To learn and apply the power rule and the sum and difference rules for differentiation..

THE POWER RULE (part 1):

- If $f(x) = x^n$, where n is a real number, then $f'(x) = nx^{n-1}$
- In Leibniz notation we say that $\frac{d}{dx}x^n = nx^{n-1}$

Note: Remember these are equivalent forms

$x^{-3} = \frac{1}{x^3}$ $\quad x^{2/3} = \sqrt[3]{x^2}$ $\quad x^{-1/2} = \frac{1}{\sqrt{x}}$

THE POWER RULE (part 2):

- If $f(x) = cx^n$, where c is a constant and n is a real number, then $f'(x) = (c)(n)x^{n-1}$
- In Leibniz notation we say that $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$

Ex #1: Find $f'(x)$ or $\frac{dy}{dx}$ of the following functions:

a) $f(x) = x^{15}$

$f'(x) = 15x^{15-1}$

$\boxed{f'(x) = 15x^{14}}$

Note: Multiply the exponent to the coefficient and subtract 1 from the exponent "Power Rule"

b) $f(x) = \frac{1}{x^3}$

$f(x) = x^{-3}$ "Re-write with negative exponent"

$f'(x) = -3x^{-3-1}$

$f'(x) = -3x^{-4}$

$\boxed{f'(x) = \frac{-3}{x^4}}$ "Re-write with a positive exponent"

c) $f(x) = \sqrt{x}$

$f(x) = x^{1/2}$ "rewrite with a fractional exponent"

$f'(x) = \frac{1}{2}x^{\frac{1}{2}-\frac{2}{2}}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$\boxed{f'(x) = \frac{1}{2\sqrt{x}}}$ "Re-write w a positive exponent and as a radical"

No negative exponents or fractional

d) $f(x) = \frac{1}{\sqrt[3]{x^2}}$

re-write in it's equivalent form

$f(x) = x^{-2/3}$

$f'(x) = -\frac{2}{3}x^{-\frac{2}{3}-\frac{3}{3}}$

$f'(x) = -\frac{2}{3}x^{-5/3}$

$f'(x) = \frac{-2}{3\sqrt[3]{x^5}}$ reduce radical

$\boxed{f'(x) = \frac{-2}{3x\sqrt[3]{x^2}}}$

NOTE: At this point we often leave negative exponents in the answers. We will sometimes leave radicals in the denominator and not rationalize the denominator.

Ex #2: Find $f'(x)$ or $\frac{dy}{dx}$ of the following functions:

a) $f(x) = -4x^{15}$

$f'(x) = -4(15)x^{15-1}$

$\boxed{f'(x) = -60x^{14}}$

b) $f(x) = (5x^4)^3$

re-write power

$f(x) = 5^3 x^{4(3)}$

$f(x) = 125x^{12}$

$f'(x) = 12(125)x^{12-1}$

$\boxed{f'(x) = 1500x^{11}}$

c) $f(x) = \sqrt{7x}$

$f(x) = (7x)^{1/2}$

$f(x) = 7^{1/2}x^{1/2} = \sqrt{7}x^{1/2}$

$f'(x) = \frac{1}{2}\sqrt{7}x^{\frac{1}{2}-\frac{2}{2}}$

$f'(x) = \frac{\sqrt{7}}{2}x^{-1/2} \Rightarrow \boxed{f'(x) = \frac{\sqrt{7}}{2\sqrt{x}}}$

d) $f(x)=\sqrt[3]{\dfrac{4}{x^2}}$

$f(x)=\left(\dfrac{4}{x^2}\right)^{1/3}$

$f(x)=\dfrac{4^{1/3}}{x^{2(1/3)}}$

$f(x)=\underbrace{\sqrt[3]{4}}_{\text{coefficient}}\,x^{-2/3}$

Re-write in it's equivalent form as a power

$f'(x)=-\dfrac{2}{3}\sqrt[3]{4}\,x^{-2/3-3/3}$

$f'(x)=-\dfrac{2\sqrt[3]{4}}{3}x^{-5/3}$

$f'(x)=\dfrac{-2\sqrt[3]{4}}{3\sqrt[3]{x^5}}$ ← Reduce Radical

$\boxed{f'(x)=\dfrac{-2\sqrt[3]{4}}{3x\sqrt[3]{x^2}}}$

e) $y=3x^3\sqrt[3]{x}$

$y=3x^3(x)^{1/3}$

* Exponent Rule: when multiplying powers with the same base add the exponents

$y=3x^{\frac{9}{3}+\frac{1}{3}}$

$y=3x^{10/3}$

$\dfrac{dy}{dx}=\cancel{3}\left(\dfrac{10}{\cancel{3}}\right)x^{\frac{10}{3}-\frac{3}{3}}$

$\dfrac{dy}{dx}=10x^{7/3}$

$\dfrac{dy}{dx}=10\sqrt[3]{x^7}$

Reduce the Radical

$\boxed{\dfrac{dy}{dx}=10x^2\sqrt[3]{x}}$

THE CONSTANT RULE: If $f(x)=c$, where c is a constant (#), then $f'(x)=0$ (Proof on P 78)

Ex #3: If $f(x)=-5$, determine $f'(x)$.

$f'(x)=0$

Ex #4: Find the equation of the tangent line to the curve $y=x^5$ at the point $(2,32)$.

