1.1 Arithmetic Sequences

SEQUENCE:

- An arithmetic sequence is an ordered list of terms in which the difference between consecutive terms is constant. This constant is called the common difference and uses the variable "d". The difference is found by subtracting the first number from the second number in the sequence. $d = t_2 - t_1$
- A geometric sequence is an ordered list of terms in which there is a common ratio between consecutive terms. The variable for the common ratio is r and can be found by: $r = \frac{t_2}{t}$

Are the following sequences arithmetic, geometric or neither.

a) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

Geometric

$r =$

b) 10, 20, 30,.....

Arithmetic

$d = 10$

c) $\frac{10}{9}, \frac{8}{7}, \frac{7}{6}, \ldots$

Neither

d) 2.5, 0.25, 0.025,...

Geometric

General Term Formula of an Arithmetic Sequence: $t_n = a + d(n-1)$ or $t_n = t_1 + d(n-1)$

"tn" is the value of a particular n^{th} term

"a" or "t_1" is the Value of the first term

"n" is the # of terms

"d" is the common difference.

Examples – Finding a particular term

1. Find the 60^{th} term of the sequence -3, 2, 7, 12, ...

a or $t_1 = -3$

$d = 5$

$n = 60$

$t_n = t_1 + d(n-1)$

$t_{60} = -3 + 5(60-1)$

$t_{60} = -3 + 5(59)$

$t_{60} = -3 + 295$

$t_{60} = 292$

The 60^{th} term in the sequence is 292.

2. A construction company hires a plumber to install pipes in new homes. The plumber will be paid $65 for the first hour of work, $110 for two hours of work, $155 for three hours of work, and so on.

a) State the first three terms of the sequence.

1h 2h 3h

\$65, \$110, \$155

b) Is there a common difference, d?

$d = 110 - 65 = 45$

d) What will the plumber get paid for 10 hours of work?

$t_{10} = 65 + 45(10-1)$

$t_{10} = 65 + 45(9)$

$t_{10} = 470$

The plumber will get paid \$470 for 10hrs of work.

e) What is the GENERAL TERM? → a formula used for this sequence to calculate any term.

$t_n = 65 + 45(n-1)$

f) Use the GENERAL TERM to determine how much did the plumber get paid for 30 hours of work?

$t_{30} = 65 + 45(30-1)$

$t_{30} = 65 + 45(29)$

$t_{30} = 65 + 1305$

$t_{30} = 1370$

The plumber will get paid \$1370 for 30 hours of work.

Examples – Determining the number of terms

1. How many terms are in the sequence 9250, 10900, 12550, ..., 100000?

$t_1 \quad t_2 \quad t_3 \quad \ldots \quad t_n$

$t_1 = 9250$

$t_n = 100000$

$d = 10900 - 9250$

$= 1650$

$t_n = t_1 + d(n-1)$ ← substitute in what you know

$100000 = 9250 + 1650(n-1)$ ← solve for "n"

$100000 = 9250 + 1650n - 1650$

$100000 = 7600 + 1650n$

$-7600 \qquad -760$

$\frac{92400}{1650} = \frac{1650n}{1650}$

$56 = n$

There are 56 terms.

Example – Manipulating the formula to find the missing part.

1. Two terms in an arithmetic sequence are $t_3 = 4$ and $t_8 = 34$. Find t_1.

Since t_1 and d are both unknown, you can use two equations and substitution to determine them.

$d = \frac{34-4}{8-3}$

$d = \frac{30}{5}$

$d = 6$

$t_3 = 4 - 6 - 6$

$= -8$

The first term is -8.

2. Find the first term and common difference of the arithmetic sequence whose third term is 1 and whose seventh term is -11. Then, find the first term.

$t_3 = 1$

$t_7 = -11$

$d = \frac{-11-1}{4}$ ← 4 terms in between 7^{th} + 3^{rd} term.

$d = \frac{-12}{4}$

$d = -3$

$t_3 = t_1 + d(n-1)$

$1 = t_1 + (-3)(3-1)$

$1 = t_1 + (-3)(2)$

$1 \overset{+6}{=} t_1 - 6 + 6$

$7 = t_1$

The first term is 7.

3. An amphitheater has 25 seats in the second row and 65 seats in the seventh row. The last row has 209 seats. The numbers of seats in the rows produces an arithmetic sequence. How many rows of seats are in the amphitheater? How many seats are in the first row?

