3.1 Factors and Multiples of Whole Numbers [GCF] (Day 1)

Composite Numbers:

A number that is not prime because it can be divided by other numbers than 1 and itself.

Ex.

4,6,8,9,10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27

Prime Numbers:

A number that is both greater than one and that is only divisible by 1 and itself. The following numbers are a list of prime number from 1 – 40

Ex.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37

Are 0 and 1 prime numbers?

No, because 0 can't be divided by itself ($\frac{0}{0}$ = undefined)

1 is only divisible by itself. Prime numbers must have 2 divisors.

Prime Factorization of a number is the number written as a product of its prime factors.

REVIEW DIVISIBILTY RULES (SEE CARD)

a) Is 3 a factor of 732?
Yes, the sum of the digits is divisible by 3

b) Is 4 a factor of 712?
Yes because the last two digits are divisible

c) Is 6 a factor of 558?
Yes, because 2 and 3 are both factors

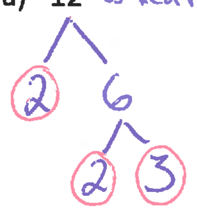
d) Is 8 a factor of 1064?
Yes, because the last three digits are divisible by 8

Example 1: Write the prime factorization of a number(Use a factor tree)

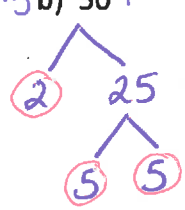
a) Leave answer as a product of it's prime factors

b) leave answer using powers as a product of its prime factors

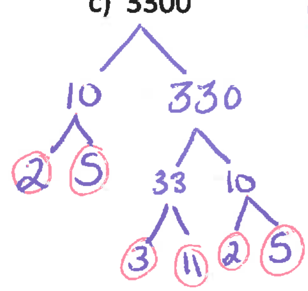
a) 12



b) 50



c) 3300



Step#1 - Create a factor tree

Step#2 - Circle prime factors

step#3 - Write as a product of prime factors

or

Write as a product of prime factors using powers

a) $12 = 2 \cdot 2 \cdot 3$

or

b) $12 = 2^2 \cdot 3$

a) $50 = 2 \cdot 5 \cdot 5$

or

b) $50 = 2 \cdot 5^2$

a) $3300 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 11$

b) $3300 = 2^2 \cdot 3 \cdot 5^2 \cdot 11$

Greatest Common Factor : the greatest common factor (the biggest number you can divide all numbers by) that 2 or more numbers have in common.

Concept #1: Determine the greatest common factor of whole numbers using prime factorization **(NC)**

Example 2: Determine the greatest common factor of:

a) 24 and 36 Method #1 "List all Factors"

24	36
1 × 24	1 × 36
2 × 12	2 × 18
3 × 8	3 × 12
4 × 6	4 × 9
	6 × 6

Find the biggest common factor

GCF = 12

b) 245, 280 and 385 Method #2 "Prime Factorization"

245: 5, 49; 49: 7, 7

$= \boxed{5} \cdot \boxed{7} \cdot 7$

280: 10, 28; 10: 2, 5; 28: 4, 7; 4: 2, 2

$= 2 \cdot 2 \cdot 2 \cdot \boxed{5} \cdot \boxed{7}$

385: 5, 77; 77: 7, 11

$= \boxed{5} \cdot \boxed{7} \cdot 11$

Step#1 - Factor each number

Step#2 - Write as a product of prime factors

Step#3 - Circle factors that are common among all numbers

Step#4 - GCF equals the product of these common factors

$\boxed{GCF = 5 \cdot 7 = 35}$

Concept #4: Solve problems that involve prime factors, greatest common factors, least common multiples, square roots or cube roots **(NC)**

Example 3: Two ropes are 48 m and 32 m long. Each rope is to be cut into equal pieces and all pieces must have the same length that is a whole number of metres. What is the greatest possible length of each piece?

