9.1 Linear Inequalities in 2 Variables

To solve linear inequalities in two variables.

The solution to a problem may be not a single value, but a range of values. A chemical engineer may need a reaction to occur within a certain time frame in order to reduce undesired pollutants. An architect may design a building to deflect less than a given distance in a strong wing. A doctor may chose a dose of medication so that a safe but effective level remains in the body after a specified time.

These situations illustrate the importance of inequalities. While there may be many acceptable values in each of the scenarios above, in each case there is a lower acceptable limit, and upper acceptable limit, or both. Even though many solutions exist, we still need accurate mathematical models and methods to obtain the solutions.

A linear inequality in two variables may be in one of the following four forms.

REVIEW: Graphing Linear Equations

1. Slope/Intercept Method ($y = mx + b$)
2. Find the x and y intercepts. (x-intercept: let y = 0, y-intercept: let x = 0)
3. Table of values (very tedious)

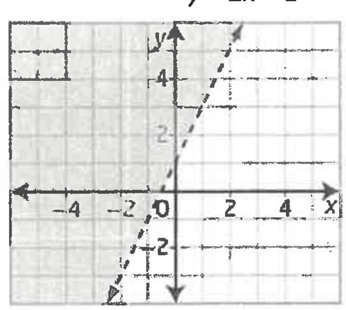
To graph inequalities: 1. Graph the **boundary line.**

- the line is *solid* if the inequality is $\leq$ or $\geq$
- the line is *dashed/dotted* if the inequality is $<$ or $>$

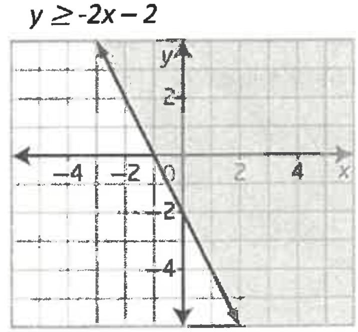
2. Choose a **check point**...a point that does *not* lie on the boundary line. If that point satisfies the equation, **shade** the portion which includes the check point. If the check point does not satisfy the equation, **shade** on the other side of the boundary line. (Note: Typically a good check point is (0,0) except when (0,0) is the y-intercept. or on the boundary line.

Examples of Solution Regions and Boundary Lines

$y > 2x + 1$



$y \geq -2x - 2$



The boundary line is $y = 2x + 1$

The line is dashed because the inequality has no equal to >

Check Pt: Check (0,0)

$0 > 2(0) + 1$

$0 > 1$ Not a true statement ∴ the area where (0,0) is located is not shaded.

The boundary line is $y = -2x - 2$

The line is solid because the inequality

Check Pt: Check (0,0)

$0 \geq -2(0) - 2$

$0 \geq -2$ TRUE ∴ Shade the area where (0,0) is located As it is apart of the solution region.

Reminder: When multiplying or dividing both sides of an inequality by a negative value, you **must** Flip the inequality.

example: Given $x - 5y > 15$, solve for "y".

$$\frac{-5y}{-5} > \frac{-x}{-5} + \frac{15}{-5}$$

Flip

$$y < \frac{1}{5}x - 3$$

Example 1: Graph $4x + 2y \geq 10$. Is (1, 3) part of the solution? Yes, because it is on the solid line.

① Solve for "y"

$$\frac{2y}{2} \geq \frac{-4x}{2} + \frac{10}{2}$$

$$y \geq -2x + 5$$

② Graph the line using the slope (m) and y int. (b)