Step 1 Find the slope of the tangent line at $x=2$ by finding the derivative $f'(2)$ or $\left.\dfrac{dy}{dx}\right|_{x=2}$

$y=x^5$

$\dfrac{dy}{dx}=5x^4$

$\left.\dfrac{dy}{dx}\right|_{x=2}\ 5(2)^4$

$=80$

Step 2 $y-32=80(x-2)$ Equation of tangent line at $(2,32)$ on $y=x^5$

THE SUM/DIFFERENCE RULE:

If $f(x)$ is the sum of 2 differentiable functions $f(x) = g(x) \pm h(x)$) then $f'(x) = g'(x) \pm h'(x)$

Ex #5: Find $f'(x)$ or $\dfrac{dy}{dx}$ of the following functions:

a) $f(x) = 2x^3 + 7x^6$

"Use the power rule for each individual term"

$\boxed{f'(x) = 6x^2 + 42x^5}$

b) $f(x) = (4x-3)^2$ Expand (Multiply)

$f(x) = (4x-3)(4x-3)$

$f(x) = 16x^2 - 24x + 9$ derivative of a constant is zero

$f'(x) = 16(2)x^{2-1} - 24x^{1-1} + 0$

$\boxed{f'(x) = 32x - 24}$

c) $f(x) = \dfrac{\pi x^6}{2} + x - \dfrac{3}{x}$

$f(x) = \dfrac{\pi}{2}x^6 + x - 3x^{-1}$

$f'(x) = 6\left(\dfrac{\pi}{2}\right)x^{6-1} + x^{1-1} - 3(-1)x^{-1-1}$

$f'(x) = 3\pi x^5 + x^0 + 3x^{-2}$

$\boxed{f'(x) = 3\pi x^5 + 1 + \dfrac{3}{x^2}}$

d) $f(x) = \dfrac{(3x-5)(3x+5)}{x^5}$ Multiply

$f(x) = \dfrac{9x^2 - 25}{x^5}$ Divide into each term

$f(x) = \dfrac{9x^2}{x^5} - \dfrac{25}{x^5}$

$f(x) = 9x^{-3} - 25x^{-5}$

$f'(x) = 9(-3)x^{-3-1} - 25(-5)x^{-5-1}$

$f'(x) = -27x^{-4} + 125x^{-6}$

$\boxed{f'(x) = \dfrac{-27}{x^4} + \dfrac{125}{x^6}}$

Ex #6: At what point on the curve $y = -x^2 + 3x + 4$ does the tangent line have a slope of 5?

Step 1 Find the derivative. $\dfrac{dy}{dx} = -2x^1 + 3$

Step 2 substitute $\dfrac{dy}{dx} = 5$ and solve for "x"

$5 - 3 = -2x + 3 - 3$

$2 = -2x$

$\dfrac{2}{-2} = \dfrac{-2x}{-2}$

$x = -1$

x-coordinate on $y = -x^2 + 3x + 4$ where slope of tangent line is 5

Step 3 Solve for y-coordinate when $x = -1$

$y = -(-1)^2 + 3(-1) + 4$

$y = -1 - 3 + 4$

$y = -4 + 4$

$y = 0$

At the point $(-1, 0)$ the slope of the tangent line is 5

Ex #7/ A ball is dropped from the upper observation deck of the CN Tower. The distance fallen, in metres, after t seconds is $s = 4.9t^2$. How fast is the ball falling at 3 seconds? Need to find the instantaneous velocity of the ball at t=3secs. Remember: First Derivative = Instantaneous velocity

$s'(t) = 4.2(2)t^{2-1}$

$s'(t) = 8.4t^1$

$s'(3) = 8.4(3)$

$s'(3) = 25.2 \, m/s$

The ball will be falling at a velocity of 25.2 m/s at 3 seconds.

Unit 3 : DAY 1 ASSIGNMENT (Section 2.2 & 2.3 in Text)

TEXTBOOK P83 #1, 2a-k, 3ace, 4bc,7,8 and P88 1a-j, 2ac, 3ab , 4 , 7

CALCULUS 30: UNIT 3: DAY 2 – THE PRODUCT RULE (SECTION 2.4)

To learn and apply the PRODUCT rule for differentiation..

When finding the derivative of a function it is definitely easier to use the power rule than it is to use the $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.
However, the power will only get us so far. If we were asked to find the derivative of
$f(x) = (2x^3 - 4x^2 + x + 8)(4x^2 + 6x - 7)$ to use the power rule we would first have to multiply everything else – which could be very time consuming and ugly!

The Product Rule

If $f(x)$ and $g(x)$ are differentiable functions, and $F(x) = f(x)\cdot g(x)$, then $F'(x) = f(x)g'(x) + f'(x)g(x)$
(first times the derivative of the second plus the second times the derivative of the first).

NOTE: It's very important to realize that the derivative of a product DOES NOT equal the product of the derivatives

$$[f(x)g(x)]' \neq f'(x)g'(x)$$

Ex #1: Find the derivative, $\frac{dy}{dx}$ if $y = (2x^3 + 7)(3x^2 - x)$. Use the product law. *(Note: these questions could also be solved by expanding and using the power rule, sum and difference rules .Using the product rule may be more efficient)*

Using the Product Rule

$$\frac{dy}{dx} = (2x^3+7)\underbrace{(6x-1)}_{\text{derivative of 2nd factor}} + \underbrace{(6x^2)}_{\text{derivative of 1st factor}}(3x^2-x)$$

$$\frac{dy}{dx} = (12x^4 - 2x^3 + 42x - 7) + 18x^4 - 6x^3$$

$$\boxed{\frac{dy}{dx} = 30x^4 - 8x^3 + 42x - 7}$$

Using Power Rule Multipy factor

$y = 6x^5 - 2x^4 + 21x^2 - 7x$

$$\boxed{\frac{dy}{dx} = 30x^4 - 8x^3 + 42x - 7}$$

Ex #2: Differentiate $f(x)=\sqrt{x}(2-3x)$ using the product law and simplify. Express your answer using a common denominator.

