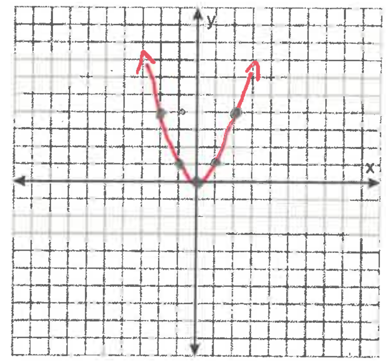
3.1 Day 1 Quadratic Functions in Vertex Form

Concept #1: To determine the coordinates of the vertex, the domain and range, the axis of symmetry, the x and y intercepts and the direction of opening of the graph of $f(x)=a(x-p)^2+q$ without the use of technology.

EX #1: Using a table of values, sketch $y = x^2$

X	y
-2	4
-1	1
0	0
1	1
2	4



REVIEW:

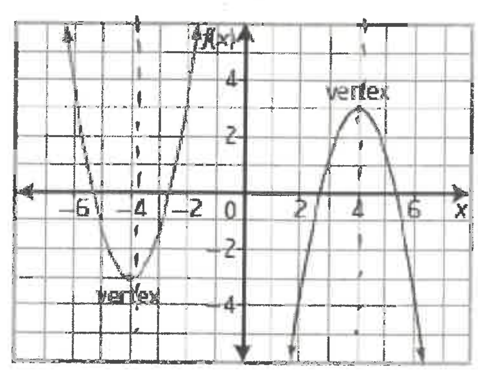
- A **QUADRATIC FUNCTION** is a function of degree two: Ex1 $y = x^2$, $y = 2x^2 - 5x + 1$, $y = 2(x-3)^2 - 3$, $y = (x+1)^2$
- The graph of a quadratic function is in the shape of a parabola
- A quadratic function is written in STANDARD FORM when it is written in the form $f(x) = ax^2 + bx + c$ and it is written in VERTEX FORM when it is in the form $f(x) = a(x-p)^2 + q$ Note: in Foundations 20 $f(x) = a(x-h)^2 + K$
- The vertex of a parabola is the lowest point in a parabola that opens upwards or the highest point of a parabola that opens downwards.
- If the parabola opens upwards then there is a minimum value. If the parabola opens down there is a maximum value. The y coordinate of the vertex defines the max or min value.
- The axis of symmetry is a line through the vertex that divides the graph of the quadratic function into two congruent halves. The x-Coordinate of the vertex defines the equation of the axis of symmetry.

EX #2: State the vertex, the max or min value and the equation of the axis of symmetry for the following:

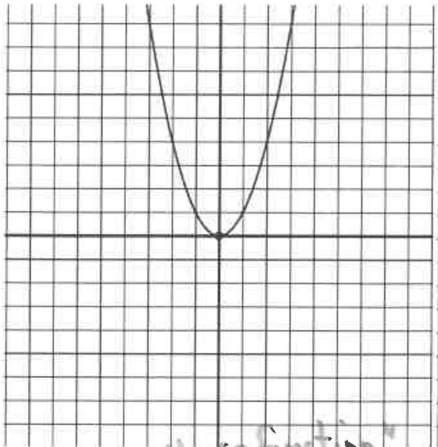
Graph A
Vertex = (-4, -3)
Min @ -3 or y = -3
E. of axis of Symmetry: x = -4

Graph B
Vertex = (4, 3)
Max at 3 or y = 3
E. of axis of symmetry: x = 4

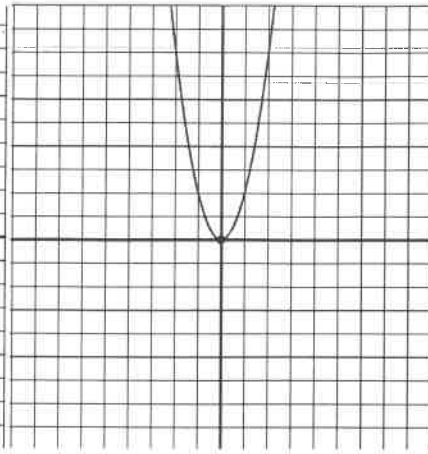
A B



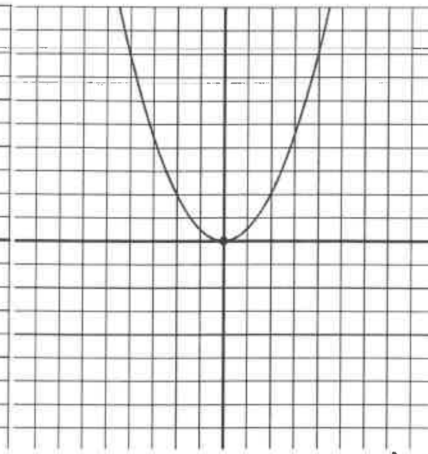
A. Compare the graphs of f(x)= x^2 and f(x)= ax^2



f(x) = x^2 "base function" "Normal Function" or "Parent Function"

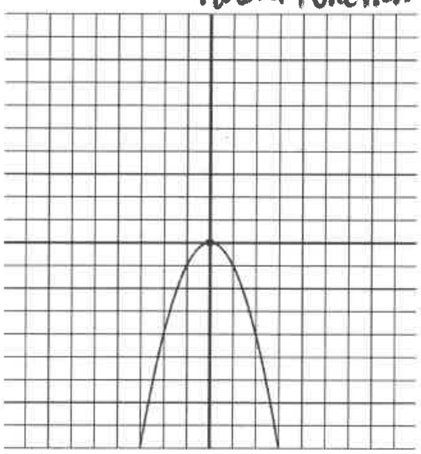


f(x) = $2x^2$

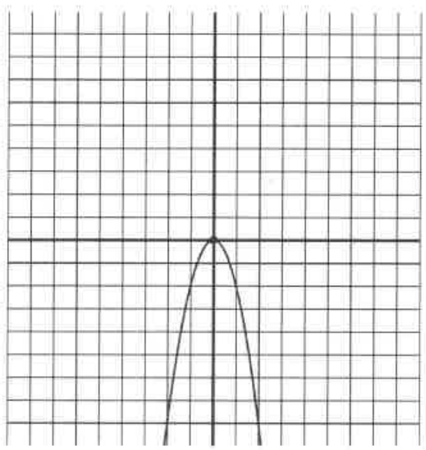


f(x) = $\frac{1}{2}x^2$

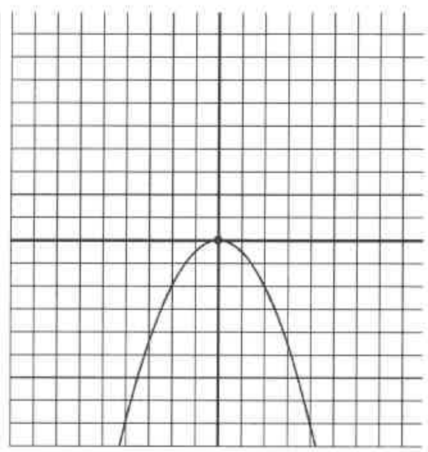
What changes in these graphs and equations?



f(x) = $-x^2$



f(x)= $-2x^2$



f(x) = $-\frac{1}{2}x^2$

How does the value of "a" affect the graph of f(x)= ax^2 ?

