Definition of a Derivative.

THE GENERAL DEFINITION OF A DERIVATIVE:

- The derivative of a function *f(x)*, is a formula that finds the slope of a tangent line to the curve f(x)at ANY POINT x = a to the curve of *f(x)*
- The derivative of the function *f(x)* is named *f'(x)* (which is read f prime of x) or can be known as $\frac{dy}{dx}$ (which is read as dee y by dee x)
- The process of finding the derivative is called **DIFFERENTIATION**
- To differentiate f(x) and find it's derivative f'(x), we can use the definition of the derivative:

$$\frac{dy}{dx} = f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Ex #1: Given f(x) = 5 + 4x – x²

a) Find f '(x) *"Can you develop a formula that will find the slope of a tangent line at any x-value for this particular function?"(note: this question could also be worded as"Find the derivative of this function?"*

$$f'(x) = \lim_{h\to 0}\frac{[5+4(x+h)-(x+h)^2]-[5+4x-x^2]}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{5+4x+4h-(x^2+2xh+h^2)-5-4x+x^2}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\cancel{5}+\cancel{4x}+4h-\cancel{x^2}-2xh-h^2-\cancel{5}-\cancel{4x}+\cancel{x^2}}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{4h-2xh-h^2}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\cancel{h}(4-2x-h)}{\cancel{h}}$$

$$f'(x) = 4-2x-0$$

$$\boxed{f'(x) = 4-2x}$$

To calculate the slope of a tangent line to any x-value on the function. $f(x) = 5+4x-x^2$ use the derivative $f'(x) = 4-2x$ or to calculate the instantaneous velocity if it was comparing distance vs time in an application.

b) Use your answer in part "a" to find f'(4)

$$f'(x) = 4-2x$$

$$f'(4) = 4-2(4)$$

$$f'(4) = 4-8$$

$$\boxed{f'(4) = -4}$$

The slope of the tangent line on $f(x) = 5+4x+x^2$ when $x=4$ is -4.

$f'(x) =$ slope of the tangent line

c) Find the coordinates of a point on the curve $f(x) = 5 + 4x - x^2$ at which the slope of the tangent line is 1. *(This could also be worded as "where the first derivative is 1")*

From previous question because we are dealing w the same function

$f'(x) = 4 - 2x$

$1 - 4 = 4 - 4 - 2x$ "substitute $f'(x) = 1$ and solve for x"

$\frac{-3}{-2} = \frac{-2x}{-2}$

$\frac{3}{2} = x$ ← x-coordinate of point

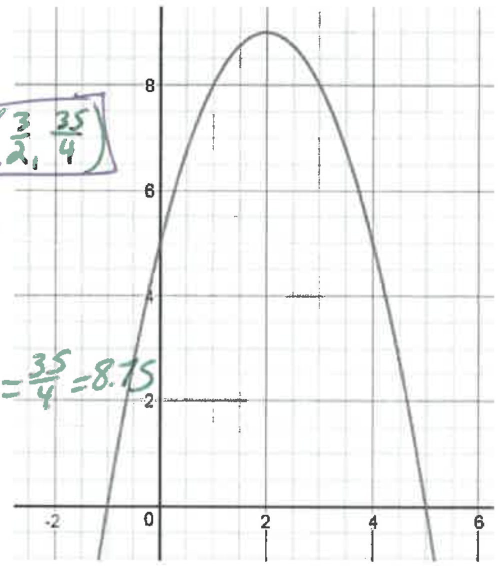
To find y-coordinate

$y = 5 + 4\left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)^2$

$y = 5 + 6 - \frac{9}{4}$

$y = \frac{35}{4} = 8.75$

$\boxed{\text{Coordinate } \left(\frac{3}{2}, \frac{35}{4}\right)}$



d) What is the difference between the answer to $f'(2)$ and $f(2)$? Use the given graph to help demonstrate your answer.

$f(2) = 9$ This is the height of the graph at $x = 2$

$f'(2) = 0$ This is the slope of the tangent line at $x = 2$ to $f(x)$

e) Where on the graph is $f'(x) = 0$? How do you know?

at $x = 2$ " Vertex to the parabola at a min or max point $f'(x) = 0$ "

f) Find the equation of the tangent line to the curve of F(x) at x=2

"Looking at the graph at $x = 2$ $y = 9$"

It is at the vertex of the parabola so the slope is zero. The tangent line is a horizontal line.

