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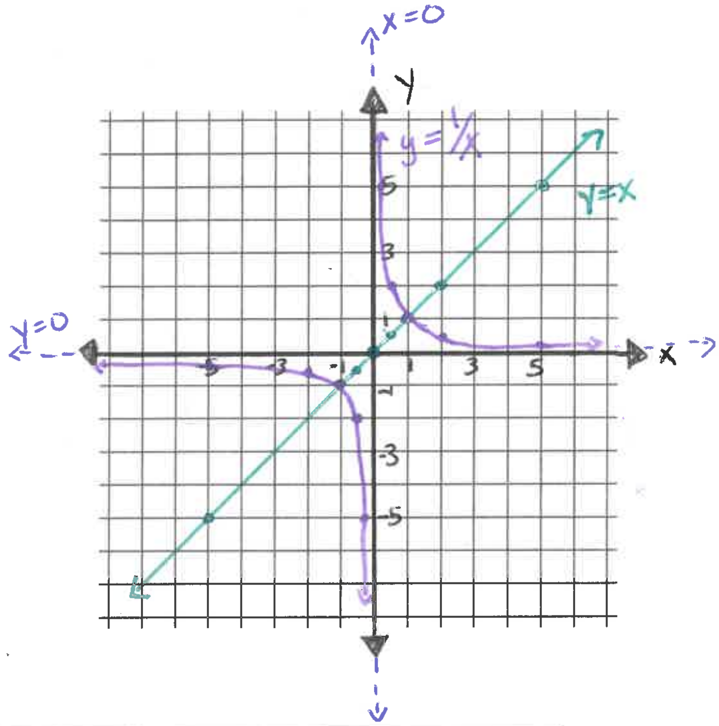
7.4 Reciprocal Functions (Day 1)- Linear Reciprocal Functions

Reciprocal Function: For any function $f(x)$, its reciprocal is $\frac{1}{f(x)}$, provided that $f(x) \neq 0$.

Note: $\left(\frac{1}{f(x)}\right)$ x (f(x)) =1

Sketch the graphs of $y = x$ and $y = \frac{1}{x}$ on the same set of axes. Use two different colors

$y = x$		$y = \frac{1}{x}$	
x	**y**	**x**	**y**
-10	-10	-10	-1/10
-5	-5	-5	-1/5
-2	-2	-2	-1/2
-1	-1	-1	-1
-1/2	-1/2	-1/2	-2
-1/5	-1/5	-1/5	-5
-1/10	-1/10	-1/10	-10
0	0	0	undefined
1/10	1/10	1/10	10
1/5	1/5	1/5	5
½	1/2	½	2
1	1	1	1
2	2	2	1/2
5	5	5	1/5
10	10	10	1/10



Characteristic	$y = x$	$y = \frac{1}{x}$
Domain	$x=\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$	$x=\{x \mid x \neq 0, x \in \mathbb{R}\}$ or $(-\infty,0) \cup (0,\infty)$
Range	$y=\{y \mid y \in \mathbb{R}\}$ or $(-\infty, \infty)$	$y=\{y \mid y \neq 0, y \in \mathbb{R}\}$ or $(-\infty,0) \cup (0,\infty)$
End Behavior The behavior of the y-values of a function as $\lvert x \rvert$ becomes very large	As $\lvert y \rvert \to \infty$ the $\lvert x \rvert \to \infty$	As $y \to 0$ (approaches) the $\lvert x \rvert \to \infty$
Behavior near the NPV's The behavior of the x-values as $\lvert y \rvert$ becomes very large.	As $\lvert x \rvert \to \infty$ the $\lvert y \rvert \to \infty$	As $\lvert x \rvert \to 0$ ← NPV the $\lvert y \rvert \to \infty$
Invariant Points of $y = x$ **and** $y = \frac{1}{x}$	(1,1) and (-1,-1)	

Invariant points are points that remain unchanged between an original function and any transformation of that function (including taking its reciprocal)

Note: Invariant points will always occur when $y = \pm 1$ on a reciprocal function

ASYMPTOTE

- a line that the graph gets infinitely close to but doesn't touch or cross
- an asymptote is drawn as a dashed line on the graph itself
- graphs can have vertical and/or horizontal asymptote(s)