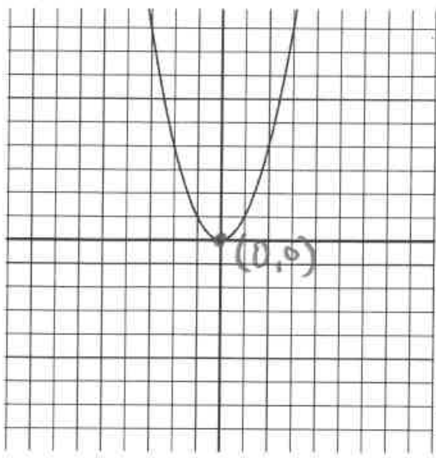
- if "*a*" is positive, graph opens up
- if "*a*" is negative, graph opens down
- if $|a|>1$ ($a>1$ or $a<-1$), the graph is narrower (fractional values are FAT)
- if $|a|<1$ ($-1<a<1$), the graph is wider or average (Larger the whole # the skinner the parabola)
- if $|a|=1$ ($a=-1$ or $a=1$), the graph is normal width

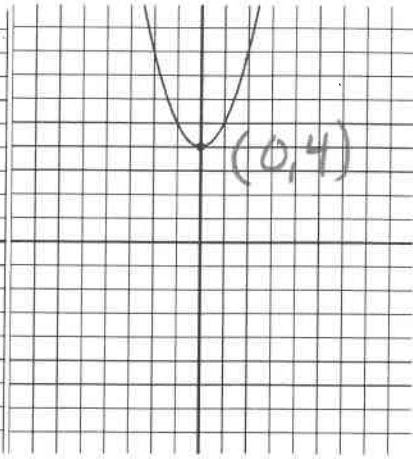
|a| the bigger it is the narrower the graph will be

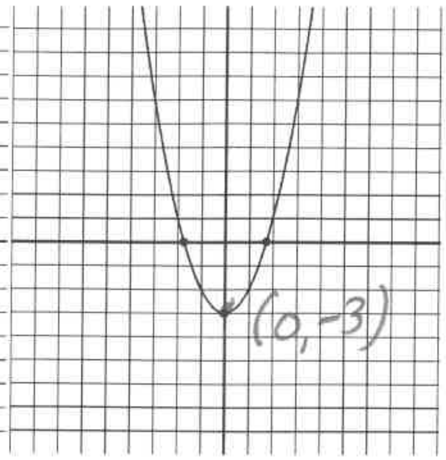
What will be the width of a parabola w/ "a" value $\frac{4}{3}$?

~ narrow because $\frac{4}{3} = 1.3$

B. *Compare the graphs of $f(x)= x^2$ and $f(x)= x^2 + q$*







f(x) = x^2

$f(x) = 1(x-0)^2+0$ "

f(x) = $x^2 + 4$

$f(x) = 1(x-0)^2+4$

f(x) = $x^2 - 3$

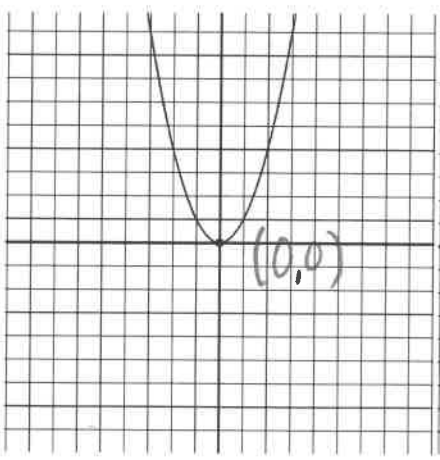
$f(x) = 1(x-0)^2-3$

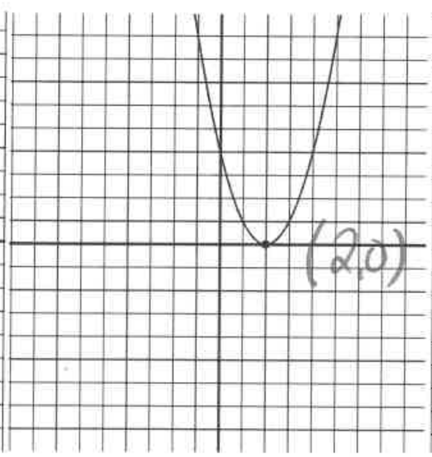
How does the value of "q" affect the graph of $f(x)= x^2 + q$? Note: q = y-coordinate of vertex

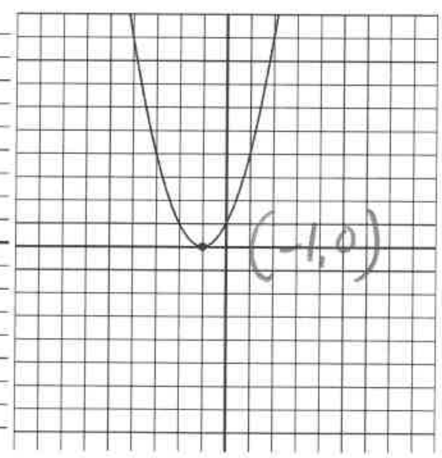
- The value of "q" indicates a vertical shift.

