Pre-Calculus 20 **8.2 Solving Systems of Equations Algebraically (Substitution)**

3 Ways to solve systems of equations:

1. Graphically 2. Substitution 3. Elimination

To verify, check ALL answers in BOTH equations.

NOTE

- If 0x = 0 (a true statement), x can be anything, so there are an infinite number of solutions.
- If 0x = 5 (a false statement), there are NO solutions

Method 1: Substitution

- To solve a system of equations in two variables using substitution,
  1. isolate one variable in one equation
  2. substitute the expression into the other equation and solve for the remaining variable
  3. substitute the value(s) into one of the original equations to determine the corresponding value(s) of the other variable
  4. verify your answer by substituting into both original equations

Solve: $5x - y = 10$ ①
$x^2 + x - 2y = 0$ ②

Step 1 Isolate the "y" in equation ①

① $5x - y = 10$ (−5x, −5x)

$-y = 10 - 5x$ (÷ −1, ÷ −1, ÷ −1)

$y = -10 + 5x$

Step 2 Substitute into equation ②

② $x^2 + x - 2(-10 + 5x) = 0$

$x^2 + x + 20 - 10x = 0$

$x^2 - 9x + 20 = 0$ * Solve by factoring *

$(x - 4)(x - 5) = 0$

$x - 4 = 0$ (+4, +4) $\quad x - 5 = 0$ (+5, +5)

$x = 4 \qquad x = 5$

Step 3 When $x = 4$ $y = ?$
Substitute $x = 4$ into either equation to find "y"

① $5(4) - y = 10$

$20 - y = 10$ (−20, −20)

$-y = -10$ (÷ −1, ÷ −1)

$y = 10$

When $x = 5$ $y = ?$ Substitute $x = 5$ into either equation

① $5(5) - y = 10$

$25 - y = 10$ (−25, −25)

$-y = -15$ (÷ −1, ÷ −1)

$y = 15$

The solutions are (4, 10) and (5, 15)

Solve: ① $2x^2 + 4y = 15$

② $4x^2 + 8y = 20$

Step 1 Isolate the "y"

① $2x^2 + 4y = 15$ $-2x^2$

$\frac{4y}{4} = \frac{15 - 2x^2}{4}$

$y = \frac{15}{4} - \frac{1}{2}x^2$

Step 2 substitute into ②

② $4x^2 + 8\left(\frac{15}{4} - \frac{1}{2}x^2\right) = 20$

$4x^2 + 30 - 4x^2 = 20$

$0x^2 + 30 = 20$ -30

$0x^2 = -10$ (False statement)

No Solutions

∴ No points of intersection

Example 2) Determine two integers that have the following relationships: Fourteen more that twice the first integer gives the second integer. The second integer increased by one is the square of the first integer.

a) Write a system of equations that relates to the problem.

b) Solve the system algebraically

Let x = 1st integer

Let y = 2nd integer

a) $2x + 14 = y$

$y + 1 = x^2$

b) solve by substitution

$(2x + 14) + 1 = x^2$

$2x + 15 = x^2 - 2x - 15$ $-2x$ -15

$0 = x^2 - 2x - 15$

*solve by factoring

$0 = (x - 5)(x + 3)$

$x - 5 = 0$ $+5$ $+5$ $\quad$ $x + 3 = 0$ -3 -3

$x = 5$ $\quad$ $x = -3$

When $x = 5$ $y = ?$

$2(5) + 14 = y$

$10 + 14 = y$

$24 = y$

$(5, 24)$

When $x = 3$ $y = ?$

$2(-3) + 14 = y$

$-6 + 14 = y$

$8 = y$

$(-3, 8)$

The numbers are 5 and 24 or -3 and 8

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