Row 1

Row 2 25 seats

⋮

Row 7 65 seats

⋮

Row n 209 seats

$d = \frac{65-25}{7-2}$

$d = \frac{40}{5}$

$d = 8$

row 1 = Row 2 − 8

Row 1 = 25 − 8

row 1 = 17

There are 17 seats in the first row.

1.1 Assignment **Part A:** 1. Determine if the sequence is arithmetic, geometric, or neither.

a) $16, 32, 48, 64, 80$

b) $\frac{3}{4}, \frac{1}{2}, \frac{1}{3}, \frac{2}{9}$

c) $-4, -7, -10, -13, -16$

d) $1, 2, 4, 7, 11, 16, 22$

e) $x, x+y, x+2y, x+3y$

f) $6, 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}$

2. Given the first term and the common difference, write the first 5 terms of the sequence.

a) $a = -30; d = -3$

b) $a = -37; d = 10$

c) $a = 3p - 5q; d = -p + 2q$

d) $a = -12; d = 200$

e) $a = 4; d = \frac{1}{5}$

f) $a = 0.4; d = -1.2$

3. Determine the general term of the sequence.

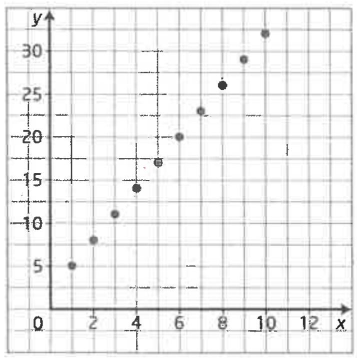
a) $22, 32, 42, 52, \ldots$

b) $-34, -234, -434, -634$

c) $\frac{7}{6}, \frac{5}{6}, \frac{1}{2}, \ldots$

d) $a = 5y; d = -3y$

e)

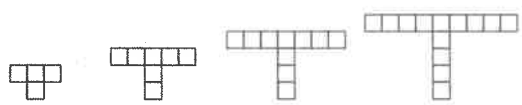


4. Determine the nth term of the arithmetic sequence.

a) 15, 21, 27, 33,…….. t_{n38}

b) 39, 32, 25, 18,…. t_{25}

5. Each square in this pattern has a side length of 1. Assume that the pattern continues.



a) Write the general form of the sequence that relates the figure number to the perimeter.

b) Determine the perimeter of figure #9.

Part B:

1. For each arithmetic sequence, determine the values of a and d. State the missing terms of the sequence.

a) __, __, __, 19, 23

b) __, __, 3, $\frac{3}{2}$

c) __, 4, __, __, 10

2. Identify which term each number is in the given arithmetic sequence. Use the formula.

a) 107 in the sequence 2, 5, 8, ...

b) -140 in the sequence 20, 16, 12

c) 21.2 in the sequence 6, 6.8, 7.6

d) $\frac{31}{2}$ in the sequence $\frac{1}{2}, \frac{7}{8}, \frac{5}{4}, \ldots$

3. What is the first term of an arithmetic sequence whose common difference is 4 and whose 24th term is 73? Use the formula.

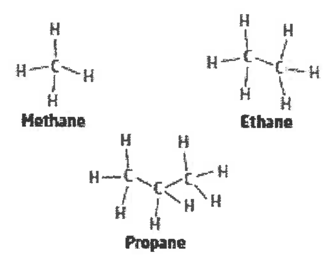
4. What is the first term of an arithmetic sequence whose common difference is 3 and whose 16th term is 48? Use the formula.

5. Find the first term and the common difference of the arithmetic sequence whose 9th term is 21 and whose 15th term is 33.

6. Find the first term and the common difference of the arithmetic sequence whose 15th term is 91 and whose 20th term is 121.

7. Susan joined a fitness class at her local gym. She incorporated a sit-up routine that followed an arithmetic sequence. On the 6th day of the program, Susan performed 11 sit-ups. On the 15th day she did 29 sit-ups.
a) Write the general term that relates the number of sit-ups to the number of days.
b) If Susan's goal is to be able to do 100 sit-ups, on which day of her program will she accomplish this?

