

Rational expression – an algebraic expression with a numerator and a denominator that are polynomials.

Example: $\frac{1}{x}$, $\frac{m}{m+1}$, $\frac{y^2-1}{y^2+2y+1}$ x^2-4 & note: has a denominator of 1

Non-permissible values – a value for a variable that makes the expression undefined. In rationals, the denominator $\neq 0$. (NPV)

Example: What are the non-permissible values for x in the following? To determine NPV set the factored denominator equal to zero

1. $\frac{6-x}{2x}$ NPV
 $\frac{2x}{2} = \frac{0}{2}$
 $x \neq 0$

2. $\frac{3}{x-7}$ NPV
 $x-7 = 0+7$
 $x \neq 7$

3. $\frac{4x-1}{x^2+4x+3}$ Step 1 Factor the denominator
 $= \frac{4x-1}{(x+3)(x+1)}$ Step 2 Set both factors equal to zero and solve for "x"
 $x+3 = 0^{-3}$ $x+1 = 0^{-1}$
 $x \neq -3$ $x \neq -1$

4. $\frac{5t}{4sr^2}$ Step 1 Set both factors equal to zero and solve for variables
 NPV
 $\frac{4s}{4} = 0$ $\sqrt{r^2} = 0$
 $s \neq 0$ $r \neq 0$

Simplifying Rational Expressions

- divide out common factors in the numerator and denominator.

Example: Simplify and state non-permissible values.

1. $\frac{9 \div 3}{12 \div 3}$
 $= \frac{1}{4}$ No variables therefore no Non-permissible values

2. $\frac{12m^2t^5}{3mt}$ Divide to simplify
 $= 4mt^4$ NPV
 $m \neq 0$ $t \neq 0$

Note: To find non permissable values always use factored original denominator before simplifying

3. $\frac{3x-6}{x-2}$ ① Factor numerator and denominator

$= \frac{3(x-2)}{x-2}$

② Find NPV

③ Divide common factors

NPV
 $x-2=0$
 $x \neq 2$

$= 3, x \neq 2$

Simplified answer Non-permissible values

5. $\frac{2y^2+y-10}{y^2+3y-10}$

Note: Do factoring work off to the side

$= \frac{(2y+5)(y-2)}{(y+5)(y-2)}$

$(y+5)(y-2)$

$= \frac{2y+5}{y+5}, y \neq 2, x \neq -5$

NPV
 $x+5=0$
 $x \neq -5$
 $y-2=0$
 $y \neq 2$

4. $\frac{x^2+2x-15}{x-3}$

$= \frac{(x+5)(x-3)}{x-3}$

$= x+5, x \neq 3$

NPV
 $x-3=0$
 $x=3$

6. $\frac{x^2-10x+24}{x^2-6x}$

$= \frac{(x-6)(x-4)}{x(x-6)}$

$= \frac{(x-4)}{x}, x \neq 0, x \neq 6$

NPV
 $x \neq 0$
 $x-6 \neq 0$
 $x \neq 6$



7. $\frac{1-t}{t^2-1}$

$= \frac{(1-t)}{(t-1)(t+1)}$

$= \frac{-1(t-1)}{(t-1)(t+1)}$

$= \frac{-1}{t+1}, t \neq 1, -1$

Notice: the numerator is the opposite of one of the factors in the denominator

Factor a -1 out of the numerator

NPV
 $t-1=0$
 $t \neq 1$
 $t+1=0$
 $t \neq -1$



8. $\frac{25-x^2}{x^2-3x-10}$

$= \frac{(5-x)(5+x)}{(x-5)(x+2)}$

Factors are opposite
 Take out a -1

$= \frac{-1(x-5)(5+x)}{(x-5)(x+2)}$

$= \frac{-1(5+x)}{(x+2)}, x \neq 5, x \neq -2$

NPV
 $x-5 \neq 0$
 $x \neq 5$
 $x+2=0$
 $x \neq -2$

NOTE:

6.2 Multiplying and Dividing Rational Expressions

- Factor numerators and denominators
- Divide out common terms

Examples: Multiply. State non-permissible values.