Vertical Asymptotes

- occur at the value(s) of x that are the non-permissible values of the domain of the rational function
- The graph will have a vertical asymptote at all non-permissible value locations (if the original function was linear the reciprocal graph will only have one vertical asymptote)
- Each vertical asymptote will have an equation where x = (a non-permissible value)

Horizontal Asymptote

- occurs when y= 0 because $\frac{1}{x} \neq 0$
- Functions will only have one horizontal asymptote
- The equation of the horizontal asymptote will always be y=0 for the reciprocal of all polynomial functions

Example 1:Graph y = 2x + 5 and its reciprocal

a) Write the reciprocal function: $y = \frac{1}{2x+5}$

b) Determine the equation of the vertical asymptote(s) of the reciprocal function. Sketch using a different color:

$2x+5=0$ -5 -5

$\frac{2x}{2}=\frac{-5}{2}$

$x = -\frac{5}{2}$

Horizontal Asymptote at $y=0$

Equation of the Vertical Asymptote = NPV

c) Find the x and y intercept of the reciprocal function.

Sketch them on the graph.

x-int when y=0

$(2x+5)\,0 = \frac{1}{2x+5}\,(2x+5)$

$0 \neq 1$

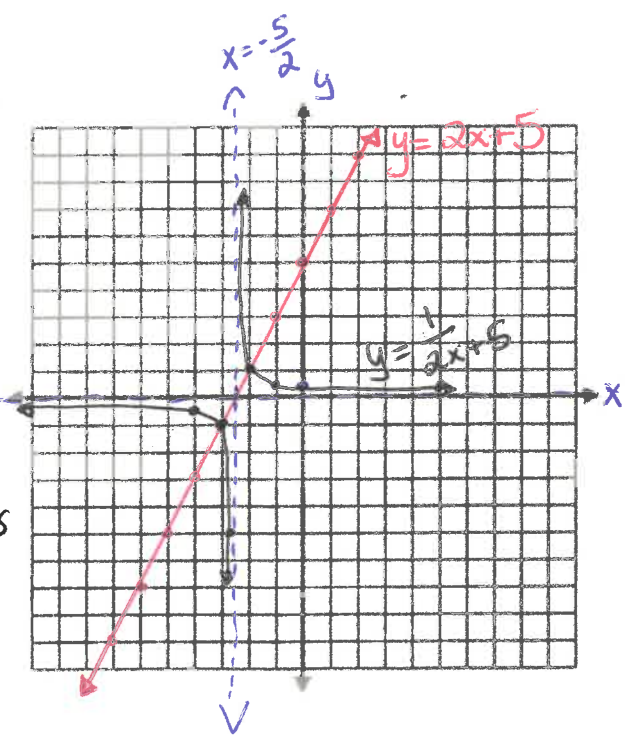
∴ No x-int.

y-int when x=0

$y = \frac{1}{2(0)+5}$

$y = \frac{1}{5}$

$(0, \frac{1}{5})$



d) Find additional points in each section so that you have 3 in each. Plot these points.

"Substitute x-values into $y=\frac{1}{2x+5}$ to find the corresponding y-value"

x	y
-4	-1/3
-3	-1
-2.6	-5
-1	1/3
1	1/7
-2	1

Pick x-values on the left side of the asymptote

Pick x-values on the right side of the asymptote

f)Using a different colour, sketch the original linear function.
Are there any invariant points? What are they?

$y = 2x+5$ (original function)

Slope = 2

y-int. = 5

Yes, the invariant points are $(-3,1)$ and $(-2,1)$

Example 2:Graph $g(x)=3x-9$ and its reciprocal.

a) Find the zeros (x-intercept) of the g(x).(original function) $\overset{+9}{0}=3x-9+9$ $\quad x=\frac{9}{3}$ $\quad x=3$

b) Determine its reciprocal function $f(x)=\frac{1}{g(x)}$. So f(x) = $\frac{1}{3x-9}$

c) Determine the equation of the vertical asymptote of the reciprocal function. How are the zeros (x-intercept) of the original function related to the non-permissible values?

$3x-9=0$

$\frac{3x}{3}=\frac{9}{3}$

$x=3$ "Equation of the vertical asymptote"

Horizontal Asymptote at $y=0$

The x-int. of the original function is the same value as the vertical asymptote.

