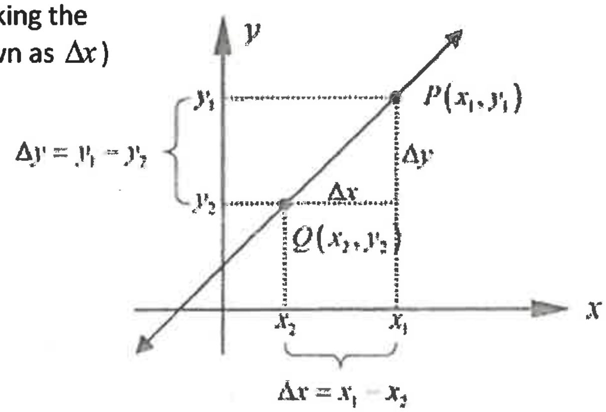
To review the concepts of slope and rate of change.

SLOPE OF A LINE:

- Is a measure of the steepness of a line and is found by taking the ratio of the rise (also known as) Δy to the run (also known as Δx)
- Steep lines have large slopes while lines that are almost horizontal have a small slope
- Horizontal lines have a slope of zero while vertical lines have undefined (infinite) slopes
- The formula for slope is: $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y - y_1}{x - x_1}$
- Point Slope Formula for a line: $y - y_1 = m(x - x_1)$
- Slope Intercept Formula for a line: y = mx + b
- Parallel lines have the same slope
- Perpendicular lines have slopes that are negative reciprocals
- When slope is given with units attached it is called a **RATE OF CHANGE**.



Ex #1:

a) Determine the slope of the line passing through the points (3, -7) and (-24, -28). What is the value of Δy and Δx?

$m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-28-(-7)}{-24-3} = \dfrac{-28+7}{-27} = \dfrac{-21}{-27}$ $\quad \Delta y = -21$, $\Delta x = -27$

b) Find the equation of the line.

Note: $m = \dfrac{-21 \div 3}{-27 \div 3} = \dfrac{7}{9}$

$y - y_1 = m(x - x_1)$

Slope point form $\boxed{y + 7 = \frac{7}{9}(x-3)}$ or Slope-int. form $\boxed{y = \frac{7}{9}x - \frac{28}{3}}$

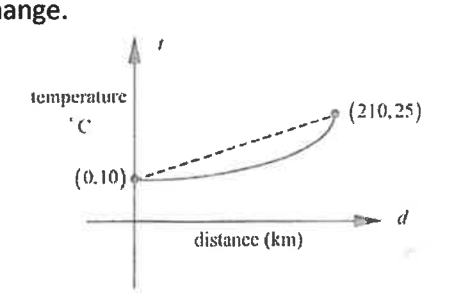
$y + 7 = \frac{7}{9}(x-3)$

$y + 7 = \frac{7}{9}x - \frac{21}{9} - \frac{7 \cdot 9}{1 \cdot 9}$

$y = \frac{7}{9}x - \frac{21}{9} - \frac{63}{9}$ $\quad y = \frac{7}{9}x - \frac{84 \div 3}{9 \div 3}$

Ex #2:

Anna's car displays the outside temperature. When she leaves her house the temp is 10°C. If the following graph shows the temperature as a function of the distance travelled, find the average rate of change.



$m = \dfrac{\Delta y}{\Delta x} = \dfrac{25-10 \text{ °C}}{210-0 \text{ Km}}$

$m = \dfrac{15 \text{ °C}}{210 \text{ Km}}$

$m = \dfrac{1 \text{°C}}{14 \text{Km}}$

The temperature rises 1°C approx every 14Km is the average rate of change.

Ex #3: For a given function f(x), $\frac{\Delta y}{\Delta x} = \frac{-4}{3}$

a) If x increased by 3, how much does y change?

$$\frac{\Delta y}{\Delta x} = \frac{-4}{3}$$

Substitute +3 in for Δx

$$(3)\frac{\Delta y}{3} = \frac{-4}{3}(3)$$

$$\Delta y = -4$$

The value of y will decrease by 4.

b) If x decreases by 18, how much does y change by?

$$\frac{\Delta y}{\Delta x} = \frac{-4}{3}$$

$$(-18)\frac{\Delta y}{-18} = \frac{-4(-18)}{3}$$

$$\Delta y = \frac{72}{3}$$

$$\Delta y = 24$$

The value of y will need to increase by 24.