$$\begin{aligned}
 1. & \frac{5}{8} \cdot \frac{2}{12} \\
 &= \frac{5 \cdot 1}{4 \cdot 12} \\
 &= \frac{5}{48}
 \end{aligned}$$

$$\begin{aligned}
 2. & \frac{4x^3}{3xy} \cdot \frac{y^4}{8} \quad \text{NPV} \\
 & \quad \quad \quad x \neq 0 \quad y \neq 0 \\
 &= \frac{xy^4}{6}, xy \neq 0
 \end{aligned}$$

$$\begin{aligned}
 3. & \left(\frac{a^2 - a - 12}{a^2 - 9} \right) \left(\frac{a^2 - 4a + 3}{a^2 - 4a} \right) \\
 &= \frac{(a-4)(a+3)}{(a-3)(a+3)} \cdot \frac{(a-1)(a-3)}{(a)(a-4)} \leftarrow \text{NPV's} \\
 &= \frac{(a-1)}{a}, a \neq 3, -3, 0, 4
 \end{aligned}$$

$$\begin{aligned}
 4. & \frac{x^2 - 25}{x^2 - 49} \cdot \frac{x^2 - 6x - 7}{x^2 + 6x + 5} \\
 &= \frac{(x-5)(x+5)}{(x-7)(x+7)} \cdot \frac{(x-7)(x+1)}{(x+5)(x+1)} \\
 &= \frac{(x-5)}{(x+7)}; x \neq 7, -7, -5, -1
 \end{aligned}$$

Step 1
Factor numerators & denominators

Step 2
Find NPV's of all factors in the denominator

Step 3
Cancel same factors that are in numerator & denominator

NPV's

$$\begin{array}{ccc}
 a-3 \neq 0 & a+3 \neq 0 & a \neq 0 \\
 a \neq 3 & a \neq -3 & a-4 \neq 0 \\
 & & a \neq 4
 \end{array}$$

NPV's

$$\begin{array}{cccc}
 x-7 \neq 0 & x+7 \neq 0 & x+5 \neq 0 & x+1 \neq 0 \\
 x \neq 7 & x \neq -7 & x \neq -5 & x \neq -1
 \end{array}$$

Dividing Rationals

- Multiply by the reciprocal of the rational following the division sign
- Divide out common terms only when multiplying
- Restrictions(Non – Permissible Values) apply to anything in the denominator at any time

Reciprocals- Interchange the numerator and denominator

• Examples: $\frac{1}{2} \rightarrow \frac{2}{1}$, $\frac{x^2}{y} \rightarrow \frac{y}{x^2}$, $\frac{a+5}{a} \rightarrow \frac{a}{a+5}$

Examples: Divide and simplify. State non-permissible values.

1. $\frac{6}{5} \div \frac{3}{2}$
 $= \frac{6^2}{5} \times \frac{2}{3}$
 $= \frac{4}{5}$

2. $\frac{3x^2}{y^2} \div \frac{x}{y}$ NPV's
 $= \frac{3x^2}{y^2} \times \frac{y}{x}$ * multiply by the reciprocal $y \neq 0$
 $x \neq 0$
 $= \frac{3x}{y}; y \neq 0, x \neq 0$

3. $\frac{x^2-4}{x^2-4x} \div \frac{x^2+x-6}{x^2+x-20}$
 $= \frac{(x-2)(x+2)}{x(x-4)} \div \frac{(x+3)(x-2)}{(x+5)(x-4)}$
 $= \frac{(x-2)(x+2)}{x(x-4)} \times \frac{(x+5)(x-4)}{(x+3)(x-2)}$ * multiply by the reciprocal
 $= \frac{(x+2)(x+5)}{x(x+3)}; x \neq -5, 4, 0, -3, 2$

NPV's
 $x \neq -5$
 $x \neq 4$
 $x \neq 0$
 $x \neq -3$
 $x \neq 2$

4. $\frac{3x+12}{3x^2-5x-12} \div \frac{12}{3x+4} \times \frac{2x-6}{x+4}$
 $= \frac{3(x+4)}{(3x+4)(x-3)} \div \frac{12}{3x+4} \times \frac{2(x-3)}{x+4}$
 $= \frac{3(x+4)}{(3x+4)(x-3)} \times \frac{3x+4}{12} \times \frac{2(x-3)}{x+4}$
 $= \frac{1}{2}; x \neq -\frac{4}{3}, 3, -4$

$3x \quad 4 \quad 4x$
 $x \quad -3 \quad -9x$
 $-5x$
NPV's
 $x \neq -\frac{4}{3}$
 $x \neq 3$
 $x \neq -4$

6.3 Adding/Subtracting Rational Expressions

Review Adding and Subtracting Fractions

a. $\frac{5}{7} - \frac{3}{7}$
 $= \frac{2}{7}$

b. $\frac{3 \cdot 2}{5 \cdot 2} + \frac{1}{10}$
 $= \frac{6+1}{10} = \frac{7}{10}$

Examples: Simplify the following rational expressions, leave answers in simplest form and state non-permissible values

Steps for Add/Subt. Rational Expressions

- Must have a common denominator (add numerators, leave denominators)
- Answer in simplest form (Reduce)
- State any non – permissible values (NPV)

1. $\frac{2a}{b} - \frac{a-1}{b}$
 $= \frac{2a - (a-1)}{b}$
 $= \frac{2a - a + 1}{b}$
 $= \frac{a+1}{b}; b \neq 0$

Note: When subtracting make sure it applies to all terms.