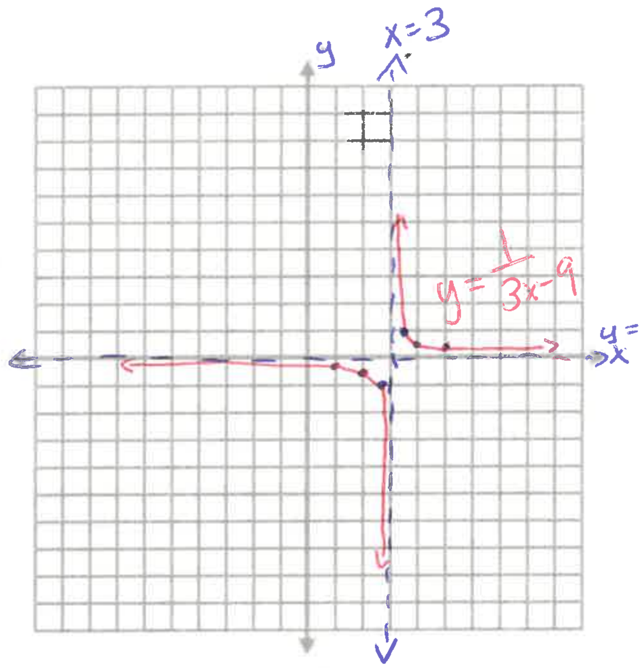
The vertical asymptote will always cross the x-axis at the x-int. of the original function.

d) Sketch the graph of the reciprocal function.

Note: Need at least 3 points on each side of the asymptote.

Find Invariant points using y=1 and y=-1

x	y
1	$-\frac{1}{6}$
2	$-\frac{1}{3}$
4	$\frac{1}{3}$
5	$\frac{1}{6}$



e) State the following:

- Domain: $x=\{x \mid x\neq 3, x\in\mathbb{R}\}$
- Range: $y=\{y \mid y\neq 0, y\in\mathbb{R}\}$
- End Behavior: As $y\to 0$ $|x|\to\infty$
- Behavior near the non-permissible value (NPV):

  As $x\to 3$ $|y|\to\infty$

Invariant points

$y=1$

$\overset{+9}{1}=3x-9+9$

$\frac{10}{3}=\frac{3x}{3}$

$\frac{10}{3}=x$

$(\frac{10}{3},1)$ or $(3.\bar{3},1)$

$y=-1$

$\overset{+9}{-1}=3x-9+9$

$\frac{8}{3}=\frac{3x}{3}$

$\frac{8}{3}=x$

$(\frac{8}{3},-1)$ or $(2.\bar{6},-1)$

7.4 Day 1 Assignment: Page 403 #1ab, 2ab, 3ab, 5ab, 7cd (Also state the domain, range, the end behavior and the behavior near the NPV's) Try #17 (is a great AP style question)

7.4 Reciprocal Functions (Day 2) – Quadratic Reciprocal Functions

EX #1: Given $f(x) = x^2 - 4$

a) What is the reciprocal function of *f(x)*?

$g(x) = \frac{1}{x^2-4}$

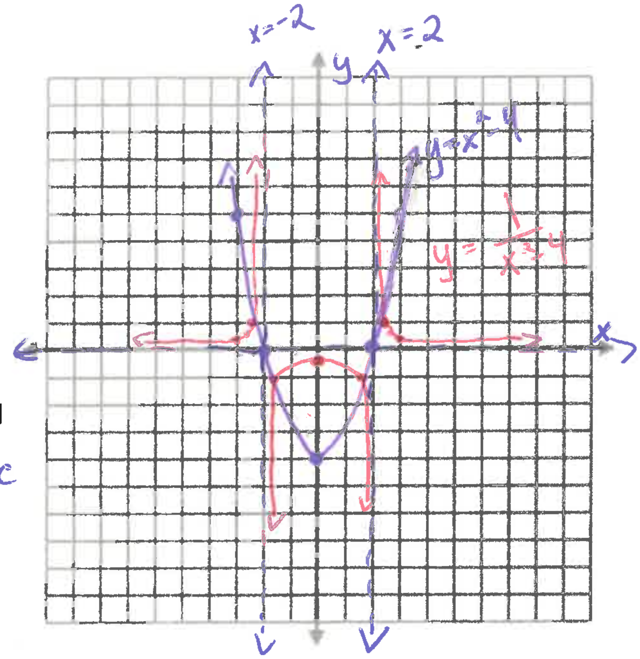
b) State the non-permissible values of x and the equation(s) of the vertical asymptotes of the reciprocal function.