$\boxed{y = 9}$

https://www.desmos.com/calculator/e5prgzf0cy

Ex#2: Given the function $f(x) = \frac{10}{x}$; determine:

a) $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{\frac{10}{x+h} - \frac{10}{x}}{h}$$

$$f'(x) = \lim_{h \to 0} \left[\frac{10(x)}{x+h(x)} - \frac{10(x+h)}{x(x+h)}\right] \div h$$

$$f'(x) = \lim_{h \to 0} \left[\frac{\cancel{10x} - \cancel{10x} - 10h}{x(x+h)}\right] \cdot \frac{1}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{-10\cancel{h}}{x(x+h)\cancel{h}}$$

$$f'(x) = \lim_{h \to 0} \frac{-10}{x(x+h)}$$

$$f'(x) = \frac{-10}{x(x+0)}$$

$$\boxed{f'(x) = \frac{-10}{x^2}}$$

Can be used to find the derivative of any x-value for $f(x) = \frac{10}{x}$

b) $f'(2)$

"Substitute $x = 2$ into $f'(x) = \frac{-10}{x^2}$"

$$f'(2) = \frac{-10}{2^2}$$

$$f'(2) = -\frac{10}{4}$$

$$f'(2) = -\frac{5}{2}$$

The derivative when $x = 2$ on $f(x) = \frac{10}{x}$ is $-\frac{5}{2}$.

or

The slope of the tangent line at $x = 2$ on $f(x) = \frac{10}{x}$ is $-\frac{5}{2}$

From previous question b) The equation of the tangent line at the point (2, f(2)). Leave answer in slope-intercept form

$\hookrightarrow m = -\frac{5}{2}$ $f(2) = \frac{10}{2} = 5$ $(2, 5)$ x y

$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{5}{2}(x - 2)$ ← point slope form You need to convert to slope-intercept

$y - 5 = -\frac{5}{2}x + 5$ (+5 both sides)

$\boxed{y = -\frac{5}{2}x + 10}$

c) The value of x at which the slope of the tangent line to the curve is −5.

"or could be worded as "The value of x at which the derivative is −5"

$f'(x) = \frac{-10}{x^2}$ f'(x) = first derivative = slope of the tangent line. $f'(x) = -5$ "substitute"

$(x^2)\cdot -5 = \frac{-10}{x^2}(x^2)$

$\frac{-5x^2}{-5} = \frac{-10}{-5}$

$\sqrt{x^2} = \sqrt{2}$

$x = \pm\sqrt{2}$

At $x = \sqrt{2}$ and $x = -\sqrt{2}$ the slope of the tangent line is −5.

d) Where or if the slope of the tangent line will ever be positive? (Graph on desmos)

"When algebraically can $-\frac{10}{x^2} > 0$ (positive) $\boxed{\text{Remember}}$: $\frac{-10}{x^2} = f'(x)$ which is the slope of the tangent line.

x^2 will always be positive no matter what you substitute in for x. Then $-10 \div$ positive # will be − #.

∴ The slope of the tangent line will never be positive.

Ex#3: Find the derivative of a) f(x) = 8 b)f(x) = 2x+1

$f'(x) = 0$ $f'(x) = 2$

Unit 2 DAY 5 ASSIGNMENT #5

DUO TANG Unit 2 : Assignment #5 : #1b-g, 2ab, 3 ,9 Extension Questions #4, 5

Definition of a Derivative.

THE DERIVATIVE:

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

NOTE: Sometimes instead of asking for f'(x) they merely say f'

There are many ways to denote the derivative of a function $y = f(x)$.

Derivative Notation	Words describing derivative notation	Important points about the derivative notation
$f'(x)$	"f prime of x"	→ very common notation → names independent variable as x
$f'(a)$	"f prime of a"	usually "a" will be replaced by a # & f'(a) = slope value
y' or f'	"y prime" Don't use This	→ poor notation & often not accepted → does not name independent variable
$y'(a)$	"y prime of a"	not often used but ok
$\boxed{\frac{d}{dx}}y = \boxed{\frac{dy}{dx}}$	"dee by dee x of y" "dee x by dee y"	Excellent notation, names both variables and often clearer than f'(x).
$\left.\frac{dy}{dx}\right\|_{x=a}$	"dee y by dee x evaluated at x = a"	Same as finding f'(a) but sometimes more clear.
$\boxed{\frac{df}{dx}} = \boxed{\frac{d}{dx}f(x)}$	"dee f by dee x" "dee by dee x of f(x)"	Similar to $\frac{dy}{dx}$ but the y is called f(x)

$$\therefore f'(x) = y' = y'(x) = \frac{d}{dx}y = \frac{dy}{dx} = \frac{d}{dx}f(x) = \frac{df}{dx}$$