$$f(x)=x^{1/2}(2-3x)$$
$$f'(x)=x^{1/2}(-3)+\tfrac{1}{2}x^{1/2-\frac{2}{2}}(2-3x)$$
$$f'(x)=-3x^{1/2}+\tfrac{1}{2}x^{-1/2}(2-3x)$$
$$f'(x)=-3x^{1/2}+x^{-1/2}-\tfrac{3}{2}x^{-1/2+\frac{2}{2}}$$
$$f'(x)=-3x^{1/2}+x^{-1/2}-\tfrac{3}{2}x^{1/2}$$
$$f'(x)=\frac{-3\sqrt{x}(2\sqrt{x})}{(2)(\sqrt{x})}+\frac{1(2)}{\sqrt{x}(2)}-\frac{3\sqrt{x}(\sqrt{x})}{2(\sqrt{x})}$$
$$f'(x)=-\frac{6x}{2\sqrt{x}}+\frac{2}{2\sqrt{x}}-\frac{3x}{2\sqrt{x}}$$
$$\boxed{f'(x)=\frac{-9x+2}{2\sqrt{x}}}$$

Ex #3: Find the equation of the tangent line to the graph of $f(x)=(3x^2+2)(2x^3-1)$ when $x=1$.

$\boxed{\text{Step 1}}$ Find the slope of the tangent line when x = 1 on f(x)

$$f'(x)=(3x^2+2)(6x^2)+(6x)(2x^3-1)$$
$$f'(1)=(3(1)^2+2)(6(1)^2)+6(1)(2(1)^3-1)$$
$$f'(1)=(5)(6)+6(1)$$
$$f'(1)=30+6$$
$$f'(1)=36$$

↳ slope of the tangent line

Note: No need to simplify since we need to calculate the slope of the tangent line when x = 1

$\boxed{\text{Step 2}}$ Find the y-coordinate

$$y=(3(1)^2+2)(2(1)^3-1)$$
$$y=(5)(1)$$
$$y=5$$
$$(1,5)$$ ← point of tangency

$\boxed{\text{Step 3}}$ $\boxed{y-5=36(x-1)}$

Equation of tangent line at x = 1 on $y=(3x^2+2)(2x^3-1)$

Ex #4: Find the slope of the tangent line to the function $y=\left(4\sqrt{x}+\frac{2}{x^2}\right)\left(\sqrt[3]{x}-x^3\right)$ at the point x = 1

$$y=(4x^{1/2}+2x^{-2})(x^{1/3}-x^3)$$
$$\frac{dy}{dx}=(4x^{1/2}+2x^{-2})\left(\tfrac{1}{3}x^{-\frac{2}{3}}-3x^2\right)+(2x^{-1/2}-4x^{-3})(x^{1/3}-x^3)$$
$$\left.\frac{dy}{dx}\right|_{x=1}\ \left(4\sqrt{1}+\frac{2}{1^2}\right)\left(\frac{1}{3\sqrt[3]{1^2}}-3(1)^2\right)+\left(\frac{2}{\sqrt{1}}-\frac{4}{(1)^3}\right)\left(\sqrt[3]{1}-1^3\right)$$
$$\left.\frac{dy}{dx}\right|_{x=1}\ (4+2)\left(\tfrac{1}{3}-\tfrac{3}{1}\right)+(2-4)(1-1)$$
$$\left.\frac{dy}{dx}\right|_{x=1}\ (6)\left(-\tfrac{8}{3}\right)+0$$
$$\left.\frac{dy}{dx}\right|_{x=1}\ \cancel{6}^{2}\left(-\tfrac{8}{\cancel{3}}\right)$$
$$\left.\frac{dy}{dx}\right|_{x=1}\ -16$$

The slope of the tangent line at x = 1 is −16

The pattern using the product rule can continue with more factors:

Ex. $(f \cdot g \cdot h)' = f' \cdot g \cdot h + f \cdot g' \cdot h + f \cdot g \cdot h'$

$(f \cdot g \cdot h \cdot j)' = f' \cdot g \cdot h \cdot j + f \cdot g' \cdot h \cdot j + f \cdot g \cdot h' \cdot j + f \cdot g \cdot h \cdot j'$

And so on . . .

Ex#5/ : If $w(x) = (2x^5)(3x^2 - 4x^{-1})(7x^3 - 6x^{1/2})$ find $w'(x)$. (Don't simplify your answer)(IF TIME PERMITS)

$w'(x) = (10x^4)(3x^2 - 4x^{-1})(7x^3 - 6x^{1/2}) + (2x^5)(6x + 4x^{-2})(7x^3 - 6x^{1/2}) + (2x^5)(3x^2 - 4x^{-1})(21x^2 - 3x^{-1/2})$

Unit 3: DAY 2 ASSIGNMENT (Section 2.4 in Text)

TEXTBOOK P 92 #1, 2abdeh, 3, 4, 5 , 6 ,9

CALCULUS 30: UNIT 3: DAY 3 – THE QUOTIENT RULE (SECTION 2.5)

To learn and apply the QUOTIENT rule for differentiation..

THE QUOTIENT RULE:

- Given a function in the form of a quotient, $F(x) = \dfrac{f(x)}{g(x)}$, then $F'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$.

  (Note that we are using a capital F(x) for the quotient function)

- In other words, the derivative of the product of two expressions will be: [(bottom)(derivative of top) – (top)(derivative of bottom)] divided by (bottom squared)
- It is customary NOT to expand the expression in the denominator when applying the quotient rule
- "Low d'high minus high d'low all over the square of what's below"

Ex #1: Differentiate $F(x)=\dfrac{x^2+2x-3}{x^3-1}$ ← High ← Low

Low — Derivative of High — High — Derivative of low

$$F'(x)=\frac{(x^3-1)(2x+2)-(x^2+2x-3)(3x^2)}{(x^3-1)^2}$$ ← All over the square of whats below