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$x-2=0 \quad x+2=0$

$\boxed{x = 2 \quad x = -2}$



c) State the x-intercept(s) and y-intercept of the reciprocal function.

- No x-int's because their is a horizontal asymptote at y=0

$\boxed{\text{y-intercept}}$ when x=0

$g(0) = \frac{1}{0^2-4}$ $(0, -\frac{1}{4})$

d) State the invariant points. (Let y = ±1 and solve for x) Note: Can use original function or reciprocal function to find the invariant points.

$1 = x^2 - 4$, $+4$ both sides: $\sqrt{5} = \sqrt{x^2}$, $\pm\sqrt{5} = x$

$-1 = x^2 - 4$, $+4$ both sides: $\sqrt{3} = \sqrt{x^2}$, $\pm\sqrt{3} = x$

Invariant points
$(\sqrt{5}, 1)$ $(\sqrt{3}, -1)$
$(-\sqrt{5}, 1)$ $(-\sqrt{3}, -1)$

Note: $\pm\sqrt{5} \approx \pm 2.236$
$\pm\sqrt{3} \approx \pm 1.732$

e) Graph the function using all the points you have (add points if you need)

Need 3 points in each section (Left + right side of asymptotes)

x	y
-3	1/5
-2.5	4/9
3	1/5
2.5	4/9

Substitute x-values into $g(x) = \frac{1}{x^2-4}$ to find y-values

$g(3) = \frac{1}{9-4}$

$g(3) = \frac{1}{5}$

f) State the following:

- Domain: $\{x \mid x \neq 2, x \neq -2, x \in R\}$
- Range: $\{y \mid y \neq 0, y \in R\}$
- End Behavior: As $y \to 0$ $|x| \to \infty$
- Behavior near the non-permissible value(s) (NPV): As $x \to 2, -2$ $|y| \to \infty$

g) Using a different colour, graph f(x).

$f(x) = x^2 - 4$

x-int = -2, 2)

vertex (0, -4)

when x = -3

$y = (-3)^2 - 4$

$y = 9 - 4$

(-3, 5)

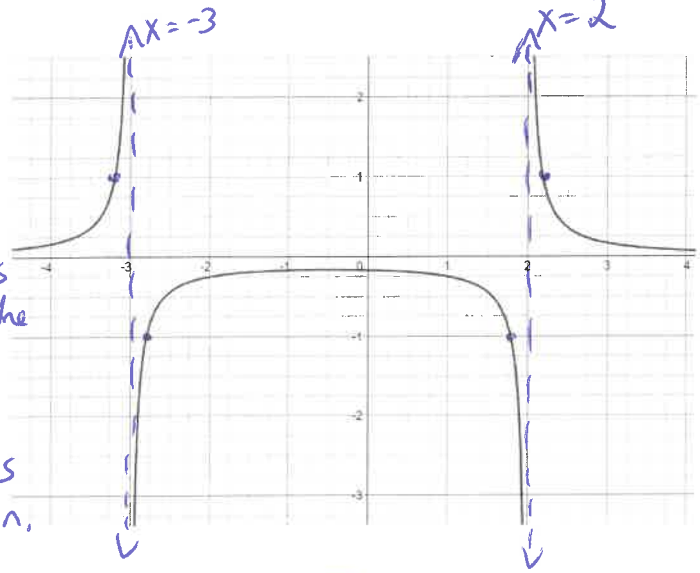
h) What is the graphical relationship between f(x) and $\frac{1}{f(x)}$?

① Original Function f(x) is a parabola that opens up and x-int. are x=2 and x=2

② G(x) the reciprocal function has asymptotes where the original functions x-int's are. It is in three sections where the middle opens down which is the opposite of the original.

EX #2: Given the graph of $y=\dfrac{1}{f(x)}$ (reciprocal function), match it with the graph of f(x) (original function).