2. $\frac{x^2}{x-2} + \frac{3x}{x-2} - \frac{10}{x-2}$
 $= \frac{x^2 + 3x - 10}{x-2}$
 $= \frac{(x+5)(x-2)}{x-2}$
 $= x+5; x \neq 2$

NPV's
 $x \neq 2$

3. $\frac{x \cdot 2x}{x \cdot xy} + \frac{4 \cdot y}{x^2 \cdot y}$
 $= \frac{2x^2}{x^2y} + \frac{4y}{x^2y}$
 $= \frac{2x^2 + 4y}{x^2y}$
 $= \frac{2(x^2 + 2y)}{x^2y}; x \neq 0, y \neq 0$

** create a common denominator by multiply to create the LCD*

4. $\frac{y^2-20}{y^2-4} + \frac{y-2}{y+2}$
 $= \frac{y^2-20}{(y-2)(y+2)} + \frac{y-2}{y+2} \cdot \frac{(y-2)}{(y-2)}$
 $= \frac{y^2-20 + [(y-2)(y-2)]}{(y-2)(y+2)}$
 $= \frac{y^2-20 + [y^2-4y+4]}{(y-2)(y+2)}$
 $= \frac{2y^2-4y-16}{(y-2)(y+2)}$
 $= \frac{2(y^2-2y-8)}{(y-2)(y+2)}$
 $= \frac{2(y-4)(y+2)}{(y-2)(y+2)}$
 $= \frac{2(y-4)}{(y-2)}; y \neq \pm 2$

- Step 1**
Factor numerators & denominators
- Step 2**
Create a common denominator
- Step 3**
Add numerators, denominators stay the same
- Step 4**
Factor & simplify
- Step 5**
State all NPV's

$$\begin{aligned}
 5. & \frac{x-1}{x^2+x-6} - \frac{x-2}{x^2+4x+3} \\
 &= \frac{x-1 \cdot (x+1)}{(x+3)(x-2) \cdot (x+1)} - \frac{x-2 \cdot (x-2)}{(x+3)(x+1) \cdot (x-2)} \\
 &= \frac{x^2-1}{(x+3)(x-2)(x+1)} - \frac{(x^2-4x+4)}{(x+3)(x-2)(x+1)} \\
 &= \frac{x^2-1-x^2+4x-4}{(x+3)(x-2)(x+1)} \\
 &= \frac{4x-5}{(x+3)(x-2)(x+1)} ; x \neq -3, 2, -1
 \end{aligned}$$

~~$$\begin{aligned}
 7. & \frac{x^2-49}{x^2-8x+7} + \frac{2-2x}{x^2-1} \\
 &= \frac{(x-7)(x+7)}{(x-7)(x-1)} + \frac{-2(1-x)}{(x-1)(x+1)} \\
 &= \frac{(x+7) \cdot (x+1)}{(x-1) \cdot (x+1)} + \frac{2(1-x)}{(x-1)(x+1)} \\
 &= \frac{(x+7)(x+1) + 2(1-x)}{(x-1)(x+1)} \\
 &= \frac{x^2+8x+7+2-2x}{(x-1)(x+1)} \\
 &= \frac{x^2+6x+9}{(x-1)(x+1)}
 \end{aligned}$$~~

$$6. \frac{1+\frac{1}{x}}{x-\frac{1}{x}}$$

$$\begin{aligned}
 &= \left(\frac{1+\frac{1}{x}}{1 \cdot x} \right) \div \left(\frac{x-\frac{1}{x}}{1 \cdot x} \right) \\
 &= \left(\frac{x+\frac{1}{x}}{x} \right) \div \left(\frac{x^2-\frac{1}{x}}{x} \right) \\
 &= \left(\frac{x+1}{x} \right) \div \left(\frac{x^2-1}{x} \right) \\
 &= \frac{(x+1)}{x} \cdot \frac{x}{x^2-1} \\
 &= \frac{(x+1) \cancel{x}}{\cancel{x} \cdot (x-1)(x+1)} \\
 &= \frac{1}{x-1} ; x \neq 0, \pm 1
 \end{aligned}$$