8. Hydrocarbons are the starting points in the formation of thousands of products, including fuels, plastics, and synthetic fibres. Some hydrocarbon compounds contain only carbon and hydrogen atoms. Alkanes are saturated hydrocarbons that have single carbon-to-carbon bonds. The diagrams below show the first three alkanes.



a) Write the general term that relates the number of hydrogen atoms to the number of carbon atoms.
b) Hectane contains 202 hydrogen atoms. How many carbon atoms are required to support 202 hydrogen atoms?

Solutions:
Part A:
1. a) arithmetic b) geometric c) arithmetic d) neither e) arithmetic f) geometric
2.a) $-30, -33, -36, -39, -42$ b) $-37, -27, -17, -7, 3$ c) $3p - 5q, 2p - 3q, p - q, q, -p + 3q$
d) $-12, 188, 388, 588, 788$ e) $4, \frac{21}{5}, \frac{22}{5}, \frac{23}{5}, \frac{24}{5}$ f) $0.4, -0.8, -2, -3.2, -4.4$
3.a) $tn = 12 + 10n$ b) $tn = 166 - 200n$
c) $tn = \frac{3}{2} - \frac{1}{3}n$ d) $tn = 8y - 3yn$ e) $2 + 3n$
4.a) 237 b) -129

Part B:

1a) a = 7, d = 4: 7, 11, 15, 19, 23 b) a = 6, d = $-\frac{3}{2}$: $6, \frac{9}{2}, 3, \frac{3}{2}$ c) a = 2, d = 2: 2, 4, 6, 8, 10
2a) 36 b) 41 c) 20 d) 41
3) a = -19 4) a = 3 5) a = 5; d = 2 6) a = 7, d = 6
7a) $tn = 2n - 1$ b) 51st day
8a) $tn = 2n + 2$ b) 100 carbon atoms

1.2 Arithmetic Series

Arithmetic Series – The sum of the terms from an arithmetic sequence.

Example: 3, 6, 9, 12, 15 18 is an arithmetic sequence
3 + 6 + 9 + 12 + 15 + 18 is the arithmetic series

Arithmetic Series Formula:

$S_n = \frac{n(a+t_n)}{2}$ or $\frac{n(t_1+t_n)}{2}$

S_n = sum of the "n" number of terms
t_1 or a = value of the 1st term in the series
t_n = value of nth term

$S_n = \frac{n[2a+d(n-1)]}{2}$ or $\frac{n}{2}[2t_1+d(n-1)]$

n = number of terms

Examples

1. Determine the sum of the first 6 terms of the arithmetic series: −75 − 69 − 63 − ⋯

★ decide which formula to use based on what information you know ★

$S_n = ?$
a or $t_1 = -75$
$d = 6$
$n = 6$

$S_n = \frac{n[2a+d(n-1)]}{2}$

$S_6 = \frac{6[2(-75)+6(6-1)]}{2}$

$S_6 = 3[-150+30]$

$\boxed{S_6 = -360}$

2. For the arithmetic series determine the value of "n". Given $t_1 = 8$ $t_n = 68$ $S_n = 608$

$t_1 = 8$
$t_n = 68$
$S_n = 608$
$n = ?$

$S_n = \frac{n(t_1+t_n)}{2}$

$(2)\ 608 = \frac{n(8+68)}{2}\ (2)$

$\frac{1216}{76} = \frac{76n}{76}$

$\boxed{16 = n}$

3. Find the indicated sum: $5 + 9 + 13 + \cdots + 149$.

☆ First find what "n" term 149 is. Use the term formula ☆

$t_1 = 5$

$n = ?$

$d = 4$

$t_n = 149$

$S_n = ?$

$t_n = t_1 + d(n-1)$

$149 = 5 + 4(n-1)$

$149 = 5 + 4n - 4$

$149 = 1 + 4n$

$\frac{148}{4} = \frac{4n}{4}$

$\boxed{37 = n}$

149 is the 37th term

$S_{37} = \frac{n(a + t_n)}{2}$

$S_{37} = \frac{37(5+149)}{2}$

$S_{37} = \frac{37(154)}{2}$

$\boxed{S_{37} = 2849}$

4. Determine the value of t_1 for each arithmetic series described d=-3 , S_n=279 n=18

☆ We don't know "t_n" so we will use the 2nd formula ☆

$t_1 = ?$

$d = -3$

$S_n = 279$

$n = 18$

$S_n = \frac{n}{2}[2t_1 + d(n-1)]$

$279 = \frac{18}{2}[2t_1 + -3(18-1)]$

$\frac{279}{9} = \frac{9[2t_1 + -51]}{9}$

$31 \overset{+51}{=} 2t_1 - 51 \; {+51}$

$\frac{82}{2} = \frac{2t_1}{2}$

$\boxed{41 = t_1}$

5. *It's About Time*, in Langley, BC, is Canada's largest custom clock manufacturer. They have a grandfather clock that, on the hours, chimes the number of times that corresponds to the time of day. For example, at 4:00 p.m., it chimes 4 times and at 4:00am it chimes 4 times. How many times does the clock chime in a 24-hour period?