If q > 0, graph moves up (above the x-axis)
If q < 0, graph moves down (below the x-axis)

C. *Compare the graphs of $f(x)= x^2$ and $f(x)= (x - p)^2$*







f(x) = x^2

f(x) = $(x - 2)^2 + 0$

f(x) = $(x + 1)^2 + 0$

How does the value of "p" affect the graph of $f(x)= (x - p)^2$? Note: p = x-coordinate of vertex

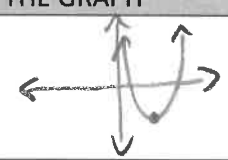
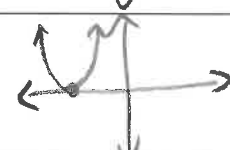
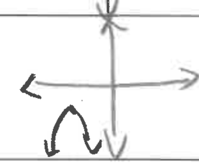
The value of "p" indicates a horizontal shift.

If p > 0, graph moves right (right of y-axis)
If p < 0, graph moves left (left of y-axis)

Summary:

- In VERTEX FORM $y=(x-p)^2+q$, the following is true:
  - The coordinates of the vertex are at (p , q)
  - The equation of the axis of symmetry is at $x=p$
  - If the value of "a" is negative, the graph will open down and have a max value at q
  - If the value of "a" is positive, the graph will open up and have a min value at q
  - The parabola will be of average width if $a=1$ or -1
  - The parabola will be narrower if $a>1$ or $a<-1$
  - The parabola will be wider if $-1<a<1$
  - The value of "p" moves the parabola horizontally (left and right)
  - The value of "q" moves the parabola vertically (up and down)
  - The domain of a parabola (with arrowheads) will be $X=\{x \mid x\in\mathbb{R}\}$ in set notation and $(-\infty,\infty)$ in interval notation ← (left, to Right)
  - The range of a parabola (with arrowheads) will be $y=\{y \mid y\geq q, y\in\mathbb{R}\}$ or $\leq$ in set notation and $[q,\infty)$ in interval notation
    $(-\infty, q]$ (low#, high#)

EX #3: Determine the number of x intercepts of each quadratic function by visualizing the graph.

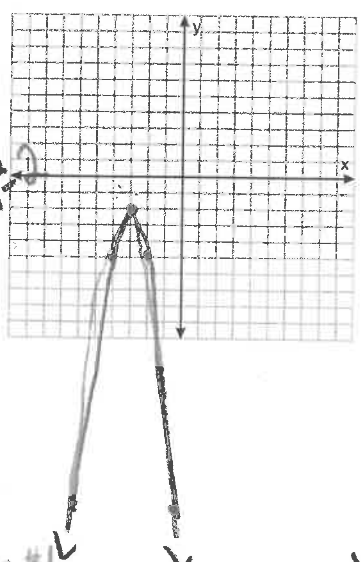
	VERTEX	DIRECTION OF OPENING	VISUALIZE & SKETCH THE GRAPH	NUMBER OF X INTERCEPTS
$f(x)=\frac{1}{2}(x-2)^2-4$	(2,-4)	up		2
$f(x)=3(x+5)^2$	(-5,0)	up		1
$f(x)=-(x+3)^2-5$	(-3,-5)	down		0

EX #4: Complete the following chart and sketch the last two functions using a table of values and key points
Note: Domain and Range you can use interval or set notation. (If you did not take Pre- Ap Foundations 20 see video on my website to teach you)

$y=2(x-0)^2+5$

	$y=3x^2$	$y=2x^2+5$	$y=(x-5)^2$	$y=-3(x+3)^2-2$	$y=\frac{1}{2}(x+3)^2-2$
Value of a	3	2	1	-3	½
Value of p	0	0	5	-3	-3
Value of q	0	5	0	-2	-2
Direction of opening	up	up	up	down	up
Width(Normal , Wide Narrow)	Narrow	Narrow	Normal	Narrow	wide
Vertex	(0,0)	(0,5)	(5,0)	(-3,-2)	(-3,-2)
Max/Min	Min = 0	Min = 5	Min = 0	Max at -2	Min at -2
Axis of Symmetry	X = 0	X = 0	X = 5	X = -3	X = -3
Domain	$(-\infty, \infty)$	$(-\infty, \infty)$	$(-\infty, \infty)$	$X=\{x \mid x \in \mathbb{R}\}$	$X=\{x \mid x \in \mathbb{R}\}$
Range	$[0, \infty)$	$[5, \infty)$	$[0, \infty)$	$Y=\{y \mid y \le -2, y \in \mathbb{R}\}$	$Y=\{y \mid y \ge -2, y \in \mathbb{R}\}$
# of x-intercepts	1	0	1	0	2
Y intercept	$y=3(0)^2$ $y=0$ (0,0)	$y=2(0)^2+5$ $y=5$ (0,5)	$y=(0-5)^2$ $y=25$ (0,25)	$y=-3(0+3)^2-2$ $y=-3(9)-2$ $y=-27-2=-29$	$y=\frac{1}{2}(0+3)^2-2$ $y=4.5-2$ $y=2.5$
Reflection of the y intercept	N/A	N/A	(10,25)	(-6,-29)	(-6,25)

$f(x)=-3(x+3)^2-2$

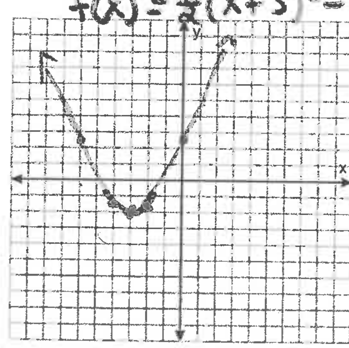


- When drawing a sketch need at least 3 points
- use a table if you need more points

X	y
-4	-5

$y=-3(-4+3)^2-2$
$y=-3(+1)-2$
$y=-3-2$
$y=-5$

$f(x)=\frac{1}{2}(x+3)^2-2$



X	y
-2	-1.5

$y=\frac{1}{2}(-2+3)^2-2$
$y=\frac{1}{2}-\frac{4}{2}$
$y=-\frac{3}{2}$
$y=-1.5$

3.1 Assign. #1

Chart assignment ← challenge

Pg 157 #1ad, 2bc, 19

↑ Have students add

3.1 Day 3 Quadratic Functions in Vertex Form

Concept: To graph quadratic functions in the form $f(x)=a(x-p)^2+q$ using transformations.

EX #1: Sketch the graph of $y = 3(x+2)^2 - 4$ using transformations.

- **STEP 1:** Describe what the numerical change to "a" is compared to its parent PARENT FUNCTION (base function, original function) $y = x^2$.
  $a=3$
  - How will this change alter the graph of the PARENT FUNCTION) $y = x^2$? The graph parabola will became narrower
  - Will this change affect the x or the y values of the ordered pairs of the parent function? The y-values.