48: 6, 8; 6: 2, 3; 8: 2, 4; 4: 2, 2

$= \boxed{2} \cdot \boxed{2} \cdot \boxed{2} \cdot \boxed{2} \cdot 3$

32: 8, 4; 8: 2, 4; 4: 2, 2; 4: 2, 2

$= \boxed{2} \cdot \boxed{2} \cdot \boxed{2} \cdot \boxed{2} \cdot 2$

$GCF = 2 \cdot 2 \cdot 2 \cdot 2$
$= 16$

The Greatest Possible length of each piece is 16m

Assignment: Pg 140 # 3ace, ~~4aef~~, 5ade, 6ace, 7, 8ace,9ad, 13 (Complete all questions with NO CALCULATOR, use divisibility rules and long division) (Note: Please label HW Assignments as follows:

[Date] 3.1 Factors and Multiples of Whole Numbers (GCF) [First and last name]

[Assignment ex. Pg 140 #3ace,4aef,5ade,6ace,7,8ace,9ad,13]

3.1 Factors and Multiples of Whole Numbers [LCM] (Day 2)

Concept #2: Determine the least common multiple of whole numbers using prime factorization (NC)

Least common multiple of two numbers is the smallest number (not zero) that is a multiple of both.

Example 1: Determine the least common multiple for:

a) 4 and 6

Method #1

List all the multiples of each number until the same number appears

4, 8, 12, 16

6, 12, 18

$\boxed{LCM = 12}$

b) 28, 42, and 63

28, 56, 84, 112, 140, 168, 196, 224, 252

42, 84, 126, 168, 210, 252

63, 126, 189, 252

$\boxed{LCM = 252}$

c) 12 and 15

Method #2

The product of the greatest power of each prime factor

12 → 4, 3; 4 → 2, 2

15 → 3, 5

$= 2^2 \cdot 3$ $= 3 \cdot 5$

Note: 3 is a factor in both #'s and it has the same power. Only take one of them

$LCM = 3 \cdot 2^2 \cdot 5$

$\boxed{= 60}$

c) 50 , 55 , 66

50 → 5, 10; 10 → 2, 5

55 → 5, 11

66 → 6, 11; 6 → 2, 3

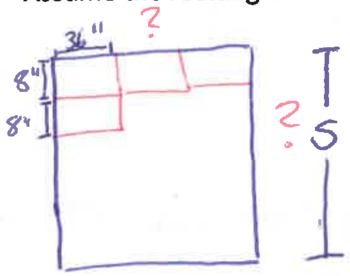
$= 2 \cdot 5^2$ $= 5 \cdot 11$ $= 2 \cdot 3 \cdot 11$

$LCM = 2 \cdot 3 \cdot 5^2 \cdot 11$

$\boxed{= 1650}$

Concept #4: : Solve problems that involve prime factors, greatest common factors, least common multiples, square roots or cube roots **(NC)**

Example 2: What is the side length of the smallest square that could be tiled with rectangles that measure 8" by 36"? Assume the rectangles cannot be cut. Sketch the square and rectangles.



S = side length

8 → 2, 4; 4 → 2, 2

36 → 6, 6; 6 → 2, 3; 6 → 2, 3

$8 = 2^3$

$36 = 2^2 \cdot 3^2$

$LCM = 2^3 \cdot \cancel{3}^2$

$= 8 \cdot \cancel{4}9$

$= \cancel{32}''$

$72''$

The side length of the smallest square that could be tiled using 8" by 36" is 72".

Example 3: The Mayan used several different calendar systems; one system used 365 days, another system used 260 days. Suppose the first day of both calendars occurred on the same day. After how many days would they again occur on the same day? About how long is this in years? Assume 1 year has 365 days.

260 → 10, 26; 10 → 2, 5; 26 → 2, 13

$= 2^2 \cdot 5 \cdot 13$

365 → 5, 73

$= 5 \cdot 73$

$LCM = 2^2 \cdot 5 \cdot 13 \cdot 73$

$LCM = 18{,}980$ Days

$$\begin{array}{r} \overset{2}{365} \\ \times\ 50 \\ \hline 000 \\ 3250 \end{array}$$

After 18,980 days which is 52 years, the two calendar systems' first day will occur on the same day.

In years,

$18{,}980 \div 365$

$= 52$ years

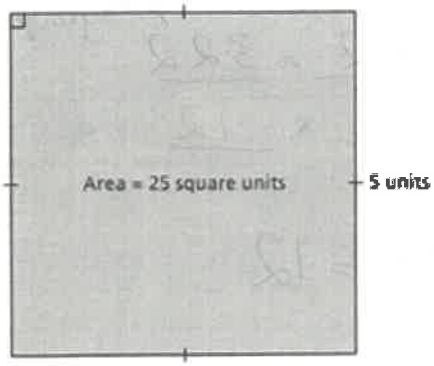
3.1 (Day 2)

Assignment Pg140 #8bd, 10bdf, 11ac, 12, 17, 19 (NO CALCULATOR)

Note: You will not use LCM to answer all questions. You need to be able to recognize when to answer the question using LCM or GCF.