$$\begin{aligned}
 8. & \frac{x+1}{x+6} - \frac{x^2-4}{x^2+2x} \div \frac{2x^2+7x+3}{2x^2+x} \quad \star \text{ Don't forget order of operations} \\
 &= \frac{x+1}{x+6} - \left[\frac{(x-2)(x+2)}{x(x+2)} \right] \div \frac{(2x+1)(x+3)}{x(2x+1)} \\
 &= \frac{x+1}{x+6} - \left[\frac{(x-2)}{x} \cdot \frac{x}{(x+3)} \right] \\
 &= \frac{x+1}{x+6} - \left[\frac{(x-2) \cdot \cancel{x} \cdot (x+6)}{\cancel{x} (x+3) \cdot (x+6)} \right] \\
 &= \frac{(x+3)(x+1) - [(x-2)(x+6)]}{(x+3)(x+6)} \\
 &= \frac{x^2+4x+3 - (x^2+4x-12)}{(x+3)(x+6)} \\
 &= \frac{15}{(x+3)(x+6)}
 \end{aligned}$$

$$x \neq 0, -2, -6, -\frac{1}{2}, -3$$

6.4 Rational Equations

- An equation containing at least one rational (fractional) expression
- TO solve , find the LCD and multiply each term by the LCD to eliminate the fractions.
- Check. There may be extraneous roots

Review

$3 \cdot \frac{1}{2}x - \frac{2}{3} = \frac{1}{2} \cdot 3$ LCD = 6

$$\frac{3x}{2} - \frac{4}{3} = \frac{3}{2}$$

$$3x - 4 = 3 + 4$$

$$3x = \frac{7}{3} \quad x = \frac{7}{9}$$

Example #1 Solve the following rational equations. Check for extraneous roots.

a) $\frac{x}{4} - \frac{7}{x} = 3$ LCD = 4x

(4x) $\frac{x}{4} - \frac{7(4x)}{x} = 3(4x)$

$$x^2 - 28 = 12x - 12x$$

$$x^2 - 12x - 28 = 0$$

$$(x - 14)(x + 2) = 0$$

$x - 14 = 0 \Rightarrow x = 14$ $x + 2 = 0 \Rightarrow x = -2$

check $x = 14$

$$\frac{14}{4} - \frac{7}{14} = 3$$

$$\frac{196}{56} - \frac{28}{56} = 3$$

$$\frac{168}{56} = 3$$

$$3 = 3 \checkmark$$

The solution are $x = 14, -2$

c) $\frac{x}{x-2} + \frac{2}{x+2} = 1$

(x-2)(x+2) $\frac{x}{x-2} + \frac{2}{x+2} = 1(x-2)(x+2)$

$$x(x+2) + 2(x-2) = (x-2)(x+2)$$

$$x^2 + 2x + 2x - 4 = x^2 - 4 + 4$$

$$4x = 0$$

$$\frac{4x}{4} = \frac{0}{4}$$

$$x = 0$$

check $x = 0$

$$\frac{0}{0-2} + \frac{2}{0+2} = 1$$

$$0 + \frac{2}{2} = 1$$

$$1 = 1 \checkmark$$

$x = \{0\}$

b) $\frac{2x}{x-4} = \frac{10}{x-4}$

$\frac{2x}{2} = \frac{10}{2}$

$$x = 5$$

check $x = 5$

$$\frac{2(5)}{5-4} = \frac{10}{5-4}$$

$$10 = 10 \checkmark$$

The solution is $x = 5$

d) $\frac{9}{y-3} - \frac{4}{y-6} = \frac{18}{y^2 - 9y + 18}$

$\frac{9(y-6)(y-3)}{(y-3)(y-6)} - \frac{4(y-3)(y-3)}{(y-3)(y-6)} = \frac{18}{(y-3)(y-6)}$