$n=12$
$a=1$
$d=1$

$$S_{12}=\frac{12[2(1)+1(12-1)]}{2}$$

$$S_{12}=\frac{12[13]}{2}$$

$$S_{12}=\frac{156}{2}$$

$$S_{12}=78$$

$S_{12}\times 2$
$=156$ times.

1.2 Assignment: Page 27 #1-5 ac and word problems below

1. The bottom row in a trapezoid had 49 cans. Each consecutive row had 4 fewer cans than the previous row. There were 11 rows in the trapezoid.
   a) How many cans were in the trapezoid?
   b) How many cans were in the 10^{th} row of the trapezoid?

2. Ryan's grandparents loaned him the money to purchase a new bike. He agreed to repay $25 at the end of the first month, $30 at the end of the second month, $35 at the end of the third month, and so on. Ryan repaid the loan in 12 months. How much money did he pay back to his grandparents?

3. Kaitlyn makes $10 per hour for her first hour of work, $10.50 for her second hour of work, $11 for her third hour and so on.
   a) How much money will she make during her 8^{th} hour of work?
   b) How much money will she make after an 8-hour day of work?

4. During a free fall, a skydiver falls 16 feet in the first second, 48 feet in the 2^{nd} second, and 80 feet in the third second. If she continues to fall at this rate:
   a) How many feet will she fall during the 8^{th} second?
   b) How many feet will she fall altogether during the 20 second free-fall?

5. The Chinese zodiac associates years with animals. Addison was born in 1994, the Year of the Dog.
   a) The Year of the Dog repeats every 12 years. List the first three years that Addison will celebrate his birthday in the Year of the Dog.
   b) In 2099, Nunavut will celebrate its 100th birthday. Will that year also be the year of the Dog? Explain.

6. Write the first four numbers that are multiples of 4 between 1 and 999.
   b) Determine the sum of all the multiples of 4 between 1 and 999.
   c) Determine the 25th multiple of 4 between 1 and 999.

Answers:

1a) 319 b) 13
2) $630
3a) $13.50/hour b) $94.00
4a) 240 ft. b) 6400 ft.
5a) 2006, 2018, 2030 b) No: 2099 is not a term in the sequence. If you solve for *n*, you will get 9.75, which is not a whole number.
6a) 4, 8, 12, 16 b) 124 500 c) 100

1.3 Geometric Sequences

Geometric sequence – a sequence in which the ratio of consecutive terms is constant.

Term Formula **Where :** t_n = the n^{th} term

$$r = \frac{t_n}{t_{n-1}}$$

$t_n = t_1(r)^{n-1}$ a or t_1 = the first term

n = the # of terms

r = the common ratio (What you multiply each term by)

$t_n = a(r)^{n-1}$

Ex#1: What is the general term for the geometric sequence that has a first term of 3 and a common ratio of 4.

$a = 3$
$r = 4$

$t_n = 3(4)^{n-1}$

Ex#2: Find the thirteenth term of the sequence -8, -4, -2, … "Finding a term value"

$n = 13$
$a = -8$
$r = \frac{-4}{-8} = \frac{1}{2}$

$t_{13} = -8\left(\frac{1}{2}\right)^{13-1}$

$t_{13} = -8\left(\frac{1}{2}\right)^{12}$

$t_{13} = (-8)\frac{1^{\div 8}}{4096^{\div 8}}$

$t_{13} = -\frac{1}{512}$

The 13^{th} term is $-\frac{1}{512}$

Ex#3: Determine the missing value for each geometric sequence with the following properties.