<u>Note:</u> Any notation using $\frac{d}{dx}$ should NOT be thought of as a fraction or as itself a derivative; it should be thought of as an operator that instructs you to take the derivative and treat x as the variable.

Ex #1: If $f(x)=\sqrt{x-2}$, find $f'(x)$ and state the domains of f and $f'(x)$ .https://www.desmos.com/calculator/rf4xj1l27b

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

"multiply by the conjugate"

$$f'(x)=\lim_{h\to 0}\frac{\sqrt{(x+h)-2}-\sqrt{x-2}}{h}\cdot\frac{(\sqrt{(x+h)-2}+\sqrt{x-2})}{(\sqrt{(x+h)-2}+\sqrt{x-2})}$$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)-2-(x-2)}{h(\sqrt{x+h-2}+\sqrt{x-2})}$$

$$f'(x)=\lim_{h\to 0}\frac{\cancel{x}+h-\cancel{2}-\cancel{x}+\cancel{2}}{h(\sqrt{x+h-2}+\sqrt{x-2})}$$

$$f'(x)=\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-2}+\sqrt{x-2})}$$

$$f'(x)=\lim_{h\to 0}\frac{1}{\sqrt{x+h-2}+\sqrt{x-2}}$$

$$f'(x)=\frac{1}{\sqrt{x+0-2}+\sqrt{x-2}}$$

$$f'(x)=\frac{1}{\sqrt{x-2}+\sqrt{x-2}}$$

← These are like radicals add

$$\boxed{f'(x)=\frac{1}{2\sqrt{x-2}}}$$

$\boxed{\text{Domain of } f(x)}$: $f(x)=\sqrt{x-2}$

Note: $(x-2)\geq 0$ because it is under a square root sign

$$\boxed{D=\{x\mid x\geq 2, x\in\mathbb{R}\} \text{ or } [2,\infty)}$$

$\boxed{\text{Domain of } f'(x)}$: $f'(x)=\frac{1}{2\sqrt{x-2}}$

Note: $2\sqrt{x-2}$ is in the denominator. so find out what makes it 0. This can't be in domain. and $x-2$ under square root must be positive.

$$\frac{2\sqrt{x-2}}{2}=\frac{0}{2}$$

$$(\sqrt{x-2})^2=(0)^2$$

$$x-2+2=0+2$$

$$x=2$$

$$\therefore x\neq 2$$

$$\boxed{D:\{x\mid x>2, x\in\mathbb{R}\} \text{ or } (2,\infty)}$$

Ex #2: If given the following limit of a function *f* at some value of *a*, state the value of *a* and the equation of f(x)

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{\overset{f(a+h)}{(4+h)^2}-\overset{f(a)}{16}}{h}$$

$\boxed{\text{Formula}}$ $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

$$\frac{dy}{dx}=\lim_{h\to 0}$$

$a=4$ $f(x)=x^2$

"you may need to complete on looseleaf.

Ex #3: Find $\frac{dy}{dx}$ ~~f'~~ if $f(x)=\dfrac{x+1}{3x-2}$. https://www.desmos.com/calculator/7brln65nth

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{\left[\frac{(x+h)+1}{3(x+h)-2}\right]-\left[\frac{x+1}{3x-2}\right]}{h}$$

$$\frac{dy}{dx}=\lim_{h\to 0}\left[\left[\frac{x+h+1}{3x+3h-2}\right]-\left[\frac{x+1}{3x-2}\right]\right]\div h$$

$$\frac{dy}{dx}=\lim_{h\to 0}\left[\left[\frac{(x+h+1)(3x-2)}{(3x+3h-2)(3x-2)}\right]-\left[\frac{(x+1)(3x+3h-2)}{(3x-2)(3x+3h-2)}\right]\right]\cdot\frac{1}{h}$$