$9(y-6) - 4(y-3) = 18$

$9y - 54 - 4y + 12 = 18$

$5y - 42 = 18 + 42$

$\frac{5y}{5} = \frac{60}{5}$

$y = 12$

$y = \{12\}$

check $y = 12$

$\frac{9}{12-3} - \frac{4}{12-6} = \frac{18}{12^2 - 9(12) + 18}$

$\frac{9 \cdot 6}{9 \cdot 6} - \frac{4 \cdot 6}{6 \cdot 6} = \frac{18}{54}$

$\frac{54}{54} - \frac{24}{54} = \frac{18}{54}$

$\checkmark \frac{18}{54} = \frac{18}{54}$

e) $\frac{3x}{x+2} - \frac{5}{x-3} = \frac{-25}{x^2-x-6}$

$(x-3)(x+2) \frac{3x}{x+2} - \frac{5}{x-3} = \frac{-25}{(x-3)(x+2)}$

check w nprv

$x \neq 3$

$x = \frac{5}{3}$

$3x(x-3) - 5(x+2) = -25$

$3x^2 - 9x - 5x - 10 = -25$

$3x^2 - 14x - 10 = -25$

$3x^2 - 14x + 15 = 0$

$3x - 5$
 $x - 3$

$(3x-5)(x-3) = 0$

$x = \frac{5}{3} \quad x = 3$

Day 1 Assign Pg 348 # 1, 2, 3, 6, 8, 11

Day 2

Example #2 Two friends share a paper route. Sheena can deliver the papers in 40min. Jeff can cover the same route in 50min. How long, to the nearest minute, does the paper route take if they work together?

Make a table to organize the information

	Time to Deliver Papers (min)	Fraction of work done in 1min	Fraction of work done in t mins
Sheena	40mins	$\frac{1}{40}$	$\frac{1}{40}(t)$ or $\frac{t}{40}$
Jeff	50mins	$\frac{1}{50}$	$\frac{t}{50}$
Together	t	$\frac{1}{t}$	$\frac{t}{t}$ or 1

The equation for Sheena & Jeff to complete the work together is:

$\frac{t}{40} + \frac{t}{50} = 1$

$5t + 4t = 200$

$\frac{9t}{9} = \frac{200}{9}$

$t = \frac{200}{9}$ or approx. 22.2 min

check solution because there are no non-permissible values.

$\left[\frac{200}{9}\right] \left[\frac{1}{40}\right] + \left[\frac{200}{9}\right] \left[\frac{1}{50}\right] = 1$

$\frac{200}{9} \left(\frac{1}{40}\right) + \frac{200}{9} \left(\frac{1}{50}\right) = 1$

$\frac{5}{9} + \frac{4}{9} = 1$

$\frac{9}{9} = 1 \checkmark$

6.4 Rational Equations Word Problems (Day 2)

Example #1 Two friends share a paper route. Sheena can deliver the papers in 40min. Jeff can cover the same route in 50min. How long, to the nearest minute, does the paper route take if they work together?

Make a table to organize the information

Sheena			
Jeff			
Together			

Example #2 A train has a scheduled run of 160km between two cities in Saskatchewan. If the average speed is decreased by 16km/h, the run will take 1/2 hour longer. What is the average speed of the train?

Note: Distance = Rate * time

Note: $t = \frac{D}{R}$

Average speed of train	D(km)	R(km/h)	T(h)
Normal Run	160km	x	$\frac{160}{x}$
Decreased Speed Run	160km	x-16	$\frac{160}{x} + \frac{1}{2}$

↑ Distances are equal

Normal Run Decreased Speed Run

$$R \times t = R \times t$$

$$x \left(\frac{160}{x} \right) = x-16 \left(\frac{160}{x-16} + \frac{1}{2} \right)$$

$$160 = x-16 \left(\frac{320}{2x} + \frac{x}{2x} \right)$$

$$(2x) 160 = x-16 \left(\frac{320+x}{2x} \right) (2x)$$

$$320x = (x-16)(320+x)$$

$$320x = 320x + x^2 - 5120 - 16x$$

$$0 = x^2 - 16x - 5120$$

$$0 = (x^2 - 16x - 5120)$$

$$0 = (x-80)(x+64)$$

x = 80 x = -64 ← extraneous solution

The average speed was 80 km/h

Example #3/ Jerry jogged 9 km in an hour. He covered the last 4 km at a speed that was 2 km/h slower than his speed over the first 5 km. What was his speed over the first 5 km? $D = R \times t$

	Distance	Rate	Time
First 5Km	5Km	x	$\frac{5}{x}$
last 4Km	4Km	$x-2$	$\frac{4}{x-2}$

$$t = \frac{D}{R}$$

$$\frac{5}{x} + \frac{4}{x-2} = 1 \quad (x)(x-2)$$

$$5(x-2) + 4x = x(x-2)$$

$$5x - 10 + 4x = x^2 - 2x - 4x$$

$$0 = x^2 - 11x + 10$$

$$0 = (x - 10)(x - 1)$$

$$\boxed{x = 10} \quad x = 1 \leftarrow \text{Extraneous solution}$$

A speed of 1 Km/h would make his rate negative in the last 4km.

Jerry jogged at a speed of 10 Km/h for the first 5Km.