Let's compare the table of ordered pairs between the parent function and the Step 1 Transformed table (which is the parent function with just the value of "a" changed – we won't worry about the values of "p" and "q" yet)

PARENT FUNCTION $y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

STEP 1 of TRANSFORMED FUNCTION $y = 3x^2$

x	y
-2	12
-1	3
0	0
1	3
2	12

Describe what changes & how:
The parabola will open up because $a>0$
The parabola will become narrower, by multiplying the y-values by a factor of 3, because $a=3$, which is $a>1$.

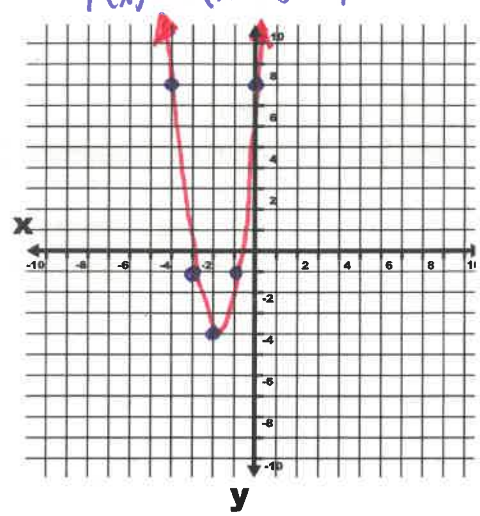
- **STEP 2:** Describe how "p" and q" have changed compared to the parent function $y = x^2$ $y=3(x+2)^2-4$
  $p=-2$ $q=-4$
  - How will this change alter the graph of the PARENT FUNCTION $y = x^2$? It will shift the parabola horizontally and vertically.
  - How will p and q affect where the parent function moves to?
    $p=-2$ so will shift the parabola 2 units to the left, $q=-4$ so will shift the parabola 4 units down.
  - Describe how x moves and how y moves. (on the table)
    We need to subtract 2 from each x-value. We need to subtract 4 from each y value.
  - Fill in the Step 2 table by moving the x and y values by the appropriate amounts
  - Sketch the graph using the Step 2 table.

STEP 2 of TRANSFORMED FUNCTION $y = 3(x+2)^2 - 4$

x	y
-4	8
-3	-1
-2	-4
-1	-1
0	8

Describe what changes & how:
The parabola will shift 2 units to the left and 4 units down because $p=-2$ and $q=-4$.
Therefore we need to subtract 2 from all x-values and subtract 4 from all y-values.

$f(x)=3(x+2)^2-4$



EX #2: Sketch the graph of $f(x) = -\frac{1}{2}(x-1)^2 + 3$ using transformations.

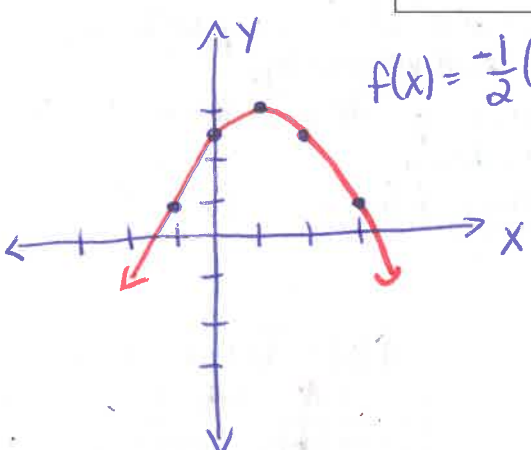
PARENT FUNCTION $y = x^2$	
x	**y**
-2	4
-1	1
0	0
1	1
2	4

STEP 1 of TRANSFORMED FUNCTION $y = -\frac{1}{2}x^2$	
x	**y**
-2	-2
-1	-1/2
0	0
1	-1/2
2	-2

Describe what changes & how:
The width of the parabola will become wider and reflect about the x-axis. (opens down) because the $a<0$.
The change in width will occur by multiplying the y-values by a factor of $-\frac{1}{2}$.

STEP 2 of TRANSFORMED FUNCTION $y = -\frac{1}{2}(x-1)^2+3$	
x	**y**
-1	1
0	2.5
1	3
2	2.5
3	1

Describe what changes & how:
$p=+1 \quad q=3$
The parabola will shift 1 unit to the right (add 1 to all x-values), and shift 3 units up. (add 3 to all y-values)



$f(x) = -\frac{1}{2}(x-1)^2+3$

3.1 Assignment #2
Pg 157 #3 & Extra Question 1 Below Pg 158 #10 (No Graphing Calculator)

1. Graph the following functions using transformations. Make sure to state transformations, the vertex and show the new tables of values. It is imperative that you use graph paper and a ruler!!

a) $y = 2x^2$

b) $y = -3(x+4)^2 + 2$

c) $y = \frac{1}{2}(x-1)^2 - 3$

d) $f(x) = -4(x+2)^2$

e) $f(x) = -\frac{1}{3}(x+1)^2 + 1$

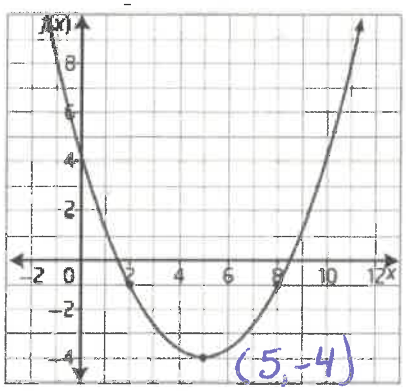
f) $f(x) = \frac{4}{3}(x-2)^2 - 7$

3.1 Day 4 – Quadratic Functions in Vertex Form

Concept: To write quadratic functions in vertex form given a graph or situation and to solve situational questions.