Simplify the numerator, Keep the denominator

$$F'(x)=\frac{2x^4+2x^3-2x-2-[3x^4+6x^3-9x^2]}{(x^3-1)^2}$$

$$F'(x)=\frac{2x^4+2x^3-2x-2-3x^4-6x^3+9x^2}{(x^3-1)^2}$$

$$\boxed{F'(x)=\frac{-x^4-4x^3+9x^2-2x-2}{(x^3-1)^2}}$$

Ex #2: Find $\dfrac{dy}{dx}$ if $y=\dfrac{\sqrt{x}}{1+2x}$. $=\dfrac{x^{1/2}}{1+2x}$

$$\frac{dy}{dx}=\frac{(1+2x)(\frac{1}{2}x^{-1/2})-(x^{1/2})(2)}{(1+2x)^2}$$

$$\frac{dy}{dx}=\frac{\frac{1}{2}x^{-1/2}+x^{2/2+(-\frac{1}{2})}-2x^{1/2}}{(1+2x)^2}$$

$$\frac{dy}{dx}=\frac{\frac{1}{2}x^{-1/2}+x^{1/2}-2x^{1/2}}{(1+2x)^2}$$

$$\frac{dy}{dx}=\frac{\frac{1}{2}x^{-\frac{1}{2}}-x^{1/2}}{(1+2x)^2}$$

$$\frac{dy}{dx}=\left[\frac{1}{2\sqrt{x}}-\frac{\sqrt{x}\cdot(2\sqrt{x})}{1\cdot(2\sqrt{x})}\right]\frac{1}{(1+2x)^2}$$

$$\boxed{\frac{dy}{dx}=\frac{1-2x}{2\sqrt{x}(1+2x)^2}}$$

Ex #3: Find the equation of the tangent line to the curve $f(x)=\dfrac{\sqrt{x}}{x+2}$ at the point $(4,\ \frac{1}{3})$. Express your answers in GENERAL FORM (Ax + By + C = 0).

No fractional Coefficients.

$f(x)=\dfrac{x^{1/2}}{x+2}$

$$f'(x)=\frac{(x+2)(\frac{1}{2}x^{-1/2})-(x^{1/2})(1)}{(x+2)^2}$$

Note: x = 4

$$f'(4)=\frac{(4+2)\left(\frac{1}{2\sqrt{4}}\right)-\sqrt{4}}{(4+2)^2}$$

$$f'(4)=\frac{(6)(\frac{1}{4})-2}{6^2}$$

$$f'(4)=\left[\frac{3}{2}-\frac{2\cdot 2}{1\cdot 2}\right]\div 36$$

$$f'(4)=\left[\frac{3}{2}-\frac{4}{2}\right]\cdot\frac{1}{36}$$

$$f'(4)=\left(-\frac{1}{2}\right)\frac{1}{36}$$

$$f'(4)=-\frac{1}{72}$$

$$y-\frac{1}{3}=-\frac{1}{72}(x-4)$$

$$y-\frac{1}{3}=-\frac{1}{72}x+\frac{4}{72}$$

$+\frac{1}{72}x$, $-\frac{4}{72}$

$$(72)\ \frac{1}{72}x+(72)\ y-(72)\ \frac{1}{3}-\frac{4(72)}{72}=0\ (72)$$

$$x+72y-24-4=0$$

$$\boxed{x+72y-28=0}$$

Ex #4: Find the coordinates of two points on the graph of the function $f(x)=\dfrac{10x}{x^2+1}$ at which the tangent line is horizontal.

$$F'(x)=\frac{(x^2+1)(10)-(10x)(2x)}{(x^2+1)^2}$$

The tangent line is horizontal when the slope is zero (a.k.a the derivative $f'(x)=0$)

$$f'(x)=\frac{10x^2+10-20x^2}{(x^2+1)^2}$$

$$f'(x)=\frac{-10x^2+10}{(x^2+1)^2}$$

$$(x^2+1)^2\ 0=\frac{-10(x^2-1)}{(x^2+1)^2}(x^2+1)^2$$

$$\frac{0}{-10}=\frac{-10(x^2-1)}{-10}$$

$$0=(x^2-1)$$

$$0=(x-1)(x+1)$$

$x-1=0$, $x=1$ $x+1=0$, $x=-1$

when $x=1$ Find y

$f(1)=\dfrac{10(1)}{1^2+1}$

$f(1)=\dfrac{10}{2}$

$f(1)=5$

$(1,5)$

when $x=-1$ Find y

$f(-1)=\dfrac{10(-1)}{(-1)^2+1}$

$f(-1)=\dfrac{-10}{2}$

$f(-1)=-5$

$(-1,-5)$

$\boxed{\text{The tangent line has a slope of zero at } (1,5) \text{ and } (-1,-5)}$

Unit 3: DAY 3 ASSIGNMENT (Section 2.5 in Text)

TEXTBOOK P 95 #1a-l, 2, 3abc, 5, 6 Plus the following

Some ice cubes were added to a cup of boiling water. The temperature of the water in degrees Celsius t minutes after the ice cubes were added, can be approximated by the function

$T(t)=\dfrac{20t^2+100t+200}{t^2+t+2}$. Round your answers to two decimal places where necessary.

(a) Find $T(0)$, $T(1)$, and $T(5)$. Interpret your answers.

(b) Find $T'(t)$.

(c) Find $T'(1)$ and $T'(5)$. Interpret your answers.

(d) Find $\lim_{t\to\infty}\dfrac{20t^2+100t+200}{t^2+t+2}$ and interpret your result.

ANSWER: 21. (a) $T(0)=100$, $T(1)=80$, $T(5)=37.5$;

initially the water was $100°C$, after 1 minute it was $80°C$, after 5 minutes it was $37.5°C$ (b) $\dfrac{-80t^2-320t}{(t^2+t+2)^2}$

21. (c) $T'(1)=-25$, $T'(5)=-3.52$; after 1 minute the temperature is falling at a rate of $25°C/\text{min}$ and after 5 minutes the temperature is falling at a rate of $3.52°C/\text{min}$. (d) 20; As time passes the temperature of the water cools towards $20°C$, likely the room temperature. 22. (a) $L(0)=12000$, $L(5)=112000$,

CALCULUS 30: UNIT 3: DAY 4 – THE CHAIN RULE (SECTION 2.6)

To learn and apply the CHAIN rule for differentiation.

