

7.4 Using Substitution to Solve a System of Linear Equations

Concept #47 Solve a system algebraically using substitution and /or elimination and verify the solutions

In lesson 7.1/2/3, you solved linear systems by graphing. This strategy can be time-consuming and sometimes you can only approximate the solution. We can use algebra to determine the exact solution. Once algebraic strategy is called solving by substitution

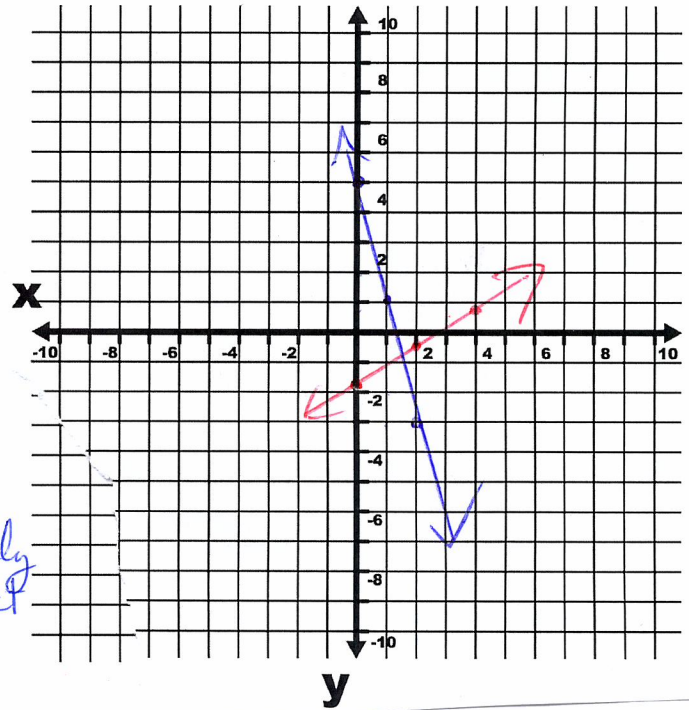
Example 1 Solve the following linear system

$$4x + y = 5 \quad 2x - 4y = 7$$

a) Solve by Graphing

$$y = -4x + 5$$

$$\begin{aligned} -4y &= -2x + 7 \\ \frac{-4y}{-4} &= \frac{-2x}{-4} + \frac{7}{-4} \\ y &= \frac{1}{2}x - \frac{7}{4} \end{aligned}$$



We would only be able to approx. the solution. So solving algebraically will give us an exact value.

b) Solve by using substitution

Step 1

$$\textcircled{1} 4x + y = 5 \quad \textcircled{2} 2x - 4y = 7$$

$$y = -4x + 5$$

Step 3
Substitute $x = \frac{3}{2}$ into equation ②

$$2\left(\frac{3}{2}\right) - 4y = 7$$

$$3 - 4y = 7 - 3$$

$$-4y = 4$$

$$y = -1$$

$$\text{Solution: } \left(\frac{3}{2}, -1\right)$$

Step 2
Substitute equation ① into ②

$$2x - 4(-4x + 5) = 7$$

$$2x + 16x - 20 = 7 \quad \text{*solve for "x"}$$

$$18x - 20 = 7 + 20$$

$$\frac{18x}{18} = \frac{27}{18} \div 9$$

$$x = \frac{3}{2}$$

STEPS FOR SOLVING BY SUBSTITUTION

- Label the equations 1 and 2
- Using either of the equations, solve for either x or y (get x or y by itself on one side of the equal sign). Hint: It is always easiest to pick an x or y that has no number or negative sign in front of it!!!
- Begin by using the equation from the above step. Use what is on the OTHER side of the equal sign opposite your single variable. Substitute this expression into the value of the appropriate variable into the equation you did NOT use in the previous step.
- Using this new equation, get the variable by itself. This is the answer to either the x or the y part of the ordered pair
- Substitute this answer for x or y into either of the original equations. This will allow you to find the answer to the other variable.

Example 2 Solve the following system using substitution

$$\begin{aligned} \textcircled{1} \quad 3x - 4y &= -15 & \textcircled{2} \quad 5x + y &= -2 \end{aligned}$$

easier to isolate "y"

$$y = -5x - 2$$

Step 2 Substitute $y = -5x - 2$ into equation $\textcircled{1}$

$$\begin{aligned} 3x - 4(-5x - 2) &= -15 \\ 3x + 20x + 8 &= -15 \\ 23x + 8 &= -15 \\ 23x &= -23 \\ x &= -1 \end{aligned}$$

Step 3 Substitute $x = -1$ into equation $\textcircled{2}$ and solve for "y"

$$\begin{aligned} 5(-1) + y &= -2 \\ -5 + y &= -2 \\ y &= 3 \end{aligned}$$

Solution: $(-1, 3)$

Example 3 Solve the following system using substitution

$$\textcircled{1} \quad \frac{1}{2}x + \frac{1}{3}y = 1 \quad \textcircled{2} \quad \frac{1}{4}x - \frac{2}{3}y = -1$$

Step 1 multiply every term by "LCD" to get rid of fractions

$$\textcircled{1} \quad 3x + 2y = 6 \quad \textcircled{2} \quad 3x - 8y = -12$$

Step 2

$$\begin{aligned} 2y &= -3x + 6 \\ y &= -\frac{3}{2}x + 3 \end{aligned}$$

Step 2 isolate ~~one~~ of x or y into one of the equations

Step 3 Substitute $y = -\frac{3}{2}x + 3$ into equation $\textcircled{2}$

Step 3

$$\begin{aligned} 3x - 8\left(-\frac{3}{2}x + 3\right) &= -12 \\ 3x + 12x - 24 &= -12 \\ 15x - 24 &= -12 \\ 15x &= 12 \\ x &= \frac{4}{5} \end{aligned}$$

Step 4 Substitute $x = \frac{4}{5}$ into equation $\textcircled{1}$ and solve for "y"

Step 4

$$\begin{aligned} 3\left(\frac{4}{5}\right) + 2y &= 6 \\ \frac{12}{5} + 2y &= 6 \\ 2y &= \frac{30}{5} - \frac{12}{5} \\ 2y &= \frac{18}{5} \\ y &= \frac{9}{5} \end{aligned}$$

Solution: $\left(\frac{4}{5}, \frac{9}{5}\right)$ or $(0.8, 1.8)$

Example 4 A math test has short answer questions and word problems. A short answer question is worth 1 mark and a word problem is worth 4 marks. There are 11 questions for a total of 30 marks.

a) Create a linear system to model this situation.

Let s = # of short answer questions
 w = # of word problems

$$\begin{aligned} s + w &= 11 \\ 2s + 4w &= 30 \end{aligned}$$

b) Solve this system using substitution. How many short answer questions and how many word problems are on the test?

$$\begin{aligned} s + w &= 11 \\ s &= -w + 11 \end{aligned}$$

$$\begin{aligned} 2(-w + 11) + 4w &= 30 \\ -2w + 22 + 4w &= 30 \\ 2w + 22 &= 30 \\ \frac{2w}{2} &= \frac{8}{2} \\ w &= 4 \end{aligned}$$

$$\begin{aligned} s + 4 &= 11 \\ s &= 7 \end{aligned}$$

Solution: There are 4 word problems and 7 short answer questions on the test.

Assignment Pg 425 #4,5,19

AT LEAST TWO OF : 10,11,12,13,14,15,16,17,18