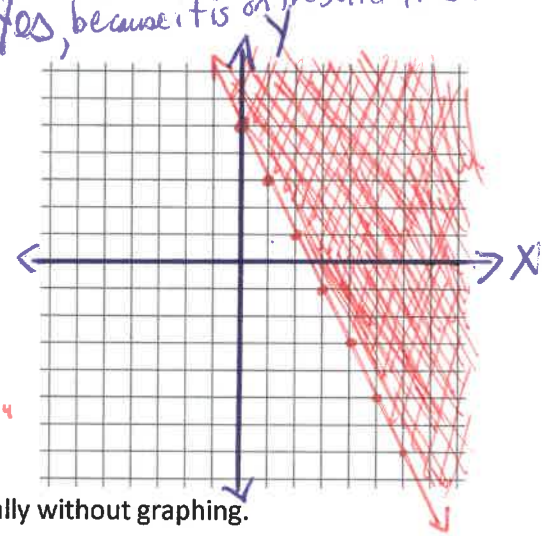
③ Use a check point to see if it is apart of the solution region.

Point (0,0)

$4(0) + 2(0) \geq 10$

$0 \not\geq 10$ "NOT TRUE"

∴ Shade other side



Example 2: Is (2,3) a solution to $-x + y \leq -5$? Solve Algebraically without graphing.

$-(2) + 3 \leq -5$

$1 \not\leq -5$

NOT True so (2,3) is not a point in the solution region for $-x+y \leq -5$

Example 3: a) Graph $5x - 20y < 0$. b) ~~Use a test point to determine what should be shaded.~~

① Solve for "y"

$5x - 20y < 0$ (−5x both sides)

$$\frac{-20y}{-20} < \frac{-5x}{-20}$$ ✱Need to Flip✱

$$y > \frac{1}{4}x$$

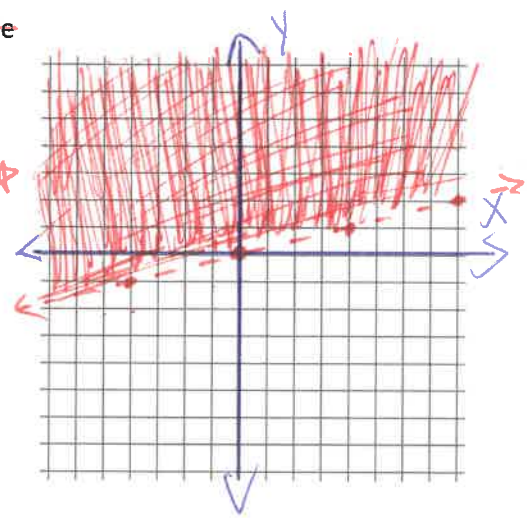
② Graph $m = \frac{1}{4}$ $b = 0$

③ Use checkpoint. Can't use (0,0) as it is not apart of the solution region because it is on a dashed boundary line

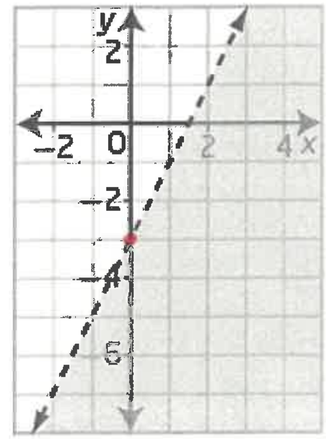
(1,1) $5(1) - 20(1) < 0$

$5 - 20 < 0$

$-15 < 0$ ✓ true



Example 4: Write an inequality to represent the graph at right.



$b = -3$ $m = \frac{rise}{run} = \frac{2}{1}$

Testpoint to determine > or <

$y = 2x - 3$

(1, -4) $-4 \; \square \; 2(1) - 3$

$-4 \; \boxed{<} \; -1$

This inequality makes it a true statement

$\boxed{y < 2x - 3}$

Example 5: Janelle has a budget of $120 for entertainment each month. She usually spends the money on a combination of movies and meals. Movie admission, with popcorn, is $15, while a meal costs $10.

a) Write an inequality to represent the number of movies and meals that Janelle can afford with her entertainment budget. b) Graph the solution c) Interpret your answer. Explain how the solution to the inequality relates to Janelle's situation

Let x = # of movies
y = # of meals

a) $15x + 10y \leq 120$

$15x - 15x + 10y \leq 120 - 15x$

$\frac{10y}{10} \leq \frac{-15x}{10} + \frac{120}{10}$

$y \leq -\frac{3}{2}x + 12$

Janelles budget options

c) The solution region represents all the possibilities of meals + movies she can attend while staying within her budget.