THE CHAIN RULE: (Think back to the composition of functions: Chapter 10 of PC30)

If $F(x)=f(g(x))$, then $F'(x)=f'(g(x))\bullet g'(x)$.

In other words: (derivative of outside function)•(derivative of inside function)

THE POWER RULE COMBINED WITH THE CHAIN RULE:

If $F(x)=[f(x)]^n$, then $F'(x)=n[f(x)]^{n-1}\bullet f'(x)$

Reminder: POWER RULE:

If $f(x)=x^n$, $f'(x)=nx^{n-1}$

Ex#1 Find $\frac{dy}{dx}$ if $y=(2x^3-4x+3)^{10}$

$$\frac{dy}{dx}=10(2x^3-4x+3)^9(6x^2-4)$$

or

$$\frac{dy}{dx}=(60x^2-40)(2x^3-4x+3)^9$$

Ex #2: a) Differentiate $y=\sqrt[3]{(2x^5-1)^2}$

$$y=(2x^5-1)^{2/3}$$

$$\frac{dy}{dx}=\frac{2}{3}(2x^5-1)^{\frac{2}{3}-\frac{3}{3}}(10x^4)$$

$$\frac{dy}{dx}=\frac{2}{3}(10x^4)(2x^5-1)^{-1/3}$$

Simplify $\frac{dy}{dx}=\frac{2(10x^4)}{3\sqrt[3]{2x^5-1}}$

$$\boxed{\frac{dy}{dx}=\frac{20x^4}{3\sqrt[3]{2x^5-1}}}$$

b) $f(x)=\frac{1}{\sqrt[3]{1-x^4}}$

$$f(x)=(1-x^4)^{-1/3}$$

$$f'(x)=-\frac{1}{3}(1-x^4)^{-1/3-\frac{3}{3}}$$

$$f'(x)=-\frac{1}{3}(1-x^4)^{-\frac{4}{3}}(-4x^3)$$

$$f'(x)=-\frac{1}{3}(-4x^3)(1-x^4)^{-4/3}$$

$$f'(x)=\frac{4}{3}x^3\frac{1}{\sqrt[3]{(1-x^4)^4}}$$ ← reduce radical

$$\boxed{f'(x)=\frac{4x^3}{3(1-x^4)\sqrt[3]{(1-x^4)}}}$$

Day 5 - The Chain Rule cont....

Ex #1) Differentiate the following.

a) $y = (x^2+1)^3(2-3x)^4$

★ This combines Product and Chain Rule to find the derivative ★

$$\frac{dy}{dx} = (x^2+1)^3\underbrace{\left(4(2-3x)^3(-3)\right)}_{\text{Derivative of 2nd factor using Chain Rule}} + \underbrace{3(x^2+1)^2(2x)}_{\text{Derivative of the first factor using Chain Rule}}(2-3x)^4$$

$$\frac{dy}{dx} = \underbrace{-12(x^2+1)^3(2-3x)^3}_{-6(x^2+1)^2(2-3x)^3} + \underbrace{6x(x^2+1)^2(2-3x)^4}_{-6(x^2+1)^2(2-3x)^3}$$

★ Think back to the crazy factoring we did in Unit 1. Factor out a GCF out of each term ★

$$\frac{dy}{dx} = \underbrace{-6(x^2+1)^2(2-3x)^3}_{\text{GCF}}\underbrace{\left[2(x^2+1) - x(2-3x)\right]}_{\text{After dividing each term by GCF.}}$$

$$\frac{dy}{dx} = -6(x^2+1)^2(2-3x)^3\left[2x^2+2-2x+3x^2\right]$$

$$\boxed{\frac{d}{dx} = -6(x^2+1)^2(2-3x)^3(5x^2-2x+2)}$$

"optional method to let u = inside function"

Note: $\frac{du}{dx}$ = The derivative of "u" function

b) $S(t) = \left(\frac{2t-1}{t+2}\right)^6$

let $u = \frac{2t-1}{t+2}$

use quotient rule → $\frac{du}{dx} = \frac{(t+2)(2)-(2t-1)(1)}{(t+2)^2}$

$$S'(t) = 6(u)^5 \cdot \frac{du}{dx}$$

$$S'(t) = 6\left(\frac{2t-1}{t+2}\right)^5 \cdot \frac{\cancel{2t}+4-\cancel{2t}+1}{(t+2)^2}$$

$$S'(t) = \frac{(2t-1)^5}{(t+2)^5} \cdot \frac{(6)\,5}{(t+2)^2}$$

← Note: When multiplying powers with the same base add the exponents

$$\boxed{S'(t) = \frac{30(2t-1)^5}{(t+2)^7}}$$

Day 5 - The Chain Rule cont.....