Finding "r"

a) If n = 7, t7 = 12288, t1 = 3, find r

$t_7 = 3(r)^{7-1}$

$\frac{12288}{3} = \frac{3r^6}{3}$

$\sqrt[6]{4096} = \sqrt[6]{r^6}$

$\boxed{4 = r}$

Note: Students may need help with finding $\sqrt[x]{y}$ button on calculator

b) If n = 5, t5= $\frac{128}{81}$, $r = \frac{4}{3}$, find t1 Finding "a or t_1"

$\frac{128}{81} = a\left(\frac{4}{3}\right)^{5-1}$

$\frac{128}{81} = a\left(\frac{4}{3}\right)^{4}$

$\left(\frac{\cancel{81}}{256}\right)\frac{128}{\cancel{81}} = a\left(\frac{\cancel{256}}{\cancel{81}}\right)\left(\frac{\cancel{81}}{\cancel{256}}\right)$

$a = \frac{128^{\div 128}}{256^{\div 128}}$

$\boxed{a = \frac{1}{2}}$

Ex#4: How many terms are in the sequence $2, 10, 50 \ldots 156250$ Finding "n"

$a = t_1 = 2$

$r = \frac{10}{2} = 5$

$t_n = 156250$

$n = ?$

$t_n = a(r)^{n-1}$

$\frac{156250}{2} = \frac{2(5)^{n-1}}{2}$

Guess and check.

$78125 = (5)^{n-1}$

$(5)^{\square} = (5)^{n-1}$

$(5)^7 = (5)^{n-1}$

$7 \overset{+1}{=} n - 1 + 1$

$8 = n$

or $\frac{\log 78125}{\log 5} = 7$

EX #5: In a geometric sequence, the third term is 54 and the sixth term is -1458. Determine the values of t1 and r and list the first 4 terms of the sequence.

$t_3 = 54$

$t_6 = -1458$

$t_1 = ?$

$r = ?$

$t_n = a(r)^{n-1}$

$54 = a(r)^{3-1}$

$\frac{54}{r^2} = \frac{a r^2}{r^2}$

$\boxed{\frac{54}{r^2} = a}$

Substitute into 6th term formula

$-1458 = \frac{54}{r^2}(r)^{6-1}$

$-1458 = \frac{54(r^5)}{r^2}$ simplify

$\frac{-1458}{54} = \frac{54r^3}{54}$

$\sqrt[3]{-27} = \sqrt[3]{r^3}$

$\boxed{-3 = r}$

$a = \frac{54}{(-3)^2}$

$\boxed{a = 6}$

$\boxed{\text{First 4 terms } 6, -18, 54, -162}$

EX #6: A ball is dropped from a height of 4m. After each bounce, it rises to 60% of its previous height.

a) Model this question as a geometric sequence

$4 \xrightarrow{\times 0.60} 2.4 \xrightarrow{\times 0.60} 1.44, 0.864 \ldots$

Bounce Bounce Bounce Bounce

b) Write a general term for the sequence.

$t_n = 4(0.60)^{n-1}$

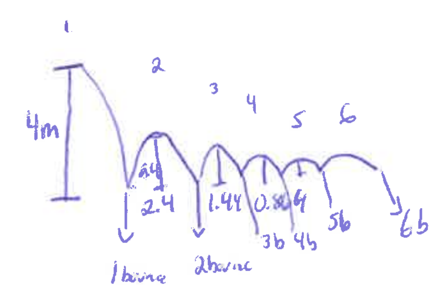
b) What height does the ball reach after the 6th bounce?

$n = 7$

$t_7 = 4(0.60)^{7-1}$

$t_7 = 4(0.60)^6$

$\boxed{t_7 = 0.186624\text{m}}$



If a sequence increases by a constant PERCENTAGE, it is geometric

- The r value will be **1 + (The Percentage as a decimal)**. For example, if property taxes increase 1% per year, the r value would be **1.01**

If a sequence decreases by a constant PERCENTAGE, it is also geometric

- The r value will be **1 - (The Percentage as a decimal)**. For example, if a car's value depreciates by 15% per year, the r value would be **0.85** because the car will only be worth 85% of what is was the year prior

EX #7: The population of a city increases by 6.5% each year for 10 years. If the initial population is 200 000, what is the population after 10 years? Note: Be careful about the value of n!! $n=11$ because the first term is the initial population

$a = 200\ 000$

$r = 1 + 0.065 = 1.065$

$n = 11$

$t_{11} = 200\ 000(1.065)^{10}$

$\boxed{t_{11} = 375427}$

1.3 Assignment Pg 39 #1ade, 3d, 4, 5cd (a.ka. General term) , 6ace, 8, 9, 17

Extra 1. Use the formula to find the required term of the given sequence.

