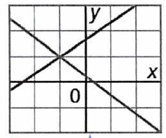
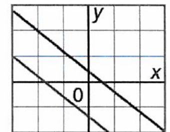
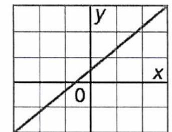
7.6 – Properties of Linear Systems (Number of solutions)

Concept # 48 – Determine the number of solutions for a linear system

Possible Solutions for a Linear System

Intersecting Lines	Parallel Lines	Coincident Line
One solution	No solution	Infinite solution
		
- lines have different slopes	- lines have same slopes and different y-intercepts	- Same slope and same y-intercept

Example 1: Determine the number of solutions of each linear system

a) ① $x + y = -2$
② $-2x - 2y = 4$

$\boxed{\text{Infinite Solutions}}$

Get equations into slope intercept form

① $x + y = -2$ $\quad -x \quad -x$

$y = -x - 2$ ← y-int (slope: $-x$)

② $-2x - 2y = 4$ $\quad +2x \quad +2x$

$-2y = 2x + 4$

$\frac{-2y}{-2} = \frac{2x}{-2} + \frac{4}{-2}$

$y = -x - 2$ ← y-int. (slope: $-x$)

b) ① $4x + 6y = -10$
② $-2x - y = -1$

① $4x + 6y = -10$ $\quad -4x \quad -4x$

$\frac{6y}{6} = \frac{-4x}{6} - \frac{10}{6}$

$y = -\frac{2}{3}x - \frac{5}{3}$ ← y-int. (slope: $-\frac{2}{3}$)

② $-2x - y = -1$ $\quad +2x \quad +2x$

$\frac{-y}{-1} = \frac{2x}{-1} - \frac{1}{-1}$

$y = -2x + 1$ ← y-int (slope: -2)

$\boxed{\text{one solution}}$

c) ① $3x + y = -1$
② $-6x - 2y = 12$

① $3x + y = -1$ $\quad -3x \quad -3x$

$y = -3x - 1$ ← y-int. (slope: -3)

② $-6x - 2y = 12$ $\quad +6x \quad +6x$

$-2y = 6x + 12$

$\frac{-2y}{-2} = \frac{6x}{-2} + \frac{12}{-2}$

$y = -3x - 6$ ← y-int. (slope: -3)

$\boxed{\text{No Solution}}$

How can you tell from the equations that the lines are parallel?	How can you tell from the equations that they represent the same line?	Why is it important to identify the y-intercepts when the equations in a linear system have the same slope?
They have the same slope & different y-intercepts	If they have the same slopes and same y-int.	Because the answer could be infinite solutions or no solution.

Example 2: Given the equation $y = 2x + 4$ write another linear equation that will form a linear system with:

a) exactly one solution (an equation that just has a different slope than 2)

$y = 3x - 2$

b) no solution (an equation that has the same slope and different y-int.)

$y = 2x - 3$

c) infinite solutions (an equation that has the same slope same y-int)

$2y = 4x + 8$ ← multiplied everything by 2 to create an equivalent equation.

Example 3:

What happens if you try to solve a system using substitution or elimination with no solution? Or with infinite solutions?

System 2

① $-2x + y = 2$

② $-2x + y = 4$

Elimination

① $-2x + y = 2$

$-\quad +2x + y = 4$

$0x + 0y = -2$

$0 \neq -2$

$\therefore$ No solution

System 3

① $-2x + y = 2$

② $-4x + 2y = 4$

Substitution

① $-2x + y = 2$ $\quad +2x \quad +2x$

$y = 2x + 2$

② $-4x + 2(2x + 2) = 4$

$-4x + 4x + 4 = 4$

$0x + 4 - 4 = 4 - 4$

$0x = 0$

$\therefore$ Infinite solutions

7.6 Assignment

1. Determine the number of solutions for each linear system

a) y = x + 2
y = x + 2

b) y = 2x – 4
y = x + 1

c) y = 3x + 2
y = 3x – 5

2. Determine the number of solutions for each linear system (you may want to rearrange into slope-intercept form first)

d) x + 3y = 6
$y = -\frac{1}{3}x + 6$

e) 3x – y = 12
4x – y = 12

f) x - 4y = 8
x + 4y = 20

3. Given the equation –6*x* + *y* = 3, write another linear equation that will form a linear system with:
   a) exactly one solution
   b) no solution
   c) infinite solutions

4. Use substitution to show that the linear system y = 2x + 5 and 2y – 4x = -15 has no solution. How do you know there is no solution?

Answers: 1a) infinite b) one c) zero

2d) infinite e) one f) one

3a)Various (Need different slopes) b) Various (Need same slopes) c)Various(need same slope and y-intercept)

4) Discussion

7.6 ASSIGNMENT:

1. Determine the number of solutions for each linear system

a)	$y = x + 2$ $y = x + 2$	b)	$y = 2x - 4$ $y = x + 1$	c)	$y = 3x + 2$ $y = 3x - 5$
d)	$y = 56 - 2x$ $y = 10 + x$	e)	$y = 60 + 3x$ $y = 60 - 5x$	f)	$y = -4x - 3$ $y = 4x - 3$

2. Determine the number of solutions for each linear system (you may want to rearrange into slope-intercept form first)

a)	$x + 2y = 6$ $x + y = -2$	b)	$3x + 5y = 9$ $6x + 10y = 18$	c)	$2x - 5y = 30$ $4x - 10y = 15$
d)	$x + 3y = 6$ $y = -\frac{1}{3}x + 6$	e)	$3x - y = 12$ $4x - y = 12$	f)	$x - 4y = 8$ $x + 4y = 20$

3. Given the equation $-6x + y = 3$, write another linear equation that will form a linear system with:
   a)exactly one solution
   b)no solution
   c)infinite solutions

4. Suppose you are given only the following pieces of information about a system of linear equations. Would you be able to predict the number of solutions to the system? Explain.

a) The slopes of the lines are the same
b) The y-intercepts of the lines are the same
c) The x-intercepts are the same, and the y-intercepts are the same.

5. Mark wrote the two equations in a linear system in slope-intercept form. He noticed that the signs of the two slopes were different. How many solutions will this linear system have? Explain.

6. Use substitution to show that the linear system $y = 2x + 5$ and $2y - 4x = -15$ has no solution. How do you know there is no solution?

SOLUTIONS:

1a)	infinite	b)	one	c)	zero
d)	one	e)	one	f)	one
2a)	one	b)	zero	c)	one
d)	zero	e)	one	f)	one
3a)	Various	b)	Various	c)	Various
4)	Discussion				
5)	One; discuss why				
6)	Discussion				