3.2 – Perfect Squares, Perfect Cubes, and Their Roots

Any whole number that can be represented as the area of a square with a whole number side length is a perfect square.
The side length of the square is the square root of the area of the square.



We write: $\sqrt{25} = 5$

25 is a perfect square and
5 is its square root.

Perfect Square a number that can be expressed as a product of two equal integers **or** a number that is the area of a square such that the side length is a whole number.

Examples: 4 because $2 \times 2 = 4$ 25 because $5 \times 5 = 25$

Principle Square Root the positive integer of the square root taken. (A.k.a positive square root)

For example, the principal square root of 9 is 3, although *both* -3 and 3 are square roots of 9. Since $3 \times 3 = 9$ $-3 \times -3 = 9$

Example 1: Determine the square root of 576

step1 Prime Factorization

576
2 288
9 32
3 3 8 4
2 4 2 2
2 2

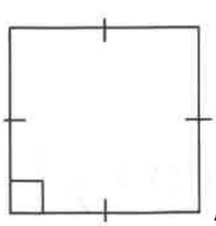
Step2 Arrange Prime factors into 2 equal groups

$= \underline{3 \cdot 2 \cdot 2 \cdot 2} \times \underline{3 \cdot 2 \cdot 2 \cdot 2}$

$= \underline{24} \times \underline{24}$

therefore $\sqrt{576} = 24$

Example 2: Determine the side length of the square using prime factorization.



Area = $225m^2$

225
5 45
9 5
3 3

$= \underline{5 \cdot 3} \times \underline{5 \cdot 3}$

$= 15 \times 15$

The side length of the square with an area of $225m^2$ is 15m.

Perfect Cube a number that can be expressed as a product of three equal integers or the volume of a cube such that the side length is a whole number

Example : 8 because $2\cdot2\cdot2=8$ 216 because $6\cdot6\cdot6=216$

Example 3: Determine the cube root of 1728 using prime factorization

Step1
Prime factorization

1728
2 864
2 432
2 216
3 72
9 8
3 3 2 4
2 2

Step2 Arrange prime factors into 3 ~~int~~ equal groups

$\underline{3\cdot2\cdot2} \times \underline{3\cdot2\cdot2} \times \underline{3\cdot2\cdot2}$

$\underline{12} \times \underline{12} \times \underline{12}$

Therefore

$\sqrt[3]{1728}=12$

Example 4: A cube has volume 5832in^3. What is the surface area of the cube?

$V=5832in^3$

9 648
3 3 9 72
3 3 9 8
3 3 2 4
2 2

$\underline{3\cdot3\cdot2} \times \underline{3\cdot3\cdot2} \times \underline{3\cdot3\cdot2}$

$\underline{18} \times \underline{18} \times \underline{18}$

Therefore the side lengths of the cube is 18in

Surface Area

S.A.= $18\times18\times6$ (# of sides)

S.A.= $1944in^2$

The Surface area of the cube is $1944in^2$.

Example 5: Use prime factorization to determine if 784 is a perfect square number, a perfect cube number, neither or both.

784
4 196
2 2 14 14
2 7 2 7

$\underline{2\cdot2\cdot7} \times \underline{2\cdot2\cdot7}$

$\underline{28} \times \underline{28}$

784 is a perfect square number because it is equal to the product of two identical integers

"Can you come up with a # that is a P.square + P.cube?"

Example ~~4~~ 6: Determine the side length of a cube with volume $125x^3y^6$

$V=125x^3y^6$

125
5 25
5 5

x^3
x x^2
x x

$y^6=y^2\cdot y^2\cdot y^2$

$\underline{5\cdot x\cdot y^2} \times \underline{5\cdot x\cdot y^2} \times \underline{5\cdot x\cdot y^2}$

Therefore the $\sqrt[3]{125x^3y^6}=5xy^2$
So the side length of the cube is represented by $5xy^2$.

Assignment Pg 146 # 4c,5a,6bcdf, 7a,8a,9, 10, 13, 17