EX #1: Determine the quadratic function in vertex form for the following graphs.

a)



$y = a(x-p)^2 + q$

vertex : $(5,-4)$ (p, q) point: $(2,-1)$ (x, y)

Substitute into equation and solve for "a"

$-1 = a(2-5)^2 - 4$

$-1 = a(-3)^2 - 4$

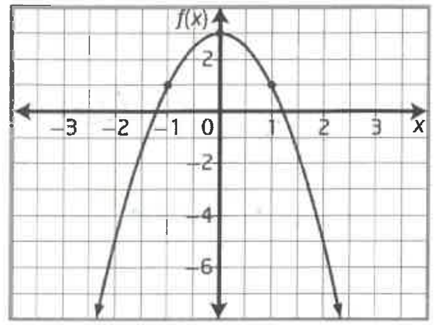
$-1 + 4 = 9a - 4 + 4$

$\frac{3}{9} = \frac{9a}{9}$

$\frac{1}{3} = a$

$\boxed{f(x) = \frac{1}{3}(x-5)^2 - 4}$

b)



vertex: $(0,3)$ (p, q) point: $(1,1)$ (x, y)

$y = a(x-p)^2 + q$

$1 = a(1-0)^2 + 3$

$1 - 3 = a(1) + 3 - 3$

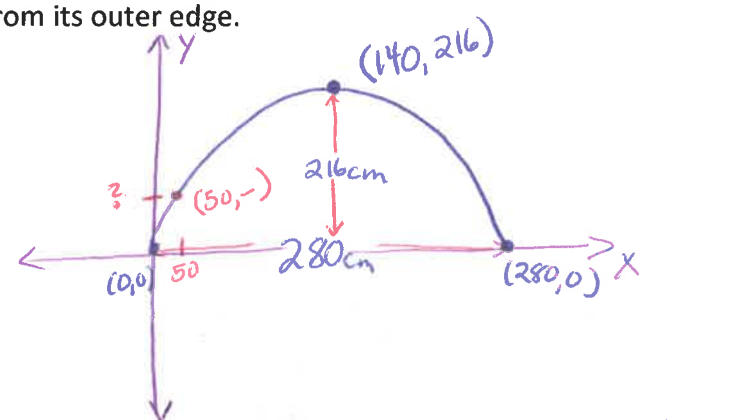
$-2 = a$

$\boxed{f(x) = -2x^2 + 3}$

EX #2: Suppose a parabolic archway has a width of 280 cm and a height of 216 cm at its highest point above the floor.

a) Write a quadratic function in vertex form that models the shape of this archway.
b) Determine the height of the archway at a point that is 50 cm from its outer edge.

c) Is there more than one answer for part "a". Why?



a) vertex: $(140, 216)$ p q point: $(280, 0)$ x y

"substitute p, q, x, y into vertex form of the quadratic equation. Solve for "a""

$$0 = a(280-140)^2 + 216$$
$$-216 = 19600a$$
$$\frac{-216}{19600} = \frac{19600a}{19600}$$
$$\frac{-27}{2450} = a$$

$$\boxed{\text{a) } y = \frac{-27}{2450}(x-140)^2 + 216}$$

b) point $(50, -)$ x y Substitute 50 in for "x" into the quadratic equation you got in part "a". Solve for y.

$$y = \frac{-27}{2450}(50-140)^2 + 216$$
$$y = \frac{-27}{2450}(8100) + 216$$
$$\boxed{y = 126.7\text{cm}}$$

c) Yes, depending on where you put the x and y axis.

EX #3: Determine a quadratic function with the following characteristics: a minimum of 12 at x = -4 and y-intercept of 60.

Minimum at 12 which is the y coordinate of the vertex $q = 12$ $p = -4$

y intercept $(0, 60)$ x y

$$y = a(x-p)^2 + q$$
$$60 = a(0+4)^2 + 12$$
$$\frac{48}{16} = \frac{a(16)}{16}$$
$$3 = a$$

$$\boxed{y = 3(x+4)^2 + 12}$$

3.1Assignment #3: Pg158 #8abc, 9abd & at least 2 of the following:13, 16, 17, 18 (No Graphing Calculator)

3.2 Quadratics in Standard Form

Concept #4 : To determine the coordinates of the vertex, the domain and range, the axis of symmetry, the x and y intercepts and the direction of opening of the graph of a function in standard form y = ax² + bx + c

A quadratic in standard form is $y = ax^2 + bx + c$ or $f(x) = ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$

- *a* determines the width of the parabola and whether the parabola opens upwards or downwards (the same as it did for vertex form)
- *b* INFLUENES the position of the graph (vertex)
- *c* determines the y intercept of the graph
- In Foundations 20, we found the x value of the vertex by calculating the x intercepts and finding the middle x value of those intercepts. We will now develop a formula that we can use

Begin with vertex form $y=a(x-p)^2+q$, Expand (multiply) to convert to standard form.

$y=a(x-p)^2+q$

$y=a(x-p)(x-p)+q$

FOIL

$y=a(x^2-px-px+p^2)+q$

Combine like terms

$y=a(x^2-2px+p^2)+q$

distribute

$y=ax^2-2apx+ap^2+q$

$y=ax^2+\underbrace{(-2ap)}_{b}x+\underbrace{ap^2+q}_{c}$

$\therefore b=-2ap$ $c=ap^2+q$

manipulate to Solve for "p" | manipulate to Solve for q

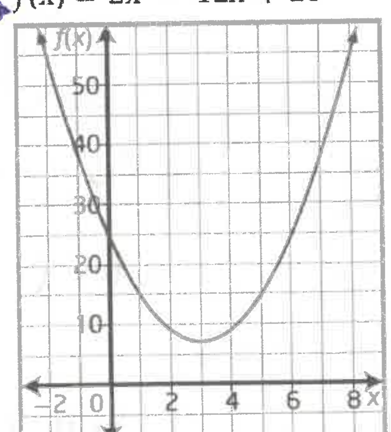
$\frac{b}{-2a}=\frac{-2ap}{-2a}$ | $c-ap^2=ap^2+q-ap^2$

$\boxed{p=\frac{-b}{2a}}$ x-coordinate of the vertex | $\boxed{c-ap^2=q}$ y-coordinate of vertex

Note: Can't tell by the graph you need to algebraically calculate

EX #1: Using the graph, determine the vertex, direction of opening, axis of symmetry, max/min value, domain, range and y intercept

$f(x) = 2x^2 - 12x + 25$



To find the vertex use $p=\frac{-b}{2a}$, because you are given the quadratic function in standard form.

