2 DAY 1 Handout assignment: EXACT VALUES and Pg 96 #1-6

# 2.2 DAY 2

**Ex#5/ Solving for Angles Given Their Sine, Cosine, or Tangent ratio**

Steps:
1. Determine which quadrant the solution(s) will be in by looking at the sign (+ or -) of the given ratio.
2. Solve for the reference angle ($\theta_R$)
3. Sketch the reference angle in the appropriate quadrant. For exact values: Use the diagram to determine the measure of the related angle in the standard position.

a) Given Exact Trig Value (Use special triangles)
Example: If $\sin\theta = \frac{1}{2}$ ($0^\circ \le \theta \le 360^\circ$), solve for $\theta$.

b) If $\cos\theta = -\frac{\sqrt{3}}{2}$ ($0^\circ \le \theta \le 360^\circ$), solve for $\theta$.

**Ex #6: Given Approximate Trig Value** (use calculator) Use $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ to find the angle.

c) If $\cos\theta = 0.6753$ ($0^\circ \le \theta \le 360^\circ$), solve for $\theta$ to the nearest tenth of a degree.

d) If $\sin\theta = 0.8090$ ($0^\circ \le \theta \le 360^\circ$), solve for $\theta$ to the nearest tenth of a degree.

e) If $\tan\theta = -0.3675$ ($0^\circ \le \theta \le 360^\circ$), solve for $\theta$ to the nearest tenth of a degree.

# 2.2 Day 2 Pg 96 #7,8,9,12,13,28,29

Pre-Calculus 20 **2.3 – The Sine Law**

1. Read Page 100 in text ( 2 paragraphs in the middle of the page)

2. Define Oblique Triangle-

3. Draw an oblique triangle. Label it $\Delta ABC$. Measure all sides and angles using a ruler and protractor. ( Remember : side "a" is opposite $\angle$A, side "b" is opposite $\angle$B, side "c" is opposite $\angle$C)

4. Find the ratio of the sine of each angle with its corresponding side: ( Round to the nearest thousandth)

$$\frac{\sin A}{a} =$$

$$\frac{\sin B}{b} =$$

$$\frac{\sin C}{c} =$$

**Sine Law:** For any triangle ΔABC where a, b, c are the sides opposite $\angle$A, $\angle$B, $\angle$C respectively:

Note: lower case letters represent the side lengths and upper case letters represent the angles.

Ex#1/ Sketch the triangle and determine the measure of the indicated side:

$\Delta ABC$ $\angle A = 50^0$ $\angle B = 50^0$ and AC = 27 cm. Find AB. Round answer to the nearest tenth of a centimeter.

Ex#2 / In $\Delta PQR$ $\angle P = 36^0$ p= 24.8m and q=23.4m. Determine the measure of $\angle R$, to the nearest degree.

Ex#3/ Determine an unknown side length Ex#1 on Pg 102.

<u>2.3 Assignment</u>: Page 108 #1ac, 2, 3b, 4bd, 5ab, 10, 12, 15

# 2.3 Sine Law Ambiguous Case

The Sine Law gives the relationship between the sides and the angles of a triangle. If you are given two angles and one side, you can use the Law of Sines to find the lengths of the two unknown sides (ASA or AAS triangles).

You can get into trouble when you are given two sides and one opposite angle (SSA triangles or ASS Triangles) in order to find the other opposite angle as the combination of side lengths and the angle measure does not always produce one unique triangle. Due to this uncertainty, this is called the Ambiguous Case.

**TRIANGLE CHALLENGE: Explore the Ambiguous case using Pipe Cleaners**

**http://teachhighschoolmath.blogspot.ca/2013/04/triangle-challenge-ambiguous-case.html**

Three different situations can arise when given ASS information: one triangle formed, two triangles formed or no possible triangle. In order to determine the number of triangles, you must first consider the type of angle then the lengths of the adjacent and opposite sides.