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{(3x^2-2x+3xh-2h+3x-2)-(3x^2+3xh-2x+3x+3h-2)}{(3x+3h-2)(3x-2)h}$$

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{\cancel{3x^2}\cancel{-2x}\cancel{+3xh}-2h\cancel{+3x}\cancel{-2}\cancel{-3x^2}\cancel{-3xh}\cancel{+2x}\cancel{-3x}-3h\cancel{+2}}{(3x+3h-2)(3x-2)h}$$

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{-5\cancel{h}}{(3x+3h-2)(3x-2)\cancel{h}}$$

$$\frac{dy}{dx}=\frac{-5}{(3x+3(0)-2)(3x-2)}$$

$$\frac{dy}{dx}=\frac{-5}{(3x-2)(3x-2)}$$

$$\boxed{\frac{dy}{dx}=\frac{-5}{(3x-2)^2}}$$

Ex #4: If $f(x) = -5x^2 + 4x$, find $f'(2)$

$$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$f'(2)=\lim_{h\to 0}\frac{-5(2+h)^2+4(2+h)-\left[-5(2)^2+4(2)\right]}{h}$$

$$f'(2)=\lim_{h\to 0}\frac{-5(4+4h+h^2)+8+4h-\left[-20+8\right]}{h}$$

$$f'(2)=\lim_{h\to 0}\frac{-20-20h-5h^2+8+4h-\left[-12\right]}{h}$$

$$f'(2)=\lim_{h\to 0}\frac{\cancel{-20}-20h-5h^2\cancel{+8}+4h\cancel{+12}}{h}$$

$$f'(2)=\lim_{h\to 0}\frac{\cancel{h}(-20-5h+4)}{\cancel{h}}$$

$$f'(2)=-20-5(0)+4$$

$$\boxed{f'(2)=-16}$$

Ex #5: The first set of graphs represent a set of original functions f(x) and the second set of graphs represent the graphs of the derivatives f'(x) of the original functions. Match the original graph f(x) with its derivative f'(x)

https://www.intmath.com/differentiation/derivative-graphs.php

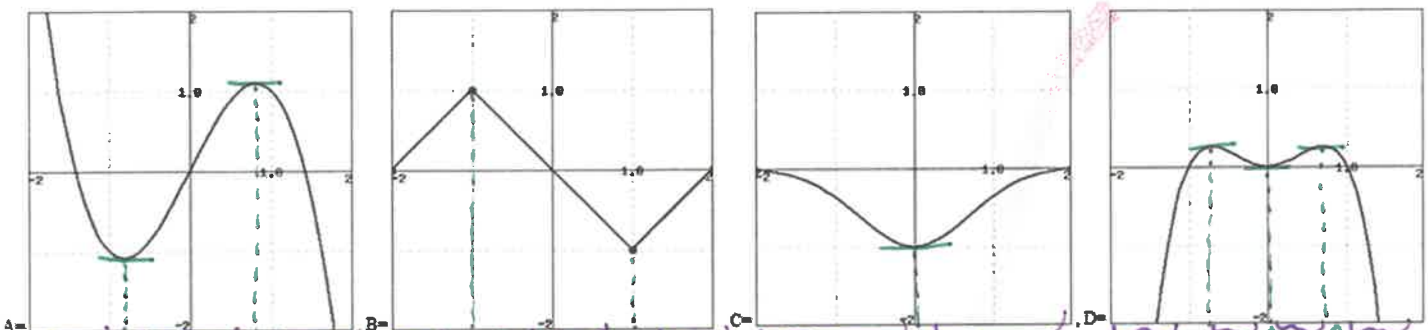
https://www.maa.org/sites/default/files/images/upload_library/4/vol4/kaskosz/derapp.html

In graph B, at what values of 'x' is the function not differentiable?
(where tangent lines do not exist)

f(x) graphs:

A **B** **C** **D**

$x = 1$; $x = -1$



A: neg slope, 0 slope, pos. slope, 0 slope, neg. slope

B: pos slope, undefined slope, neg. slope, ud slope, pos slope

C: neg slope, 0 slope, pos. slope

D: pos, 0 slope, neg slope, 0 slope, pos slope, 0 slope, neg

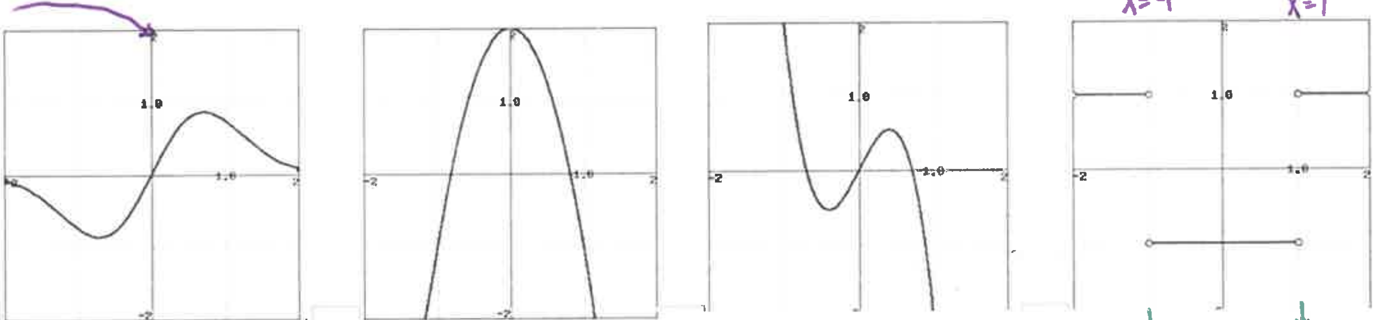
$\boxed{\text{Note}}$: Slopes on f(x) = position (y-value) on f'(x)

f'(x) graphs

C A D B

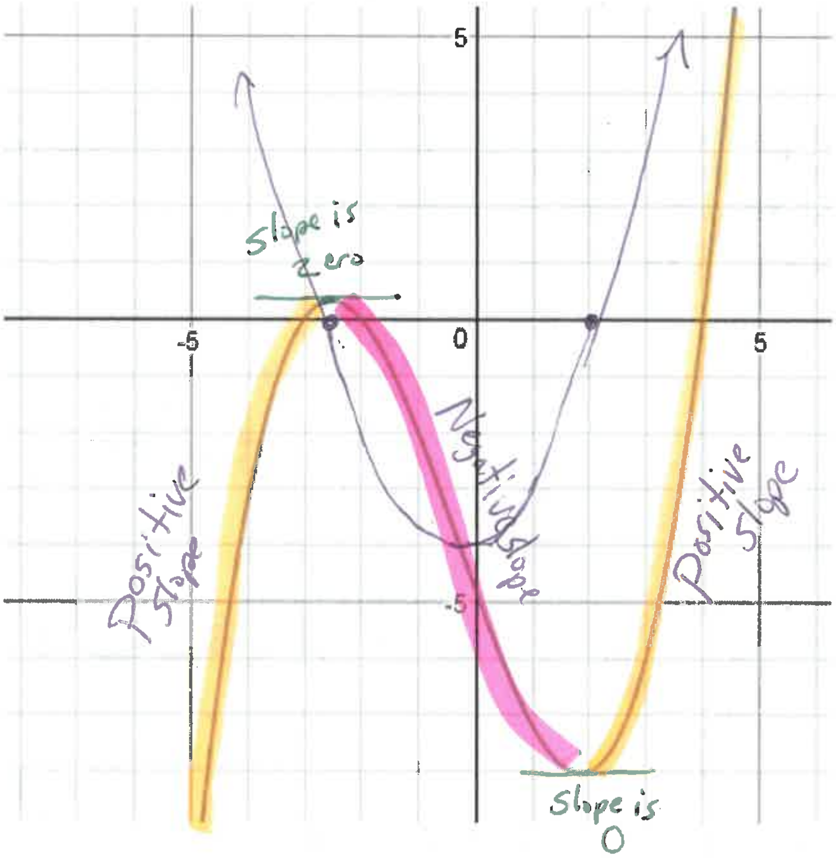
y = slope of tangent lines on f'(x)

$x=-1$ $x=1$



undefind Undefind

Ex #6: Given the following graph of f(x), sketch the graph of f′(x)



https://www.desmos.com/calculator/rmzuqwiyho

UNIT 2 DAY 6 ASSIGNMENT #6

Textbook Page 75 #1b-d, 2, 3, 5, 6, 8bc, 10b, 11a, 12cd, 13,14