Hint: Determine the equations of the vertical asymptotes

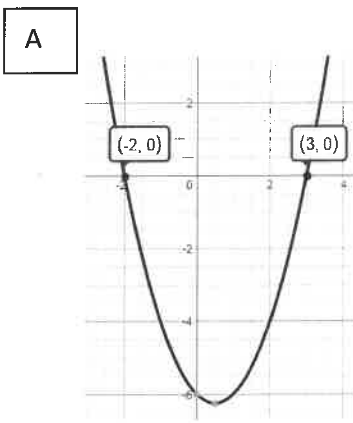


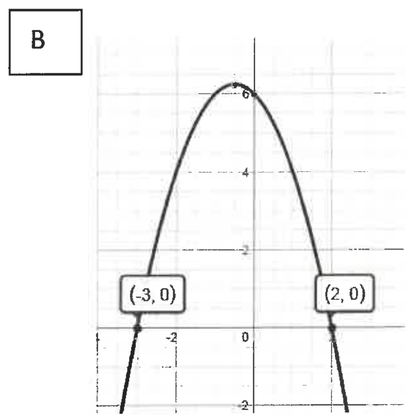
$X=-3$

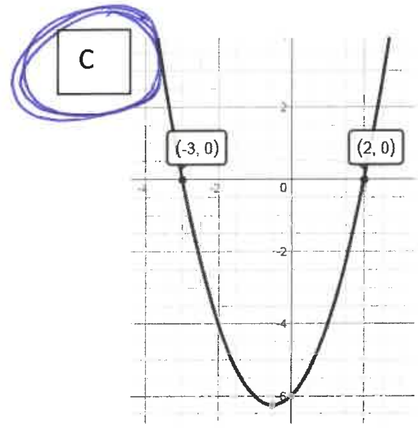
$X=2$

Invariant points are points on the original function as the reciprocal function and occur at $y=1,-1$.

The vertical asymptotes cross the x-axis at the x-int's of the original function.







EX #: Given $f(x) = -x^2 - x + 6$

a) What is the reciprocal function?

$g(x)=\dfrac{1}{-x^2-x-6}$

b) What are the x and y intercept(s) of f(x) ?

No x-int $y\text{-int} = \frac{1}{6}$

b) What are the asymptote(s)?

Vertical Asymptotes

$-x^2-x+6=0$

$-1(x^2+x-6)=0$

$-1(x+3)(x-2)=0$

$x=-3 \quad x=2$

Horizontal Asymptote

$y=0$

$y=0$

d) Sketch the graph of f(x). and $\frac{1}{f(x)}$

Need At least 3 points in each section.

x	y

$\frac{1}{f(x)}$

e) State the following: about $\frac{1}{f(x)}=g(x)$

- Domain:

$\{x \mid x\neq -3, x\neq 2, x\in\mathbb{R}\}$

- Range:

$\{y \mid y\neq 0, y\in\mathbb{R}\}$

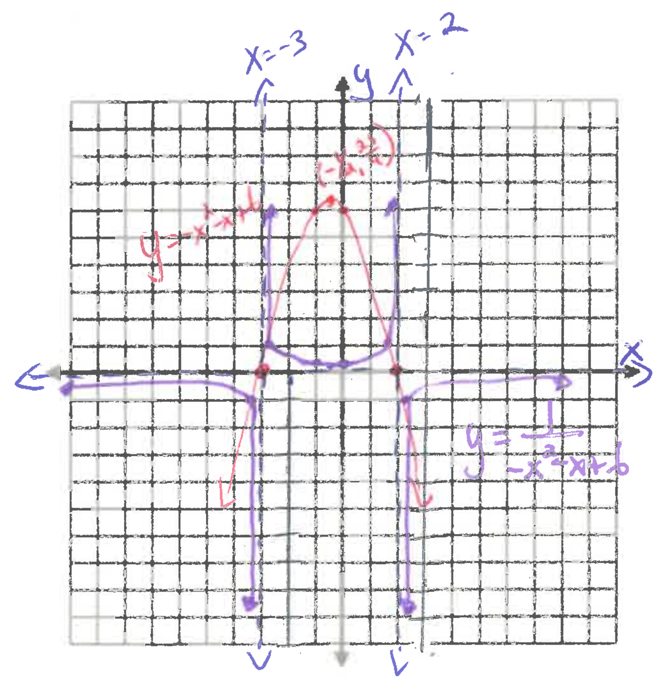
- End Behavior:

As $y \to 0$, $|x| \to \infty$

- Behavior near the non-permissible value(s) (NPV):