9.1 Assignment: Page 472 #1bc,2ac, 3ace, 4 ,8ce, 9 (11abc, 12ab, 13)

9.3 Quadratic Inequalities in Two Variables

- Graphically these inequalities represent a solution region and a **PARABOLIC BOUNDARY CURVE**
- ~~The solution is the set of points (x,y) that satisfy the inequality~~
- ~~In order to solve quadratic inequalities that contain two variables we are going to solve by GRAPHING and using a test point~~

Steps: 1. Graph the quadratic equation (parabola)

2. Decide if the boundary is solid ($\leq$, $\geq$) or dashed (<, >)

3. Use a test point to determine which region is the solution. Shade either ***inside*** the parabola or ***outside*** of the parabola.

Example#1: Graph $y < -2(x-3)^2 + 8$. Determine if (2, -4) is a solution graphically and algebraically.

Vertex (3,8)

X-intercepts

$0 = -2(x-3)^2 + 8$

$0 = -2(x-3)(x-3) + 8$

$0 = (-2x+6)(x-3) + 8$

$0 = -2x^2 + 6x + 6x - 18 + 8$

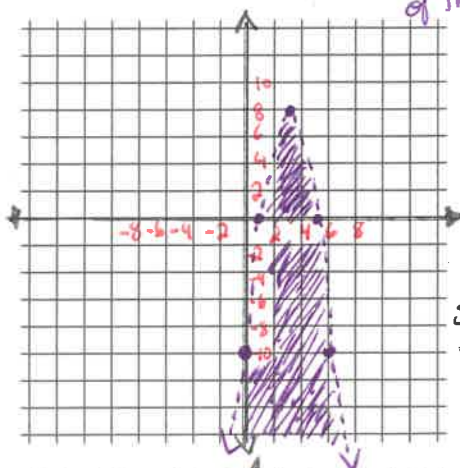
$0 = -2x^2 + 12x - 10$

$0 = -2(x^2 - 6x + 5)$

$0 = -(x+5)(x-1)$

$\boxed{x=5 \quad x=1}$

$\boxed{y\text{-int} = -10}$



c/o (2,-4) apart of the solution region

$-4 < -2(2-3)^2+8$

$-4 < -2(1)+8$

$-4 < -2+8$

$-4 < 6$

True

Shade inside the parabola.

Example#2: Graph $y \leq x^2 - 4x - 5$. Identify one ordered pair that is a solution.

y-int = -5

vertex (2,-9)

Complete the square to find the vertex

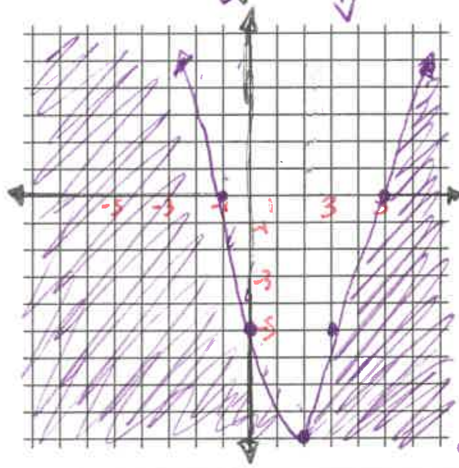
$y + 5 \overset{+4}{=} x^2 - 4x + 4$

$y + 9 \overset{-9}{=} (x-2)^2 - 9$

$y = (x-2)^2 - 9$

x-int. $0 = (x-5)(x+1)$

$\boxed{x=5 \quad x=-1}$



Test point (0,0)

$0 \leq (0)^2 - 4(0) - 5$

$0 \leq -5$

False

(0,0) is not apart of the solution region

Shade the outside

Example#3: Write the inequality that describes the following graph.