$y=2x^2-12x+25$

$a=2$ $b=-12$ $c=25$

x-coordinate of vertex: $p=\frac{-(-12)}{2(2)}$

$p=\frac{12}{4}$

$p=3$

y-coordinate of vertex

Method #1

$c=25$ $a=2$ $p=3$

Substitute into $q=c-ap^2$

$q=25-(2)(3)^2$

$q=25-18$

$q=7$

Method #2

substitute into "p value" into function for x to find y-value

$y=2(3)^2-12(3)+25$

$y=18-36+25$

$y=7$

To find y-int set x=0 and solve for y. or c = y int

Vertex: (3,7) y-int (0,25)

D. of opening: up

a. of sym.: x=3

Min value at y=7

Domain $(-\infty,\infty)$

Range $[7,\infty)$

$p=-\frac{b}{2a}$ $q=c-ap^2$

EX #2: Without looking at a graph, determine the same information as in example 1 for the following:

a) $y = x^2 + 6x + 5$

Vertex: (3,-4)
Directions of opening: up
Equation of axis of symmetry: x=3
Min or max Value: Min at y=-4
Domain: $(-\infty,\infty)$
Range: $[-4,\infty)$
Y- int.: (0,5)

$p=\frac{-6}{2}$ $q=5-(3)^2$

$p=3$ $q=5-9$

$q=-4$

vertex (3,-4)

b) $y = -x^2 + 2x + 3$

Vertex: (1,4)
Direction of opening: down
Equations of the axis of symmetry: x = 1
Min or Max Value: Max at y=4
Domain: $(-\infty,\infty)$
Range: $(-\infty,4]$
Y- Int.: (0,3)

$p=\frac{-2}{2(-1)}$ $q=3-(-1)(1)^2$

$p=1$ $q=4$

or

$y=-(1)^2+2(1)+3$
$y=-1+2+3$
$y=1+3$
$y=4$

Concept #5: Solve situational problems involving Quadratics in standard form $y = ax^2 + bx + c$

EX #3:
A diver jumps from a 3-m springboard with an initial vertical velocity of 6.8 m/s and hits the water after approx.. 1.74 secs. Her height, *h*, in metres, above the water *t* seconds after leaving the diving board can be modelled by the function $h(t) = -4.9t^2 + 6.8t + 3$.

a) Graph the function by finding the vertex, x intercept(s), y intercept and it's reflection. Calculate using the axis of symmetry
b) What does the *y*-intercept represent?
c) What maximum height does the diver reach? When does she reach that height?
d) What domain and range are appropriate in this situation?
e) What is the height of the diver 0.6 s after leaving the board?

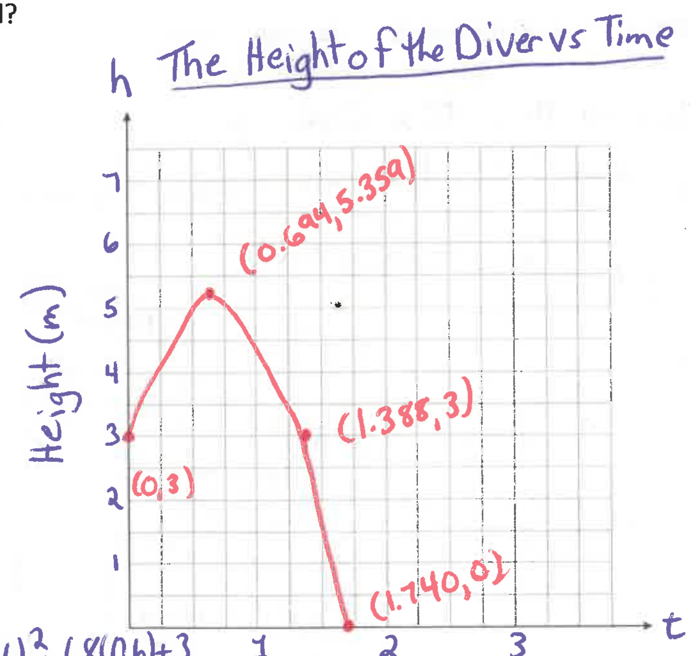
a) Vertex

$p=\frac{-6.8 \times 10}{2(-4.9) \times 10}$ $q=3-(-4.9)\left(\frac{34}{49}\right)^2$

$q\approx 5.359$

$p=\frac{68 \div 2}{98 \div 2}$ vertex=(0.694,5.359)

$p=\frac{34}{49}\approx 0.694$



b) The y-int. represents the height the diver began at.

c) The diver reaches a max. height of 5.359m at 0.694secs

3.2 Assignment Pg 174 #1,2,3,6,7a-d,11

d) Domain [0, 1.740]
Range [0, 5.359]

f) $h(0.6)=-4.9(0.6)^2+6.8(0.6)+3$
$=5.316$
The height of the diver after 0.6secs is 5.316m

3.3 Changing from Standard Form to Vertex form (Completing the Square)

Concept: To change the form of a quadratic function from Standard Form, $y = ax^2 + bx + c$, to Vertex Graphing Form, $y = a(x - p)^2 + q$ (note that it is sometimes called $y = a(x - h)2 + k$

Quadratic functions are often written in ***standard form***: $y = ax^2 + bx + c$. While it is easy to find the *y*-intercept when the function is given in this form, it is more difficult to graph the function.

We will use a procedure called ***completing the square*** to rewrite the function in vertex form, $y = a(x-p)^2 + q$, so that we can more easily identify the characteristics and graph the function.

Completing The Square

Ex. 1/ Rewrite the function $y = x^2 + 6x - 4$ to VERTEX form (From the form $y = ax^2 + bx + c$ to the form $y = a(x - p)^2 + q$)

What is the vertex of this function?

$y \overset{+4}{=} x^2 + 6x - 4 + 4$

$y + 4 \overset{+9}{=} x^2 + 6x + 9$

$y + 13 \overset{-13}{=} (x+3)^2 - 13$

$y = (x+3)^2 - 13$

Vertex: $(-3, -13)$

$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$

STEPS FOR WRITING IN VERTEX FORM WHEN a = 1

1. Move the value of "c" to the left side of the function.
2. When the value of a = 1, calculate the value of $\left(\frac{b}{2}\right)^2$ and add this value to both sides of the function.
3. Simplify the left side of the function and factor the right side of the function. Note that the function will now always factor into a perfect square of the form $\left(x \pm \frac{\square}{\square}\right)^2$
4. Move the constant on the left side to the right side.

