4.3 Solving Quadratic Equations by Completing the Square and Square Root Property

To solve equations that are ***non-factorable (yet may have x-intercepts)***, complete the square (if necessary) and then:

1. Isolate the squared term
2. Take the $\pm$ square root of both sides.
3. Solve. If necessary, write 2 equations.
4. Check. Watch for ***extraneous*** roots (an answer not satisfying the restrictions on the variable)
5. Write a solution set x={#,#} or x=#,#

REVIEW SIMPLIFYING RADICALS

Simplify the following:

a) $\sqrt{75}$
$= \sqrt{25 \cdot 3}$
$= 5\sqrt{3}$

b) $-\sqrt{98}$
$= -\sqrt{49 \cdot 2}$
$= -7\sqrt{2}$

$\sqrt{48}$
$= \sqrt{16 \cdot 3}$
$= 4\sqrt{3}$

EX #1: Solve each equation using the square root method. Leave your answers in exact form.

a) $\sqrt{(x-4)^2} = \sqrt{16}$

$(x-4) = \pm 4$

$x - 4 + 4 = \pm 4 + 4$

$x = 4+4 \quad x = -4+4$

$x = 8 \quad x = 0$

$x = \{8, 0\}$

b) $2x^2 - 1 \overset{+1}{=} 5^{+1}$ Verify your solution

$\frac{2x^2}{2} = \frac{6}{2}$

$\sqrt{x^2} = \sqrt{3}$

$x = \pm\sqrt{3}$

$x = \{\pm\sqrt{3}\}$

$x = \sqrt{3}$
$2(\sqrt{3})^2 - 1 = 5$
$2(3) - 1 = 5$
$6 - 1 = 5$
$5 = 5 \checkmark$

$x = -\sqrt{3}$
$2(-\sqrt{3})^2 - 1 = 5$
$2(3) - 1 = 5$
$6 - 1 = 5$
$5 = 5 \checkmark$

c) $x^2 + 6x + 16 = 0$

★complete the square★

$x^2 + 6x + 9 = -16 + 9$ $\quad \left(\frac{6}{2}\right)^2 = 9$

$\sqrt{(x+3)^2} = \sqrt{-7}$

$x = \{\ \}$ ← empty or null set

$x = \emptyset$ No solution

d) $x^2 - 10x = 3$ (approx. to the nearest tenth)

★Complete the square★

$x^2 - 10x - 3 + 3 = 0 + 3$

$x^2 - 10x + 25 = 3 + 25$ $\quad \left(\frac{-10}{2}\right)^2 = 25$

$\sqrt{(x-5)^2} = \pm\sqrt{28}$

$x - 5 + 5 = \pm\sqrt{28} + 5$

$x = \pm\sqrt{4 \cdot 7} + 5$

$x = \pm 2\sqrt{7} + 5$

$x \approx 10.3 \quad x \approx -0.3$

~~e) $3x^2 + 12 = 0$~~

~~$-12 \quad -12$~~

~~$\frac{3x^2}{3} = \frac{-12}{}$~~

$\frac{3}{2} \times \frac{1}{2}$

g) $9(x-2)^2 = 27$

$\frac{9(x-2)^2}{9} = \frac{27}{9}$

$\sqrt{(x-2)^2} = \sqrt{3}$

$x - 2 \overset{+2}{=} \pm\sqrt{3} + 2$

$x = \pm\sqrt{3} + 2$

h) $2x^2 = 12x - 3$ ★complete the square★

$2x^2 - 12x + 3 = 0$

$2x^2 - 12x = -3$

$2(x^2 - 6x + 9) = -3 + 9(2)$

$2(x-3)^2 = -3 + 18$

$\frac{2(x-3)^2}{2} = \frac{15}{2}$

$\sqrt{(x-3)^2} = \pm\sqrt{\frac{15}{2}}$

$x - 3 \overset{+3}{=} \pm\sqrt{\frac{15}{2}} + 3$

$x = \left\{\pm\sqrt{\frac{15}{2}} + 3\right\}$

$x \approx 0.261,\ 5.739$

i) $-2x^2 - 3x + 7 = 0$ -7 -7

$-2\left(x^2 + \frac{3}{2}x + \frac{9}{16}\right) = -\frac{7}{1} + \frac{9(-2)}{16\,8}$

$-2\left(x + \frac{3}{4}\right)^2 = -\frac{56}{8} + -\frac{9}{8}$

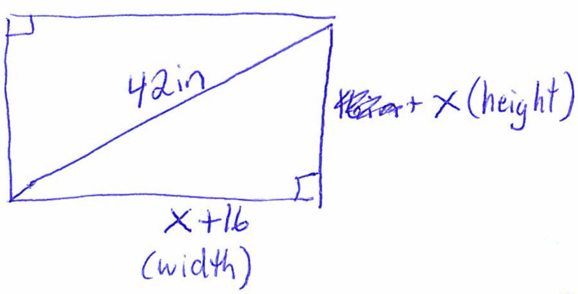
$\frac{-2\left(x + \frac{3}{4}\right)^2}{-2} = -\frac{65}{8} \div -2$

$\sqrt{\left(x + \frac{3}{4}\right)^2} = \frac{\sqrt{65}}{\sqrt{16}}$

$x + \frac{3}{4} \overset{-3/4}{=} \pm\frac{\sqrt{65}}{4} - \frac{3}{4}$

$x = \left\{\frac{\pm\sqrt{65} - 3}{4}\right\}$

Ex.#3 (Ex. 1 on pg 236) A wide-screen television has a diagonal measure of 42 in. The width of the screen is 16 in. more than the height. Determine the dimensions of the screen, to the nearest tenth of an inch.



$42^2 = x^2 + (x+16)^2$

$1764 = x^2 + (x+16)(x+16)$

$1764 - 1764 = x^2 + x^2 + 32x + 256 - 1764$

$0 = 2x^2 + 32x - 1508$

$\frac{0}{2} = \frac{2(x^2 + 16x - 754)}{2}$

$0 + 754 = x^2 + 16x - 754 + 754$

$\left(\frac{16}{2}\right)^2 = 64$

$64 + 754 = x^2 + 16x + 64$

$\sqrt{818} = \sqrt{(x+8)^2}$

$\pm\sqrt{818} - 8 = x + 8 - 8$

$x = \pm\sqrt{818} - 8$

$x = +\sqrt{818} - 8$ or $x = -\sqrt{818} - 8$

$x \approx \cancel{36.6}$ 20.6 $\qquad x \approx -36.6$

extraneous root

The dimensions of the screen are ~~Hein and~~ 20.6in and 36.6in.

4.3 Assignment Pg 240 #4bcd, 5bcef, 6bcde,7abde, 8,9