a) 7^{th} term of $1, 3, 9, \dots$ b) 20^{th} term of $1, -2, 4, \dots$ c) 13^{th} term of $-8, -4, -2, \dots$
d) 7^{th} term of $64, -16, 4, \dots$ e) 6^{th} term of $64, 48, 36, \dots$ f) 18^{th} term of $x, 2x^2, 4x^3, \dots$

2. Liam just started a new job earning $45 000 per year. Assuming good performance, he is promised a raise of 2%. How much money will Liam be making in his 10^{th} year on the job?

3. A certain type of new car depreciates 20% per year. How much will the car be worth when it is 8 years old, if it is $35 000 brand new?

4. Identify what term each number is in the given geometric sequence. Use the nth term formula.

a) 156250 in the sequence 2, 10, 50, … d) 262144 in the sequence $\frac{1}{64}, \frac{1}{16}, \frac{1}{4}, \dots$

b) $\frac{1}{32}$ in the sequence 32, -16, 8, … e) $\frac{4}{81}$ in the sequence $\frac{81}{64}, \frac{27}{32}, \frac{9}{16}, \dots$

c) -1458 in the sequence -2, 6, -18, … f) -10240 in the sequence -5, -10, -20, …

5. In a certain region, the number of highway accidents increased by 5% per year. How many accidents will there be in 2018 if there were 5120 in 1978?

SOLUTIONS TO EXTRA QUESTIONS

1a) 729 b) -524288 c) $-\frac{1}{512}$ d) $\frac{1}{64}$ e) $\frac{243}{16}$ f) $131072x^{18}$

2. $53 779.17

3. $5872.03

4a) 8^{th} term b) 11^{th} term c) 7^{th} term d) 13^{th} term e) 9^{th} term f) 12^{th} term

5. about 36 045

1.4 Geometric Series

GEOMETRIC SERIES FORMULA:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Where: S_n is the sum of the first n terms

a is the first term
r is the common ratio
n is the number of terms

Remember Geometric Sequence Formula

$t_n = a(r)^{n-1}$

EX #1: Determine the sum of the first 10 terms of each geometric series.

a) 5+15+ 45+ …

$n = 10$
$t_1 = 5$
$r = 3$
$S_{10} = ?$

$S_{10} = \frac{5(3^{10} - 1)}{3 - 1}$

$S_{10} = \frac{5(59049 - 1)}{2}$

$S_{10} = \frac{5(59048)}{2}$

$S_{10} = \frac{295240}{2}$

$\boxed{S_{10} = 147620}$

b) $t_1 = 5$ $r = \frac{1}{2}$

$n = 10$
$t_1 = 5$
$r = \frac{1}{2}$

$S_{10} = \frac{5\left(\frac{1}{2}^{10} - 1\right)}{\frac{1}{2} - 1}$

$S_{10} = \frac{5\left(\frac{1}{1024} - \frac{1024}{1024}\right)}{\frac{1}{2} - \frac{2}{2}}$

$S_{10} = \frac{5\left(\frac{-1023}{1024}\right)}{-\frac{1}{2}}$

$S_{10} = 5\left(-\frac{1023}{1024}\right) \cdot -2$

$S_{10} = -\cancel{10}^{5}\left(-\frac{1023}{\cancel{1024}_{512}}\right)$

$S_{10} = -5\left(\frac{-1023}{512}\right)$

$\boxed{S_{10} = \frac{5115}{512}}$

EX #2: Determine the sum of each geometric series.

a) −2+ 4 − 8 + …− 8192

$a = t_1 = -2$
$r = -2$
$t_n = -8192$
$n = ?$

Step 1 Need to find what number in the sequence −8192 is. Find "n". Use the Geometric Sequence Formula.

$t_n = a(r)^{n-1}$

$\frac{-8192}{-2} = \frac{-2(-2)^{n-1}}{-2}$

$4096 = (-2)^{n-1}$

$(-2)^{12} = (-2)^{n-1}$

$12 + 1 = n - 1 + 1$

$13 = n$

Step 2 Now Find the Sum of 13 terms.