Ex2. Rewrite the following in vertex form. What is the max/min of each parabola?

a) $y = x^2 + 8x - 5$

$y \overset{+5}{=} x^2 + 8x - 5 + 5$

$y + 5 \overset{+16}{=} x^2 + 8x + 16$

$y + 21 \overset{-21}{=} (x+4)^2 - 21$

$y = (x+4)^2 - 21$

Min. at $y = -21$

$\left(\frac{8}{2}\right)^2 = 4^2 = 16$

b) $y = x^2 + 9x - 1$

$y \overset{+1}{=} x^2 + 9x - 1 + 1$

$y + 1 \overset{+\frac{81}{4}}{=} x^2 + 9x + \frac{81}{4}$

$y + \frac{4}{4} + \frac{81}{4} = \left(x^2 + 9x + \frac{81}{4}\right)$

$y + \frac{85}{4} \overset{-85/4}{=} \left(x + \frac{9}{2}\right)^2 - 85/4$

$y = \left(x + \frac{9}{2}\right)^2 - \frac{85}{4}$

Min. at $-\frac{85}{4}$ or 21.25

$\left(\frac{9}{2}\right)^2 = \frac{81}{4}$

STEPS WRITING IN VERTEX FORM WHEN $a \neq 1$

1. Move the value of "c" to the left side of the function.
2. Factor out the value of "a" on the right side (even if it doesn't factor out evenly you must factor it out! To factor it out of "b" when it isn't divisible by "a" you will turn the term into $\frac{b}{a}x$) **Leave a short space inside the brackets on the right end.**
3. Calculate the value of $\left(\frac{b}{2}\right)^2$ and add this value to THE RIGHT SIDE side of the function only (add this value where you left the space inside the brackets)
4. Now calculate the value of $a \bullet \left(\frac{b}{2}\right)^2$ and add this value to the LEFT side the function.
5. Simplify the left side of the function and factor the right side of the function. Note that the function will now always factor into a perfect square of the form $a\left(x \pm \frac{\square}{\square}\right)^2$
6. Move the constant on the left side to the right side.

Ex3. Rewrite the following in Vertex form:

a) y = 2x² – 16x + 11

$y \overset{-11}{=} 2x^2 - 16x + 11^{-11}$

$y - 11 = 2x^2 - 16x$

$\quad +16(2)$

$y - 11 = 2(x^2 - 8x + 16)$

$y - 11 + 32 = 2(x-4)^2$

$y + 21 \overset{-21}{=} 2(x-4)^2 - 21$

$\boxed{y = 2(x-4)^2 - 21}$

$\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

b) $y = -3x^2 - 27x + 13$

$y \overset{-13}{=} -3x^2 - 27x + 13^{-13}$

$y - 13 = -3x^2 - 27x$

$\quad +\frac{81}{4}(-3)$

$y - 13 = -3\left(x^2 + 9x + \frac{81}{4}\right)$

$y - 13 - \frac{243}{4} = -3\left(x + \frac{9}{2}\right)^2$

$y - \frac{295}{4} \overset{+295/4}{=} -3\left(x + \frac{9}{2}\right)^2 + \frac{295}{4}$

$\boxed{y = -3\left(x + \frac{9}{2}\right)^2 + \frac{295}{4}}$

$\left(\frac{9}{2}\right)^2 = \frac{81}{4}$

c) $y = \frac{1}{3}x^2 + 2x - 9$ $+9$ $+9$

$y + 9 = \frac{1}{3}x^2 + 2x$ $\div\frac{1}{3}$ $\div\frac{1}{3}$

$\quad +9\left(\frac{1}{3}\right)$

$y + 9 = \frac{1}{3}(x^2 + 6x + 9)$

$2 \div \frac{1}{3}$
$= 2 \times \frac{3}{1}$
$= 6$

$\left(\frac{6}{2}\right)^2 = 3^2 = 9$

$y + 9 + \frac{9}{3} = \frac{1}{3}(x^2 + 6x + 9)$

$y + 12 \overset{-12}{=} \frac{1}{3}(x+3)^2 - 12$

$\boxed{y = \frac{1}{3}(x+3)^2 - 12}$

d) y = -5x² – 8x

$y = \frac{-5x^2}{-5} - \frac{8x}{-5}$

$\quad +\frac{64}{100}(-5)$

$y = -5\left(x^2 + \frac{8}{5}x + \frac{64}{100}\right)$

$y - \frac{320}{100} \overset{+\frac{320}{100}}{=} -5\left(x + \frac{8}{10}\right)^2 + \frac{320}{100}$

$\boxed{y = -5\left(x + \frac{4}{5}\right)^2 + \frac{16}{5}}$

$\left(\frac{8}{5} \div 2\right)^2$
$= \left(\frac{8}{5} \times \frac{1}{2}\right)^2$
$= \left(\frac{8}{10}\right)^2$
$= \frac{64}{100}$

3.3 Assignment Pg 193 #5d, 7e, 8c, and #1 and 2 on next page

1. Rewrite the following functions in Vertex Form by Completing the Square.

a) $y = x^2 - 4x - 5$ b) $y = x^2 + 6x - 16$ c) $y = x^2 - 8x + 18$

d) $y = x^2 + 10x$ e) $y = x^2 + 9x$ f) $y = x^2 + 3x - 10$

g) $y = -x^2 + 4x + 12$ h) $y = 3x^2 + 12x - 15$ i) $y = 2x^2 + 5x - 3$

j) $y = \frac{1}{3}x^2 + 2x - 4$ k) $y = \frac{1}{2}x^2 + x - 8$

2. Find each of the following answers for each question in #1 above:

i) Vertex **ii)** Axis of Symmetry **iii)** Direction of Opening **iv)** Max/Min **v)**Y intercept **vi)**Domain **vii)** Range

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Answers to #1 and #2:

1a) $y = (x-2)^2 - 9$ b) $y = (x+3)^2 - 25$ c) $y = (x-4)^2 + 2$ d) $y = (x+5)^2 - 25$

e) $y = \left(x + \frac{9}{2}\right)^2 - \frac{81}{4}$ f) $y = \left(x + \frac{3}{2}\right)^2 - \frac{49}{4}$ g) $y = -(x-2)^2 + 16$ h) $y = 3(x+2)^2 - 27$

i) $y = 2\left(x + \frac{5}{4}\right)^2 - \frac{49}{8}$ j) $y = \frac{1}{3}(x+3)^2 - 7$ k) $y = \frac{1}{2}(x+1)^2 - \frac{17}{2}$