vertex = (2,5)

point = (0,1)

$y = a(x-p)^2 + q$

$1 = a(0-2)^2 + 5$

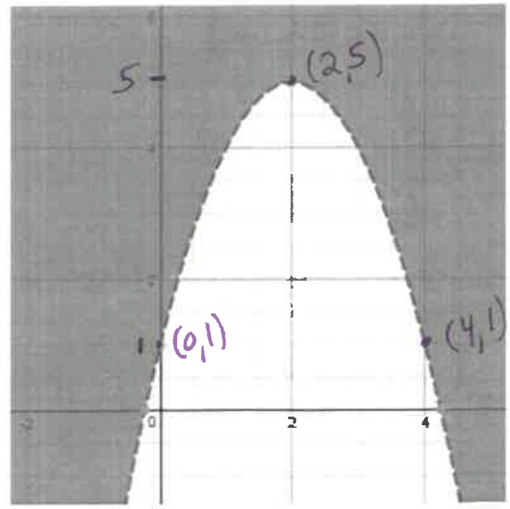
$1 - 5 = 4a + 5 - 5$

$-4 = 4a$

$\frac{-4}{4} = \frac{4a}{4}$

$-1 = a$

$\boxed{y > -1(x-2)^2 + 5}$



Example#4: Graph the following quadratic inequality using transformations to sketch the boundary parabola.

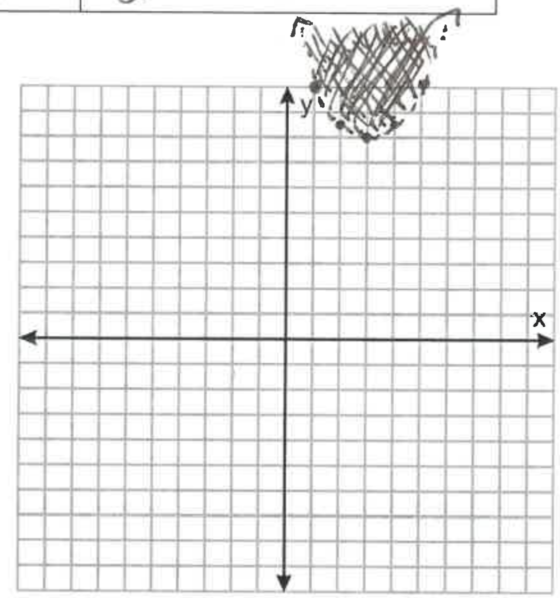
$$y > \frac{1}{2}(x-3)^2 + 8$$

Don't have to graph by transformations

PARENT FUNCTION $y = x^2$	
x	**y**
-2	4
-1	1
0	0
1	1
2	4

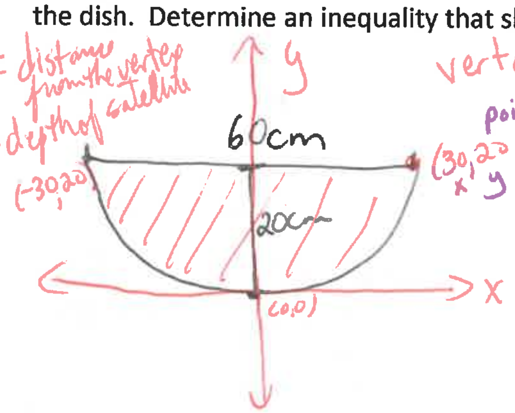
STEP 1 of TRANSFORMED FUNCTION – The Value of "a" $y = \frac{1}{2}x^2$	
Describe what changes & how: The parabola gets wider. The y-values are $\frac{1}{2}$ the size	
x	**y**
-2	2
-1	0.5
0	0
1	0.5
2	2

STEP 2 of TRANSFORMED FUNCTION – The Values of "p" and "q" $y = \frac{1}{2}(x-3)^2 + 8$		
Describe what changes & how: The parabola shift 3 units to the right (add 3 to x-values). The parabola shifts 8 units up (add 8 to the y-values)		
x	**y**	**Final Ordered Pair**
1	10	
2	8.5	
3	8	
4	8.5	
5	10	



Example#5: A satellite dish is 60cm in diameter and 20cm deep. The dish has a parabolic cross section. Locate the vertex of the parabolic cross- section at the origin, and sketch the parabola that represents the dish. Determine an inequality that shows the region from which the dish can receive a signal.