Type of Angle

↓

Relationship of the Given Sides

↓

Number of Triangles that can be formed

↓

Number of Solutions

Ex #1/ Given $\Delta ABC$ $\angle C = 31°$ , a = 5.6cm and c=3.9cm

a) Draw a diagram.
b) Decide how many solutions.
c) Determine the unknown sides and angles. ( round angles to the nearest degree and sides to the nearest tenth)

Ex #2/ Given $\Delta ABC$ $\angle A = 30°$, a = 24cm and c=42cm. Determine the measures of the other sides and angles. Round your answers to the nearest unit.

https://www.desmos.com/calculator/jcrvqcc1y2

Assignment: Handout & pg 108 #11

**Assignment 2.3 Sine Law Ambiguous Case**

Determine the number of solutions for the given SSA information:

a) $\Delta ABC$ where $\angle A = 42°$, $a = 30\,\text{cm}$ and $b = 25\,\text{cm}$

b) $\Delta ABC$ where $\angle C = 99°$, $a = 30\,\text{cm}$ and $c = 37\,\text{cm}$

c) $\Delta ABC$ where $\angle B = 27°$, $b = 25\,\text{cm}$ and $c = 30\,\text{cm}$

d) $\Delta ABC$ where $\angle B = 127°$, $b = 25\,\text{cm}$ and $c = 25\,\text{cm}$

e) $\Delta ABC$ where $\angle C = 37.3°$, $b = 90\,\text{cm}$ and $c = 85\,\text{cm}$

f) $\Delta ABC$ where $\angle A = 30°$, $a = 25\,\text{cm}$ and $b = 50\,\text{cm}$

g) $\Delta ABC$ where $\angle C = 38.7°$, $a = 25\,\text{cm}$ and $c = 25\,\text{cm}$

h) $\Delta ABC$ where $\angle A = 139°$, $a = 15\,\text{cm}$ and b= 22cm

Solve the following triangles if possible. Round all angle measures and side lengths to the nearest tenth.

a) $\Delta ABC$ where $\angle A = 42°$, $a = 10.2\,\text{cm}$ and $b = 8.5\,\text{cm}$

b) $\Delta PQR$ where $\angle P = 56°$, $p = 1.92\,\text{m}$ and $q = 2.35\,\text{m}$ ( round height to the nearest Hundredth)

c) $\Delta BAD$ where $\angle B = 37.7°$, $b = 30\,\text{cm}$ and $d = 42\,\text{cm}$

## **2.4 Cosine Law**

Given ABC is any oblique triangle:

$$c^2 = a^2 + b^2 - 2ab\text{Cos}C$$

$$b^2 = a^2 + c^2 - 2ac\text{Cos}B$$

$$a^2 = b^2 + c^2 - 2bc\text{Cos}A$$

If given SAS use the Cosine law to find the unknown side

If given SSS use the Cosine law to find a particular angle

Use the Sine law to solve the rest of the unknowns in the triangles

Hint: 1) Given all 3 sides (SSS), find the size of the largest angle first using Law of Cosines.
2) When using Law of Sines for finding angles, find smallest or second smallest angle first as you know they will be acute. The largest angle may be obtuse or acute.
3) The sum of the lengths of any 2 sides in a triangle must always be greater than the $3^{rd}$ side.

Your Turn (page 117) Ex#1/ Nina wants to find the distance between two points , A and B, on opposite sides of a pond. She locates a point C that is 35.5m from A and 48.8m from B. If the angle at C is $54^0$, determine AB, to the nearest tenth of a metre.

Ex/ #2 (page 117) The lions gate bridge has been a Vancouver landmark since it opened in 1938. It is the longest suspensions bridge in Western Canada. The bridge is strengthened by triangular braces. Suppose one brace has side lengths 14m, 19m, and 12.2 m. Determine the measure of the angle opposite the 14m side to the nearest degree.

2.4 Assignment: Page 119 #1ab, 2ab, 3ab, 12, 15