Ex#1) c) $F(x)=\sqrt{x+\sqrt{x^2+1}}$

Rewrite as a power with rational exponents $F(x)=\left(x+(x^2+1)^{1/2}\right)^{1/2}$

derivative of "x"

using chain rule derivative of $(x^2+1)^{1/2}$

Derivative of inside function x^2+1

$$F'(x)=\frac{1}{2}\left(x+(x^2+1)^{1/2}\right)^{-1/2}\cdot\left(1+\frac{1}{2}(x^2+1)^{-1/2}(2x)\right)$$

$$F'(x)=\frac{1}{2\left(x+(x^2+1)^{1/2}\right)^{1/2}}\cdot\left(1+\frac{2x}{2(x^2+1)^{1/2}}\right)$$

"Simplify, leave with no rational or negative exponents"

$$F'(x)=\frac{1}{2\sqrt{(x+\sqrt{x^2+1})}}\underbrace{\left(1+\frac{2x}{2\sqrt{x^2+1}}\right)}_{\text{create a common denominator to add}}$$

$$F'(x)=\frac{1}{2\sqrt{x+\sqrt{x^2+1}}}\left(\frac{\cancel{2}\sqrt{x^2+1}}{\cancel{2}\sqrt{x^2+1}}+\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}\right)$$

"Cancel 2's in each term"

multiply

$$F'(x)=\frac{1}{2\sqrt{x+\sqrt{x^2+1}}}\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right)$$

$$\boxed{F'(x)=\frac{\sqrt{x^2+1}+x}{2\sqrt{x+\sqrt{x^2+1}}\left(\sqrt{x^2+1}\right)}}$$

Day 5 - The Chain Rule cont....

Ex#1d) $y = \dfrac{3}{\sqrt{4x-3}}$ or $y = 3(4x-3)^{-1/2}$

Using chain rule

$y' = -\frac{3}{2}(4x-3)^{-3/2} \cdot \underbrace{(4)}_{\text{derivative of inside}}$

one degree less using chain rule

$y' = \dfrac{-12}{2(4x-3)^{3/2}}$ ← rewrite with positive exponent.

$y' = \dfrac{-6}{\sqrt{(4x-3)^3}}$ ← Simplify the radical

$\boxed{y' = \dfrac{-6}{4x-3\sqrt{4x-3}}}$

Using Quotient Rule

Ex#1d) $y = \dfrac{3}{\sqrt{4x-3}}$ or $\dfrac{3}{(4x-3)^{1/2}}$

$y' = \dfrac{(4x-3)^{1/2}(0) - 3\left(\frac{1}{2}(4x-3)^{-1/2}(4)\right)}{\left[(4x-3)^{1/2}\right]^2}$

$y' = \dfrac{-12\left(\frac{1}{2}\right)(4x-3)^{-1/2}}{(4x-3)^1}$

$y' = \dfrac{-6}{(4x-3)(4x-3)^{1/2}}$

$y = \dfrac{-6}{(4x-3)^{3/2}}$

$y = \dfrac{-6}{\sqrt{(4x-3)^3}}$

$\boxed{y = \dfrac{-6}{(4x-3)\sqrt{4x-3}}}$

Notice: Answers are the same no matter what method you use

REVIEW: Converting negative exponents to positive exponents

Ex #1: $\left(\frac{2x-1}{x^2+1}\right)^{-5} = \left(\frac{x^2+1}{2x-1}\right)^5$

Ex #2: $\frac{(x+1)^{-\frac{2}{3}}}{3x^{-4}(x-2)^2} = \frac{3x^4}{(x+1)^{2/3}(x-2)^2}$

Ex #3: $\frac{1}{2}(x+1)^{-\frac{3}{2}} \cdot 3x^{-3}(2x-3)^2 = \frac{3(2x-3)^2}{2(x+1)^{3/2}x^3}$

Factoring (with rational/negative exponents):

- Find the common factor (if there is one)
- Find the smallest exponent
- Remember that when you divide by the common factor, you subtract your exponent.

Ex #4: $2x^{\frac{3}{2}} + 4x^{\frac{1}{2}} - 6x^{-\frac{1}{2}}$

$2x^{-1/2}(x^2+2x-3) = 2x^{-1/2}(x+3)(x-1)$

Ex #5: $2x^3(x-2)^{-1}(x+1)^{\frac{3}{4}} - 4x^2(x-2)(x+1)^{-\frac{1}{4}}$

$2x^2(x-2)^{-1}(x+1)^{-1/4}\left[x(x+1) - 2(x-2)^2\right]$

Ex #1: Certain functions can be solved using different methods. To find the derivative of the following functions, what rules could you use? Choose the easiest option and differentiate the functions. (COMPLETE ON LOOSELEAF) See looseleaf

a) $y = \frac{x^2}{\sqrt{4x-3}}$

Quotient Rule or Product Rule if you rewrite the function $y = x^2(4x-3)^{-1/2}$

b) $f(x) = \frac{x(x^2+3)}{(x-2)^4}$ ← Quotient Rule w Product rule for numerator

$f(x) = x(x^2+3)(x-2)^{-4}$ ← Product Rule
$= (x^3+3x)(x-2)^{-4}$

Challenge Question:

c) $f(x) = \frac{x^5(x^2+3)^5}{(x-2)^4}$

Unit3 Day6 - Combining Rules

a) $y = \dfrac{x^2}{\sqrt{4x-3}}$

<u>Note:</u> You could use quotient rule with chain rule or Re-write as $y = x^2(4x-3)^{-1/2}$ use product rule with chain rule in it.

$\boxed{\text{Using The Product Rule}}$

$$y = x^2(4x-3)^{-1/2}$$

using the chain rule to find the derivative of the 2nd factor

$$\frac{dy}{dx} = 2x(4x-3)^{-1/2} + x^2\left(-\tfrac{1}{2}(4x-3)^{-3/2}(4)\right)$$

$$\frac{dy}{dx} = \frac{2x}{\sqrt{4x-3}} + \frac{-1\overset{2}{\cancel{(4)}}(x^2)}{\cancel{2}\sqrt{(4x-3)^3}}$$

reduce radical

$$\frac{dy}{dx} = \frac{2x\,(4x-3)}{\sqrt{4x-3}\,(4x-3)} - \frac{2x^2}{(4x-3)\sqrt{4x-3}}$$