$S_{13} = \frac{-2((-2)^{13} - 1)}{-2 - 1}$

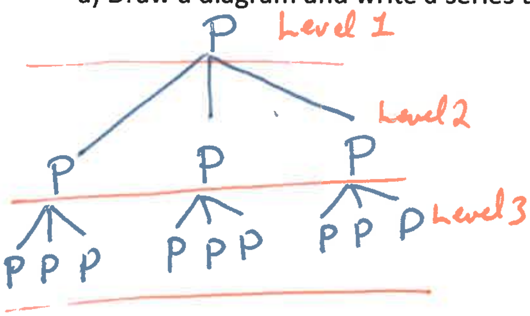
$S_{13} = \frac{-2(-8192 - 1)}{-3}$

$S_{13} = \frac{-2(-8193)}{-3}$

$\boxed{S_{13} = -5462}$

EX #3: A phone tree is used to contact a large number of people in a short period of time. In a particular phone tree, the first person contacts 3 people, who each contact 3 more people, and so on.

a) Draw a diagram and write a series to represent the total number of people in the phone tree.



$n =$ 1, 2, 3, 4

$1 + 3 + 9 + 27 \ldots\ldots$

$r = 3$

$t_1 = 1$

b) How many people are contacted after 6 levels of the tree (assuming the first level has 1 person)?

$n = 6$

$S_6 = \frac{1(3^6 - 1)}{3 - 1}$

$S_6 = \frac{(729-1)}{2}$

$S_6 = \frac{728}{2}$

$S_6 = 364$

There would have been 364 people contacted after 6 levels of the tree.

c) After how many levels will the total number of people contacted reach 2,391,484?

$S_n = 2391484$

$r = 3$

$t_1 = 1$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$2391484 = \frac{1(3^n - 1)}{3 - 1}$

$(2)\, 2391484 = \frac{3^n - 1}{2}\, (2)$

$4782968 {+1} = 3^n - 1 + 1$

$4782969 = 3^n$

$3^{14} = 3^n$

$14 = n$

There would be 14 levels when 2 391 484 people had been contacted.

1.4 Assignment pg 53 ##2,3,4,5,6,9

1.5 Infinite Series

Video on You Tube " Zeno's Paradox – numberphile" 1:33min – 6:22min Watch

Approximate the sum of the following Geometric series *show with calculator*

$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}\ldots\ldots$

$1+0.5+0.25+0.125+0.0625+0.03125+0.015625\ldots\ldots \approx 2$

Convergent Series

- The series does not have a last term. It has an infinite number of terms
- The sum of the terms approaches a fixed value.
- The value of the common ratio "r" is between $-1<r<1$.

Ex: $4+2+1+0.5\ldots\ldots \quad r=0.5$

To find the sum of an infinite geometric series:

$$\boxed{S_\infty=\frac{t_1}{1-r} \quad \text{where } -1<r<1}$$

You can only use formula if infinite geometric series is convergent

Divergent Series

- The series does not have a last term. It has an infinite number of terms
- The sum of the terms does not approach a fixed value. Therefore you cannot find the sum.
- The value of the common ratio is r > 1 or r <-1
- You cannot find the sum of an infinite geometric divergent series.

Ex. $4+8+16+32\ldots\ldots \quad r=2$

No Sum

Example #1/ Convergent or Divergent?

Find the sum of the following series if possible – explain why you can or cannot for each of them.

$\boxed{\text{Step1}}$ Find "r", the ratio first. If $-1<r<1$ (Convergent). If $r>1$ or $r<-1$ (Divergent).

$\boxed{\text{Step2}}$ Find the infinite geometric series sum.

1. $1-\frac{1}{3}+\frac{1}{9}-\cdots$

$\boxed{\text{Step1}}$ $r=\frac{1}{9}\div\frac{-1}{3}$

$r=\frac{1}{9}\cdot -3$

$r=-\frac{1}{3}$ ∴ Convergent.

$\boxed{\text{Step2}}$ $S_\infty=\frac{1}{1-(-\frac{1}{3})}$

$S_\infty=\frac{1}{\frac{3}{3}+\frac{1}{3}}$

$S_\infty=1\div\frac{4}{3}$

$S_\infty=1\cdot\frac{3}{4}$

$\boxed{S_\infty=\frac{3}{4}}$

2. $2-4+8-\cdots$

$\boxed{\text{Step1}}$ $r=\frac{8}{-4}$

$r=-2$ ∴ This is a divergent series and you cannot find the sum. It is not approaching a fixed value.