Ex #4: A linear function is given by $y = 6 - 5x$. If x increases by 2, how does y change?

slope

$$slope = \frac{\Delta y}{\Delta x} = -5, \text{ so...}$$

$$\frac{\Delta y}{\Delta x} = \frac{-5}{1}$$

$$(2)\frac{\Delta y}{2} = \frac{-5}{1}(2)$$

$$\Delta y = -10$$

Ex #5: Find the equation of the line passing through (1, -2) with a slope of $2/3$.

$$y - (-2) = \frac{2}{3}(x + 1)$$

slope-point form

$$\boxed{y + 2 = \frac{2}{3}(x - 1)}$$

$$y + 2 = \frac{2}{3}x - \frac{2}{3} \quad -2 \quad \frac{-2 \cdot 3}{1 \cdot 3}$$

$$\boxed{y = \frac{2}{3}x - \frac{8}{3}}$$

Slope-intercept form.

Ex #6: Find the equation of a secant line that passes through the points (-3, 5) and (-6, 7). Express your answer in slope intercept form.

Google secant line to show pics.

$$m = \frac{\Delta y}{\Delta x} = \frac{7-5}{-6-(-3)} = \frac{2}{-3}$$

$$y - 7 = -\frac{2}{3}(x + 6)$$

$$y - 7 = -\frac{2}{3}x - \frac{12}{3} + \frac{7}{1} \quad +7$$

$$y = -\frac{2}{3}x - 4 + 7$$

$$\boxed{y = -\frac{2}{3}x + 3}$$

UNIT 2: DAY 1 ASSIGNMENT #1

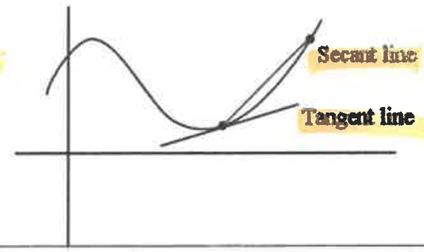
Textbook: **Page P9 #1 - 6, 12**

CALCULUS 30: UNIT 2 DAY 2 - SLOPES OF SECANT AND TANGENT LINES

To introduce the tangent line and to estimate the slope at the tangent line

TANGENT LINE – A line that touches a curve in one place

SECANT LINE – A line touches a curve in two or more places



Link to Desmos Graph for a Tangent line

Link to Desmos graph for a Secant line

USING THE SLOPE OF A SERIES OF SECANT LINES TO FIND THE SLOPE OF THE TANGENT LINE:

1. If we are given the following diagram where line l is tangent to the given curve at point P

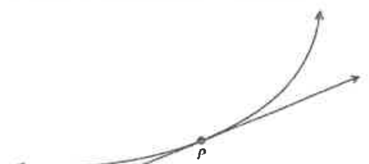


Figure 4

2. Suppose we find an infinite sequence of points Q_1, Q_2, Q_3.... that lie on the curve to the right of P, each one closer to P than it's neighbour.

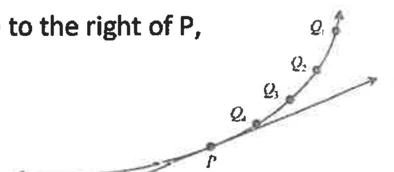


Figure 5

3. Now draw a sequence of secant lines $\overleftrightarrow{PQ_1}$, $\overleftrightarrow{PQ_2}$, $\overleftrightarrow{PQ_3}$,..., as shown in figure 6.

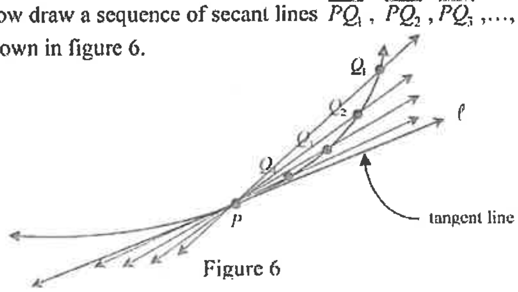


Figure 6

Notice in figure 7 that if we had taken our infinite sequence of points Q_n to the left of P and drawn the corresponding secant lines, they would also have a limiting position that would coincide with that of the tangent line.