* create a common denominator to subtract both fractions *

multiply

$$\frac{dy}{dx} = \frac{2x(4x-3) - 2x^2}{(4x-3)\sqrt{4x-3}}$$

$$\frac{dy}{dx} = \frac{8x^2 - 6x - 2x^2}{(4x-3)\sqrt{4x-3}}$$

$$\frac{dy}{dx} = \frac{6x^2 - 6x}{(4x-3)(\sqrt{4x-3})} = \frac{6x(x-1)}{(4x-3)(\sqrt{4x-3})}$$

Unit 3: Day 6 – Combining Rules

Ex#1) a) $y = \dfrac{x^2}{\sqrt{4x-3}}$

$\boxed{\text{Using The Quotient Rule}}$

$$\frac{dy}{dx} = \frac{(4x-3)^{1/2}(2x) - x^2\left(\frac{1}{2}(4x-3)^{-1/2}(4)\right)}{\left[(4x-3)^{1/2}\right]^2}$$

$$\frac{dy}{dx} = \frac{2x(4x-3)^{1/2} - \frac{4x^2}{2}(4x-3)^{-1/2}}{(4x-3)^1}$$

$$\frac{dy}{dx} = \frac{2x(4x-3)^{1/2} - 2x^2(4x-3)^{-1/2}}{(4x-3)}$$ ← ★ FACTOR numerator using CRAZY GCF factoring ★

$$\frac{dy}{dx} = \frac{\overbrace{2x(4x-3)^{-1/2}}^{\text{GCF}}\overbrace{\left[(4x-3) - x\right]}^{\text{what's left after factoring GCF}}}{4x-3}$$ combine like terms

$$\frac{dy}{dx} = \frac{2x(4x-3)^{-1/2}(3x-3)}{(4x-3)^1}$$ same base and they are being divided so subtract the exponents

$$\frac{dy}{dx} = 2x(4x-3)^{-3/2}(3x-3)$$ multiply

$$\frac{dy}{dx} = \frac{6x^2-6x}{\sqrt{(4x-3)^3}}$$ → reduce radical

$$\frac{dy}{dx} = \frac{6x^2-6x}{(4x-3)\sqrt{4x-3}} = \frac{6x(x-1)}{(4x-3)\sqrt{4x-3}}$$

★ Notice the derivative is the same if you use the product or Quotient Rule ★

"See previous page"

b) $f(x) = \dfrac{x(x^2+3)}{(x-2)^4}$

Using the Product Rule

$f(x) = x(x^2+3)(x-2)^{-4}$ * Re-write as a product

$f(x) = (x^3+3x)(x-2)^{-4}$

$f'(x) = (3x^2+3)(x-2)^{-4} + (x^3+3x)\underbrace{(-4(x-2)^{-5}(1))}_{\text{use chain rule to find derivative of the 2nd Factor}}$ Find Derivative using Product Rule

$f'(x) = (x-2)^{-5}\left[(3x^2+3)(x-2) - 4(x^3+3x)\right]$ Re-write with positive exponent

$f'(x) = \dfrac{[(3x^2+3)(x-2) - 4(x^3+3x)]}{(x-2)^5}$ ← multiply numerator

$f'(x) = \dfrac{3x^3-6x^2+3x-6-4x^3-12x}{(x-2)^5}$ ← combine like terms in the numerator

$f'(x) = \dfrac{-x^3-6x^2-9x-6}{(x-2)^5}$ ← Factor out GCF of -1

$f'(x) = -\dfrac{(x^3+6x^2+9x+6)}{(x-2)^5}$ Final Answer

Notice: It is the same answer if you use Quotient or Product Rule

b) $f(x) = \dfrac{x(x^2+3)}{(x-2)^4}$

Using the Quotient Rule

multiply,

$$f(x) = \frac{x(x^2+3)}{(x-2)^4}$$

$$f(x) = \frac{x^3+3x}{(x-2)^4}$$

$$f'(x) = \frac{(x-2)^4\overbrace{(3x^2+3)}^{\text{derivative of top}} - (x^3+3x)\overbrace{(4(x-2)^3(1))}^{\text{using chain rule derivative of bottom}}}{[(x-2)^4]^2}$$

← multiply exponents, using the exponent law when raising a power to a power

$$f'(x) = \frac{(x-2)^4(3x^2+3) - 4(x^3+3x)(x-2)^3}{(x-2)^8}$$

$$f'(x) = \frac{(x-2)^3[(x-2)(3x^2+3) - 4(x^3+3x)]}{(x-2)^8}$$ ★ Factor GCF

subtract exponents, using expont law

$$f'(x) = \frac{[(x-2)(3x^2+3) - 4(x^3+3x)]}{(x-2)^5}$$ multiply

$$f'(x) = \frac{(3x^3+3x-6x^2-6) - 4x^3-12x}{(x-2)^5}$$ ← combine like terms in the numerator

$$f'(x) = \frac{-x^3-6x^2-9x-6}{(x-2)^5}$$ ← Take out GCF of -1

$$f'(x) = \frac{-1(x^3+6x^2+9x+6)}{(x-2)^5}$$ FINAL ANSWER

Ex#1/ Unit 3: Day 6 - Combining Rules

c) Challenge Question

$$f(x) = \frac{x^5(x^2+3)^5}{(x-2)^4}$$

Using the Quotient Rule overall and the product rule for derivative of the Top.

$$f'(x) = \frac{(x-2)^4\left[\overbrace{5x^4(x^2+3)^5 + x^5(5(x^2+3)^4(2x))}^{\text{Derivative of Top}}\right] - (x^5(x^2+3)^5)\,4(x-2)^3}{\left[(x-2)^4\right]^2}$$