As $x \to -3$ $|y| \to \infty$

and As $x \to 2$ $|y| \to \infty$



Original function Vertex

$x=\dfrac{-b}{2a}$

$x=\dfrac{-(-1)}{2(-1)}$

$x=\dfrac{1}{-2}$

y value

$y=-(-\frac{1}{2})^2-(-\frac{1}{2})+6$

$y=-\frac{1}{4}+\frac{1}{2}\cdot\frac{2}{2}+\frac{6}{1}\cdot\frac{4}{4}$

$y=-\frac{1}{4}+\frac{2}{4}+\frac{24}{4}$

$y=\frac{25}{4}$

NEW

EX #4 Graph $y = \dfrac{1}{x^2+3}$ **(Show graph on desmos after)**

① Are there any vertical Asymptotes? Horizontal Asymptotes?

① No, because there is no value that would make the denominator zero. Yes $y = 0$

② Are there any invariant points?

② $1 = \dfrac{1}{x^2+3}$

$x^2+3 = 1 \quad (-3)$

$\sqrt{x^2} = \sqrt{-3}$

$x = \sqrt{-3}$

NO solution.

$-1 = \dfrac{1}{x^2+3}$

$-(x^2+3) = 1$

$-x^2-3 = 1 \quad (+3)$

$\dfrac{-x^2}{-1} = \dfrac{4}{-1}$

$\sqrt{x^2} = \sqrt{-4}$

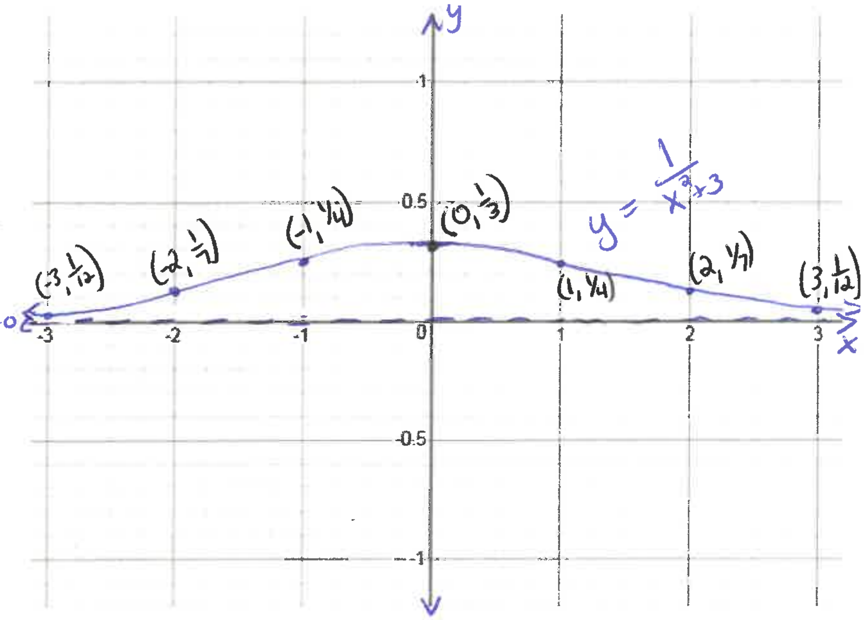
$x = \sqrt{-4}$

No solution

∴ No invariant points

③ Use a table of values.

x	y
-3	1/12
-2	1/7
-1	1/4
0	1/3
1	1/4
2	1/7
3	1/12



STEPS TO FINDING & GRAPHING RECIPROCAL FUNCTIONS

- Given an f(x) is a function where $f(x) \neq 0$, then it's reciprocal is $\dfrac{1}{f(x)}$
- Factor the denominator to find the vertical asymptotes. The horizontal asymptote is at y = 0. Sketch the asymptotes using dashed lines
- Find any intercept(s) and plot them
- Find test points in each "zone" so that you have at least three points to draw each section (invariant points are usually helpful to find). Connect your dots in a smooth curve so that they approach (but don't cross or touch) the asymptotes.
- **Domain:** will be all reals except for the non-permissible values

  Range : will be all reals except for y = 0

  End Behavior: as $y \to 0,\ |x| \to \infty$

  Behavior near NPV Points: as $x \to \text{each NP Value},\ |y| \to \infty$ (You need to describe the behavior near each NP Value)

<u>7.4 Day 2 Assignment:</u> **Pg403 # 2cd, 5cd, 6bc, 8bcd (also state the domain, the range, the end behavior and the behavior near the NPV), & #9**