4.4 The Quadratic Formula and the Discriminant

Show how the quadratic formula is derived by taking standard form and solve by completing the square and square root property.

STEPS to solving equations using the QUADRATIC FORMULA.

1. Make sure your quadratic equation is in standard form $ax^2 + bx + c = 0$
2. To find the roots use the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
3.. Write the solution by either listing each root using x =# or by listing all roots in a solution set {#,.... }

EX #1: Solve the following using the quadratic formula. Write your answer in exact form.

a) $x^2 + 4x - 1 = 0$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(-1)}}{2(1)}$

$x = \dfrac{-4 \pm \sqrt{16+4}}{2}$

$x = \dfrac{-4 \pm \sqrt{20}}{2}$ ← simplify the radical $\sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$

$x = \dfrac{-4 \pm 2\sqrt{5}}{2}$ ← reduce $\dfrac{\div 2}{\div 2}$

$x = -2 \pm \sqrt{5}$

b) $x^2 + 6x + 9 = 0$

$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(9)}}{2(1)}$

$x = \dfrac{-6 \pm \sqrt{36-36}}{2}$

$x = \dfrac{-6 \pm 0}{2}$

$x = -3$

c) $2x = 3(x-1)(x+1)$

$2x = (3x-3)(x+1)$

$2x = 3x^2 + 3x - 3x - 3$ $-2x$ $-2x$

$0 = 3x^2 - 2x - 3$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-3)}}{2(3)}$

$x = \dfrac{2 \pm \sqrt{4+36}}{6}$

$x = \dfrac{2 \pm \sqrt{40}}{6}$ ← simplify the radical

$x = \dfrac{2 \pm \sqrt{4 \cdot 10}}{6}$

$x = \dfrac{2 \pm 2\sqrt{10}}{6}$ reduce $\dfrac{\div 2}{\div 2}$

$x = \dfrac{1 \pm \sqrt{10}}{3}$

d) $\dfrac{2}{3}x^2 + 1 = \dfrac{5}{6}x$ (6) (6) (6)

$4x^2 + 6 = 5x$ $-5x$ $-5x$

$4x^2 - 5x + 6 = 0$

$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(6)}}{2(4)}$

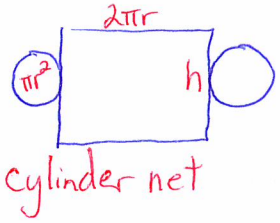
$x = \dfrac{5 \pm \sqrt{25-576}}{8}$

$x = \dfrac{5 \pm \sqrt{-551}}{8}$

No solution $x = \emptyset$

EX #2: The surface area of a cylinder is 250 cm^2. The height of the cylinder is 7 cm. What is the radius of the cylinder to the nearest thousandth of a centimetre?

S.A = 250cm^2



cylinder net

$S.A = 2\pi r^2 + 2\pi rh$

$250 = 2\pi r^2 + 2(7)\pi r$ -250 -250

$0 = 2\pi r^2 + 14\pi r - 250$

$a = 2\pi \quad b = 14\pi \quad c = -250$

$x = \dfrac{-14\pi \pm \sqrt{(-14\pi)^2 - 4(2\pi)(-250)}}{2(2\pi)}$

$x = \dfrac{-14\pi \pm \sqrt{196 + 2000\pi}}{4\pi}$

$x = -10.714$ and 3.714

The radius of the cylinder is 3.714 cm

Discriminant – the expression $b^2 - 4ac$ from the quadratic formula.

The discriminant tells us how many roots (or solutions, or zeroes, or x-intercepts) there are.

- If $b^2 - 4ac < 0$ There are no real roots, solutions, zeros or x-intercepts Ex/ $x = \dfrac{-3 \pm \sqrt{-2}}{3}$
- If $b^2 - 4ac = 0$ There is 1 real root, solution, zero or x-intercept. Ex/ $x = \dfrac{-3 \pm \sqrt{0}}{3}$ $x = \dfrac{-3}{3}$ $x = \{-1\}$
- If $b^2 - 4ac > 0$ There are 2 real roots, solutions, zeros or x-intercepts Ex/ $x = \dfrac{-3 \pm \sqrt{4}}{3}$ $x = -\dfrac{1}{3}, -\dfrac{5}{3}$

Ex 3: How many roots/solutions/x-intercepts do the following have? (Determine the nature of the roots)

1. $x^2 - 5x + 4 = 0$

$b^2 - 4ac$
$= (-5)^2 - 4(1)(4)$
$= 25 - 16$
$= 9$ ∴ There are 2 real roots

2. $3x^2 + 4x + \dfrac{4}{3} = 0$

$4^2 - 4(3)\left(\frac{4}{3}\right)$
$16 - 12\left(\frac{4}{3}\right)$
$16 - 16$
$= 0$
∴ 1 real rool

3. $2x^2 - 8x = -9$

$2x^2 - 8x + 9 = 0$
$(-8)^2 - 4(2)(9)$
$64 - 72$
$= -8$
∴ No real roots

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