← multiply exponents

$$f'(x) = \frac{(x-2)^4\left[\underbrace{5x^4(x^2+3)^5 + 10x^6(x^2+3)^4}_{\text{Factor}}\right] - 4x^5(x^2+3)^5(x-2)^3}{(x-2)^8}$$

$$f'(x) = \frac{(x-2)^4\left[5x^4(x^2+3)^4\left[\underbrace{(x^2+3)+2x^2}_{\text{combine like terms}}\right]\right] - 4x^5(x^2+3)^5(x-2)^3}{(x-2)^8}$$

$$f'(x) = \frac{(x-2)^4\;5x^4(x^2+3)^4(3x^2+3) - 4x^5(x^2+3)^5(x-2)^3}{(x-2)^8}$$

Factor numerator with GCF

$$f'(x) = \frac{x^4(x-2)^3(x^2+3)^4\left[\overbrace{5(x-2)(3x^2+3) - 4x(x^2+3)}^{\text{Multiply and combine like terms}}\right]}{(x-2)^8}$$

simplify by subtracting exponents

$$f'(x) = \frac{x^4(x^2+3)^4\left[\overbrace{(5x-10)(3x^2+3) - 4x^3 - 12x}^{\text{combine like terms}}\right]}{(x-2)^5}$$

$$f'(x) = \frac{x^4(x^2+3)^4(15x^3+15x-30x^2-30-4x^3-12x)}{(x-2)^8}$$

$$f'(x) = \frac{x^4(x^2+3)^4(11x^3-30x^2+3x-30)}{(x-2)^5}$$

Same Answer if you used the product rule

Unit 3: Day 6 – Combining Rules

Ex#1/

Using the Product Rule

c) $f(x) = \dfrac{x^5(x^2+3)^5}{(x-2)^4}$

$h(x) = fgh$ ★ When their are 3 factors★

Derivative $h'(x)$ $= f'gh + fg'h + fgh'$

$f(x) = x^5(x^2+3)^5(x-2)^{-4}$

$f'(x) = 5x^4(x^2+3)^5(x-2)^{-4} + x^5(5(x^2+3)^4(2x))(x-2)^{-4} + x^5(x^2+3)^5(-4(x-2)^{-5}$

$f'(x) = 5x^4(x^2+3)^5(x-2)^{-4} + 10x^6(x^2+3)^4(x-2)^{-4} - 4x^5(x^2+3)^5(x-2)^{-5}$ ★ FACTOR GCF★

Multiply

$f'(x) = x^4(x^2+3)^4(x-2)^{-5}\left[5(x^2+3)(x-2) + 10x^2(x-2) - 4x(x^2+3)\right]$

Keep multiplying

$f'(x) = \dfrac{x^4(x^2+3)^4\left[(5x^2+15)(x-2) + 10x^3 - 20x^2 - 4x^3 - 12x\right]}{(x-2)^5}$

Combine Like terms

$f'(x) = \dfrac{x^4(x^2+3)^4\left[5x^3 - 10x^2 + 15x - 30 + 10x^3 - 20x^2 - 4x^3 - 12x\right]}{(x-2)^5}$

$f'(x) = \dfrac{x^4(x^2+3)^4(11x^3 - 30x^2 + 3x - 30)}{(x-2)^5}$

Unit 3 : DAY 6 ASSIGNMENT : the circled numbers

Find the derivative of each of the following functions, writing your answers in factored form where possible.

1. $f(x)=2x^3+15x^2-36x+12$

2. $f(x)=-2x^{-3}-\frac{1}{2}x^{-2}+x^{-1}+11$

3. $y=\frac{1}{x}+4x$

~~4.~~ $y=\sqrt{\frac{x}{5}}+\frac{5}{\sqrt{x}}-\frac{x}{\sqrt{5}}$

5. $f(x)=(2x-3)^3(x+1)^2$

6. $f(x)=x^2\sqrt{1-x^2}$

7. $y=(x-2)\sqrt{x^2-3x-1}$

8. $y=4\sqrt{x-1}-6\sqrt{x+1}$

9. $f(x)=\frac{x^2-3x}{x^2+3}$

10. $f(x)=\frac{6}{\sqrt[3]{x^3-2}}$

11. $y=x^3(2x-1)(3x+2)$

12. $y=\frac{x(2x-3)}{x^2+2}$

13. $f(x)=\left(\frac{2x}{x+2}\right)^2$

14. $f(x)=\frac{(x+1)^2}{x^2-2}$

15. $y=\frac{\sqrt{x}}{x^2+1}$

16. $f(x)=\frac{\sqrt{3-x}}{x^4}$

17. $f(x)=\frac{1}{(x^2-2)\sqrt{2x+3}}$

18. $f(x)=\sqrt{\frac{x+4}{x-4}}$

Answers:

1. $6(x-1)(x+6)$ 2. $-x^{-4}(x-3)(x+2)$ 3. $x^{-2}(2x-1)(2x+1)$ 4. $\frac{1}{2\sqrt{5}}x^{-1/2}-\frac{5}{2}x^{-3/2}-\frac{1}{\sqrt{5}}$

5. $10x(x+1)(2x-3)^2$ 6. $-x(3x^2-2)(1-x^2)^{-1/2}$ 7. $\frac{1}{2}(4x^2-13x+4)(x^2-3x-1)^{-1/2}$

8. $2(x-1)^{-1/2}-3(x+1)^{-1/2}$ 9. $\frac{3(x-1)(x+3)}{(x^2+3)^2}$ 10. $-6x^2(x^3-2)^{-4/3}$ 11. $2x^2(15x^2+2x-3)$

12. $\frac{3x^2+8x-6}{(x^2+2)^2}$ 13. $\frac{-(x+2)}{x^3}$ 14. $\frac{-2(x+1)(x+2)}{(x^2-2)^2}$ 15. $\frac{1-3x^2}{2x^{1/2}(x^2+1)^2}$ 16. $\frac{1}{2}x^{-5}(7x-24)(3-x)^{-1/2}$

17. $-(5x^2+6x-2)(x^2-2)^{-2}(2x+3)^{-3/2}$ 18. $-4(x+4)^{-1/2}(x-4)^{-3/2}$