Pre-Calculus 20

8.2 Solving Systems of Equations Algebraically (Elimination)

Method 2: Elimination

- To solve a system of equations in two variables using elimination,
  - Step 1 { if necessary, rearrange the equations so that the like terms align
  - if necessary, multiply one or both equations by a constant to create equivalent equations with a pair of variable terms with opposite coefficients
  - Step 2/3 add or subtract to eliminate one variable and solve for the remaining variable
  - Step 4 substitute the value(s) into one of the original equations to determine the corresponding value(s) of the other variable
  - verify your answer(s) by substituting into both original equations

Solve: $6x^2 - x - y = -1$ ①
$4x^2 - 4x - y = -6$ ②

Step 1

Multiply equation ① by -1 to create an equivalent equation with a pair of variable terms with opposite coefficients.

① $(6x^2 - x - y = -1)(-1)$

Step 2 Add both equations to eliminate a variable

① $-6x^2 + x + y = 1$

\+ ② $4x^2 - 4x - y = -6$

$-2x^2 - 3x + 0y = -5$ ☆ solve for "x" ☆

$-2x^2 - 3x + 5 = -5 + 5$

$-2x^2 - 3x + 5 = 0$ ☆ Solve using the quadratic formula

Step 3 $a = -2$, $b = -3$, $c = 5$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(-2)(5)}}{2(-2)}$$

$$x = \frac{3 \pm \sqrt{9 + 40}}{-4}$$

$$x = \frac{3 \pm \sqrt{49}}{-4}$$

$$x = \frac{3+7}{-4} \qquad x = \frac{3-7}{-4}$$

$$x = -\frac{5}{2} \qquad x = 1$$

Step 4 when $x = 1$, $y = ?$

Substitute $x = 1$ into either equation

① $6(1)^2 - (1) - y = -1$

$6 - 1 - y = -1$

$5 - 5 - y = -1 - 5$

$\frac{-y}{-1} = \frac{-6}{-1}$

$y = 6$

→

when $x = -\frac{5}{2}$ $y = ?$

substitute $x = -\frac{5}{2}$ into either equation and solve for "y"

① $6(-2.5)^2 - (-2.5) - y = -1$

$6(6.25) + 2.5 - y = -1$

$37.5 + 2.5 - y = -1$

$40 - 40 - y = -1 - 40$

$-y = -41$

$\frac{-y}{-1} = \frac{-41}{-1}$

$y = 41$

The solutions are $(-\frac{5}{2}, 41)$ and $(1, 6)$

Solve: ① $3x^2 - x - y = 2$
② $6x^2 - 2x - 2y = 4$

Step 1 Multiply equation ① by −2

① $-2(3x^2 - x - y = 2)$

Step 2 Add the two equations

① $-6x^2 + 2x + 2y = -4$
② $+\ 6x^2 - 2x - 2y = 4$

$0x^2 + 0x + 0y = 0$ ← True statement x + y can be any number

$\boxed{\text{There are infinite solutions}}$

Example 3) During a basketball game Ben completes an impressive "alley-oop," with teammate Luke. The path of the ball thrown by Luke can be modelled by the equation $\boxed{d^2 - 2d + 3h = 9}$, where *d* is the horizontal distance of the ball from the centre of the hoop, in metres, and h is the height of the ball above the floor, in metres. The path of Ben's jump can be modelled by the equation $\boxed{5d^2 - 10d + h = 0}$, where d is the horizontal distance from the centre of the hoop, in metres, and h is the height if the hands above the floor, in metres.

a) Solve the system of equations algebraically. Give your solution to the nearest hundredth.
b) Interpret your result. What assumptions are you making?

① $d^2 - 2d + 3h = 9$
② $(5d^2 - 10d + h = 0)\,-3$
$-15d^2 + 30d - 3h = 0$

① $d^2 - 2d + 3h = 9$
② $+\ -15d^2 + 30d - 3h = 0$
$-14d^2 + 28d + 0h = 9 - 9$
$-9 \quad -9$

★ Solve using the quadratic formula → $-14d^2 + 28d - 9 = 0$

$X = \dfrac{-28 \pm \sqrt{784 - 4(-14)(-9)}}{2(14)}$

$X = \dfrac{-28 \pm \sqrt{784 - 504}}{28}$

$X = \dfrac{-28 + \sqrt{280}}{-28}$

$X = 0.40 \quad X = 1.60$

When x = 0.40 y = ?
$h = -5(0.40)^2 + 10(0.40$
$h = 3.2$

when x = 1.6 y = ?
$h = -5(1.6)^2 + 10(1.6)$
$h = 3.2$

The solutions are (0.40, 3.2) and (1.6, 3.2)

b) Ben will complete the "allegoop" if he catches the ball 0.40m from the hoop. Assuming Ben jumps at the correct time, ~~and~~ is able to make the shot and not blocked by another player.

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