2a)	i)(2,-9)	ii)x=2	iii)up	iv)min at y=-9	v)(0,-5)	vi)domain $(-\infty,\infty)$	vii)Range [-9, ∞)
b)	i)(-3,-25)	ii) x=-3	iii)up	iv)min at y=-25	v) (0,-16)	vi) $(-\infty,\infty)$	vii) [-25, ∞)
c)	i)(4,2)	ii)x=4	iii)up	iv)min at y=2	v) (0,18)	vi) $(-\infty,\infty)$	vii) [2, ∞)
d)	i) (-5,-25)	ii)x=-5	iii)up	iv) min at y=-25	v)(0,0)	vi) $(-\infty,\infty)$	vii) [-25, ∞)
e)	i) $\left(-\frac{9}{2}, \frac{-81}{4}\right)$	ii) $x = -\frac{9}{2}$	iii)up	iv)min at y = $\frac{-81}{4}$	v)(0,0)	vi) $(-\infty,\infty)$	vii) [$\frac{-81}{4}$, ∞)
f)	i) $\left(\frac{-3}{2}, \frac{-49}{4}\right)$	ii)x = $\frac{-3}{2}$	iii)up	iv)min at y= $\frac{-49}{4}$	v) (0,-10)	vi) $(-\infty,\infty)$	vii) $\left[\frac{-49}{4}, \infty\right)$
g)	i)(2,16)	ii) x=2	iii) down	iv) max at y=16	v)(0,12)	vi) $(-\infty,\infty)$	vii) $(-\infty, 16]$
h)	i)(-2, -27)	ii)x = -2	iii)up	iv)min at y= -27	v) (0,-15)	vi) $(-\infty,\infty)$	vii) $[-27, \infty)$
i)	i) $\left(\frac{-5}{4}, \frac{-49}{8}\right)$	ii) $x = \frac{-5}{4}$	iii)up	iv) min at y= $\frac{-49}{8}$	v) (0,-3)	vi) $(-\infty,\infty)$	vii) $\left[\frac{-49}{8}, \infty\right)$
j)	i)(-3, -7)	ii) x = -3	iii)up	iv)min at y=-7	v)(0,-4)	vi) $(-\infty,\infty)$	vii) $[-7, \infty)$
k)	i) $\left(-1, \frac{-17}{2}\right)$	ii)x=-1	iii)up	iv) min at y =-17/2	v) (0,-8)	vi) $(-\infty,\infty)$	vii) $\left[-\frac{17}{2}, \infty\right)$

3.3 Day 2 Completing the square

Concept: To solve situational questions involving maximums and minim ums of a quadratic functions.

Ex.1/ Two numbers have a sum of 29 and a product that is a maximum. Determine the two numbers and the maximum product.

Let $x = 1^{st}$ #
$y = 2^{nd}$ #
$P =$ product

$x + y = 29$ $\quad -x \quad -x$

$y = 29 - x$ substitute

$P = xy$

$P = x(29 - x)$

$P = 29x - x^2$ complete the square to find the maximum Product

$P = -x^2 + 29x$

$P + \frac{841}{4}(-1) = -1\left(x^2 - 29x + \frac{841}{4}\right)$ $\quad \left(\frac{29}{2}\right)^2 = \frac{841}{4}$

$P - \frac{841}{4} + \frac{841}{4} = -1\left(x - \frac{29}{2}\right)^2 + \frac{841}{4}$

$P = -1(x - 29/2)^2 + \frac{841}{4}$

The maximum product is $\frac{841}{4} = 210.25$ and the two #'s are $\frac{29}{2} = 14.5$ and 14.5

Ex2./A sporting goods store sells reusable sports water bottles for \$8. At this price their weekly sales are approximately 100 items. Research says that for every \$2 increase in price, the manager can expect the store to sell five fewer water bottles. Determine the maximum revenue the manager can expect based on these estimates. What selling price will give that maximum revenue?

Let $R =$ revenue
$x =$ # of \$2 increases

$R = \underbrace{(8 + 2x)}_{\text{Price of water bottle}}\underbrace{(100 - 5x)}_{\text{# of water bottles}}$

$R = 800 - 40x + 200x - 10x^2$

$R - 800 = -10x^2 + 160x + 800 - 800$

complete the square to convert to vertex form to find the max. revenue

$R - 800 = -10x^2 + 160x$

$R - 800 + 64(-10) = -10(x^2 - 16x + 64)$ $\quad \left(\frac{-16}{2}\right)^2 = 64$

$R - 800 - 640 = -10(x - 8)^2$

$R - 1440 + 1440 = -10(x - 8)^2 + 1440$

$R = -10(x - 8)^2 + 1440$

vertex $(8, 1440)$
8: # of water bottles
1440: max. revenue

With 8 - \$2 increases so an increase to \$24 a water bottle the max. revenue will be \$1440 weekly

The problem assumes that price affects the # of sales and therefore, the revenue. Other factors that may affect sales include quality of the product, safety of the materials used in the production of the product, weather, and motivation of ppl to buy waterbottles

Enriched Pre-Calc 20 (SUNDEEN) Outcome P20.7and 6 Chapter 3 – Quadratic Functions

1. Solve the following problems algebraically using the vertex formula and completing the square.
   a) What is the maximum product that two numbers can have if their sum is 100? What are the two numbers?

   b) One number is 10 larger than another. What is the smallest possible value for the sum of their squares? What are the two numbers?

2. A farmer wishes to fence in a rectangular pen using his barn as one of the sides of the rectangle. If the farmer has 40 m of fencing, what is the largest area that can be enclosed? What are the dimensions of the rectangle?

3. The managers of a business are examining costs. It is more cost-effective for them to produce more items. However, if too many items are produced, their costs will rise because of factors such as storage and overstock. Suppose that they model the cost, *C*, of producing *n* thousand items with the function: $C(n) = 75n^2 - 1800n + 60\ 000$. Determine the number of items that will minimize their costs.

4. A gymnast is jumping on a trampoline. His height, *h*, in metres, above the floor on each jump is roughly approximated by the function $h(t) = -5t^2 + 10t + 4$, where t represents the time, in seconds, since he left the trampoline. Determine his maximum height on each jump.

5. Sandra is practicing at an archery club. The height, *h*, in feet, of the arrow on one of her shots can be modelled as a function of time, t in seconds, since it was fired using the function $h(t) = -16t^2 + 10t + 4$. What is the maximum height of the arrow, in feet, and when does it reach that height?

Answers:

1. a) Max product is 2500 when the numbers are 50 and 50 b) Min sum is 50 when the numbers are -5 and 5
2. Max area is 200 m² when the rectangle is 10m by 20m.
3. 12 000 items
4. 9m
5. Max height is 5.56ft after 0.31 seconds being shot

Extension
Pg 197 #28