3. $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots$

$\boxed{\text{Step1}}$ $r=\frac{1}{64}\div\frac{1}{16}$

$r=\frac{1}{64}\cdot\frac{16}{1}$

$r=\frac{1}{4}$ ∴ It is a Convergent series

$\boxed{\text{Step2}}$ $S_\infty=\frac{\frac{1}{4}}{1-(\frac{1}{4})}$

$S_\infty=\frac{\frac{1}{4}}{\frac{4}{4}-\frac{1}{4}}$

$S_\infty=\frac{\frac{1}{4}}{3/4}$

$S_\infty=\frac{1}{4}\div 3/4$

$S_\infty=\frac{1}{\cancel{4}}\cdot\frac{\cancel{4}}{3}$

$\boxed{S_\infty=\frac{1}{3}}$

Example #2/ We use the S_∞ formula to convert repeating decimals to fractions.

1. Convert $0.\overline{3}$ to a fraction. ★ You can express $0.\overline{3}$ as an infinite geometric series ★

$0.\overline{3}=0.3+0.03+0.003\ldots\ldots$

$t_1=0.3$

$r=\frac{0.03}{0.3}=0.1$

$r=\frac{1}{10}$

$S_\infty=\frac{t_1}{1-r}$ Note: use fractional values

$S_\infty=\frac{\frac{3}{10}}{1-1/10}$

$S_\infty=\frac{3/10}{\frac{10}{10}-1/10}$

$S_\infty=\frac{3/10}{9/10}$

$S_\infty=\frac{\cancel{3}^{1}}{\cancel{10}_{1}}\cdot\frac{\cancel{10}^{1}}{\cancel{9}_{3}}$

$\boxed{S_\infty=\frac{1}{3}}$

2. Convert $0.\overline{17}$ to a fraction $0.\overline{17} = 0.17 + 0.0017 + 0.000017 \ldots$

$r = \frac{0.0017}{0.17}$

$r = 0.01$

$r = \frac{1}{100}$

$t_1 = \frac{17}{100}$

$S_\infty = \frac{\frac{17}{100}}{1 - \frac{1}{100}}$

$S_\infty = \frac{\frac{17}{100}}{\frac{100}{100} - \frac{1}{100}}$

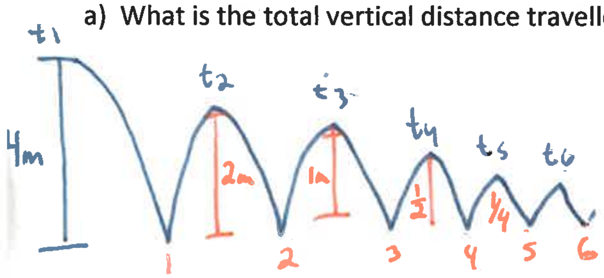
$S_\infty = \frac{\frac{17}{100}}{\frac{99}{100}}$

$S_\infty = \frac{17}{100} \cdot \frac{100}{99}$

$\boxed{S_\infty = \frac{17}{99}}$

EX #3: A ball is dropped from a height of 4m to the floor. After each bounce, the ball rises to 50% of its previous height.

a) What is the total vertical distance travelled after 6 bounces?



$S_6 = \left[\frac{4(0.5^6 - 1)}{0.5 - 1}\right] \times 2$ ← To count the up and down

$S_6 = \left[\frac{4(-0.984375)}{-0.5}\right] \times 2$

$S_6 = \left(\frac{-3.9375}{-0.5}\right) \times 2$

$S_6 = 15.75 - 4$ ← because the ball was dropped so we need to subtract 4m that would have been the upward motion

$\boxed{S_6 = 11.75m}$

The total Vertical distance travelled was 11.75m.

b) What is the vertical distance travelled when the ball comes to rest?

$r = \frac{1}{2}$

$S_\infty = \frac{4}{1 - \frac{1}{2}}$

$S_\infty = \frac{4}{\frac{2}{2} - \frac{1}{2}}$

$S_\infty = \frac{4}{\frac{1}{2}}$

$S_\infty = 4 \cdot 2$ To count the upward + downward distances

$S_\infty = [8 \times 2] - 4$ ← subtract the initial

$S_\infty = 12m$

The total vertical distance the ball has travelled when it comes to rest is 12m

1.5 Assignment: Pg 63 #1, 2 ace, 3,6,7 ,13,16