Let x = distance from the vertex
y = depth of satellite



vertex = (0,0) p q

point

① Find the quadratic equation

$y = a(x-p)^2 + q$

$20 = a(30-0)^2 + 0$

$\frac{20}{900} = \frac{900a}{900}$

$\frac{1}{45} = a$

$\boxed{y \geq \frac{1}{45}x^2}$

← show them how to demonstrate on the graphing calc.

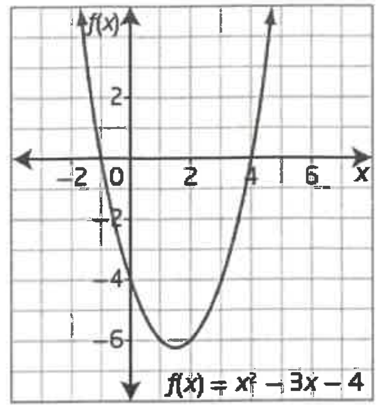
① Press Apps (enter

② Y=

③ Press Alpha

9.3 Assignment: Page 497 #1b, 2a,3, 4cd, 5a, 6abd ,8,9, pg 505 #9

9.2 Quadratic Inequalities in 1 variable



Example #1: Given $f(x) = x^2 - 3x - 4$ (graphed at right):

a) what are the ***zeroes*** of this function?
(what are the x-intercepts?) $x = -1 \quad x = 4$

b) for what values of x will $x^2 - 3x - 4 > 0$? $f(x) > 0$ or $y > 0$
(where is the graph greater than 0?)

Set Notation $\{x \mid x < -1 \text{ or } x > 4, x \in \mathbb{R}\}$ Interval $(-\infty, -1) \cup (4, \infty)$

c) for what values of x will $x^2 - 3x - 4 < 0$
(where is the graph less than 0?)

Set Notation $\{x \mid -1 < x < 4, x \in \mathbb{R}\}$

Interval notation $(-1, 4)$

d) for what values of x will $x^3 - 3x - 4 \le 0$

$\{x \mid -1 \le x \le 4, x \in \mathbb{R}\}$

$[-1, 4]$

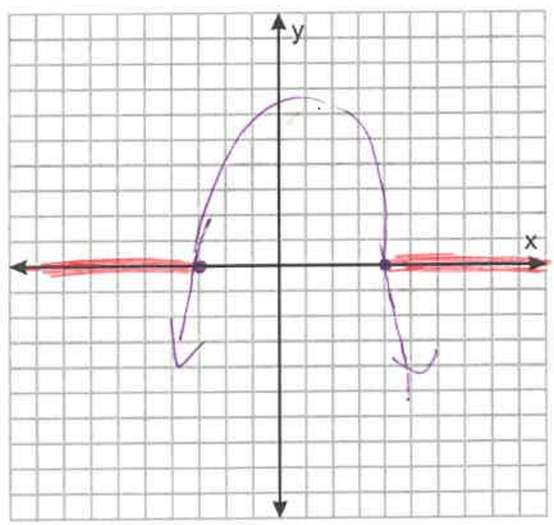
Example #2: Solve $-x^2 + x + 12 < 0$ by graphing.

$f(x) = -x^2 + x + 12$ Step 1 Graph the function

$f(x) = -(x^2 - x - 12)$

$f(x) = -(x-4)(x+3)$

$x = 4 \quad x = -3$



$x = \{x \mid x < -3 \text{ or } x > 4, x \in \mathbb{R}\}$

$(-\infty, -3) \cup (4, \infty)$

Example#4: Solve $2x^2 - 5x > 12$ using sign analysis. Write your solution using set notation and Union of intervals notation.

Steps: 1) Rewrite the inequality with 0 on one side
2) Factor (make sure that factors have a positive *x coefficient*)
3) Draw a number line with ***all*** the roots listed
4) Decide which values make the factor 0, + and –
5) Multiply the signs together
6) Write a solution using proper notation

① $2x^2 - 5x - 12 > 0$ $\quad 2x \quad 3$ / $1x \quad -4$

$(2x+3)(x-4)$

$2x + 3 = 0 \quad x - 4 = 0$

$\frac{2x}{2} = \frac{-3}{2} \quad x = 4$

$x = -\frac{3}{2}$

$x < -\frac{3}{2}$	$x = -\frac{3}{2}$	$-\frac{3}{2} < x < 4$	$x = 4$	$x > 4$	
−		+		+	$(2x+3)$
−		−		+	$(x-4)$
+		−		+	$(2x+3)(x-4)$
	zero		zero		

$X = \{x \mid x < -\frac{3}{2} \text{ or } x > 4, x \in \mathbb{R}\}$

$(-\infty, -\frac{3}{2}) \cup (4, \infty)$

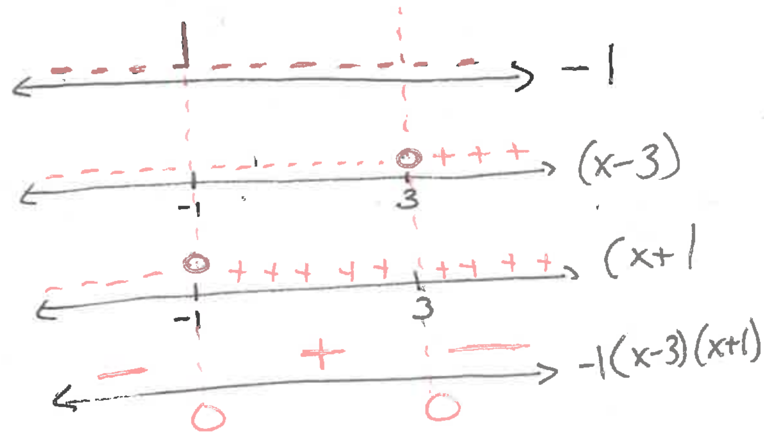
Example #5: Solve using sign analysis. Write your solution in set or interval notation.

$-x^2+2x+3\le 0$

$-1(x^2-2x-3)\le 0$

$-1(x-3)(x+1)$

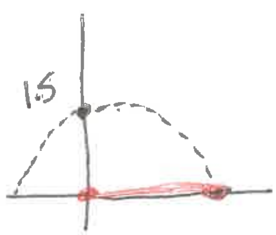
$x=3 \quad x=-1$



$x=\{x \mid x\le -1 \text{ or } x\ge 3, x\in\mathbb{R}\}$

$(-\infty,-1]\cup[3,\infty]$

Example #6: A baseball thrown from a height of 1.5m has an equation of $-4.9t^2+17t+1.5>0$ where t is the time in seconds, that the ball is in flight. What time interval is the ball in flight?



$x=\dfrac{-17\pm\sqrt{17^2-4(-4.9)(1.5)}}{2(-4.9)}$

$x=\dfrac{-17\pm\sqrt{289+29.4}}{-9.8}$

$x=\dfrac{-17\pm\sqrt{318.4}}{-9.8}$

$x\approx-0.086 \quad x\approx 3.56$

The interval of the ball in flight is $\{x \mid 0\le x\le 3.56, x\in\mathbb{R}\}$

$[0, 3.56]$

9.2 Assignment (Look at #2c together) #1,2,3,5,7(Graph by hand), 9bcd,10

No solution.

Pg 503 #16 word problem > example.

Pg 487
#17 (Extension question)
Look at together on Day 2