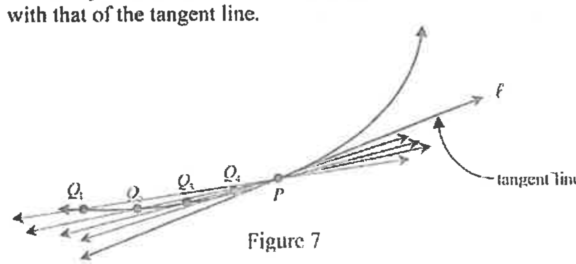


Figure 7

NOTE: Finding each individual slope between PQ_1, PQ_x, etc is finding an AVERAGE RATE OF CHANGE. Refer back to Ex#2 P.1

A Tangent line EXISTS if :

- the secant lines $\overleftrightarrow{PQ_n}$ approach the same unique line regardless as to whether the points Q_n are on the left or the right side of P. The unique line that is formed is called the TANGENT line to the curve at point P, and we say that the tangent line exists.

Animation of secant line approaching tangent line

https://www.youtube.com/watch?v=9aWZGbBSwas

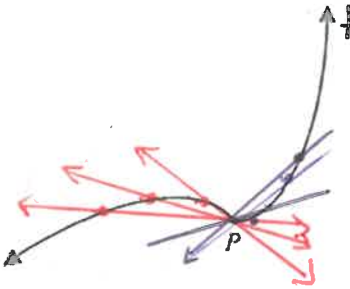
https://www.desmos.com/calculator/8ubngtz3ei

A Tangent line DO NOT EXIST if:

- At points where the slope of the tangent lines approached from the left do not equal the slope of the tangent lines approached from the right. This occurs as at a cusp or a corner.
- At points of discontinuity (Jump, removable, infinite)

Ex #1: Explain where the following graphs do not have tangent lines and why.

a)



This is the slope of the secant lines when point P is approached from the right.

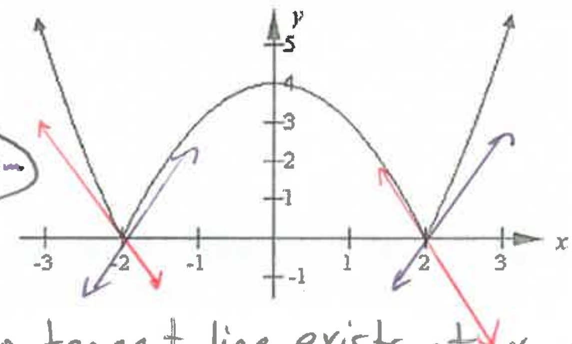
This is the slope of the secant lines when approached from the left of point P

No tangent line exists at point P because

$\lim_{x \to p^-} (\text{slope of } f(x)) \neq \lim_{x \to p^+} (\text{slope of } f(x))$

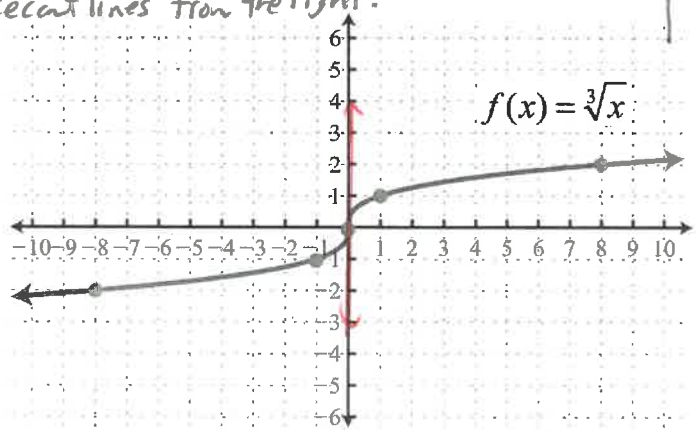
The secant lines from the left do not approach the same line as the secant lines from the right.

B)



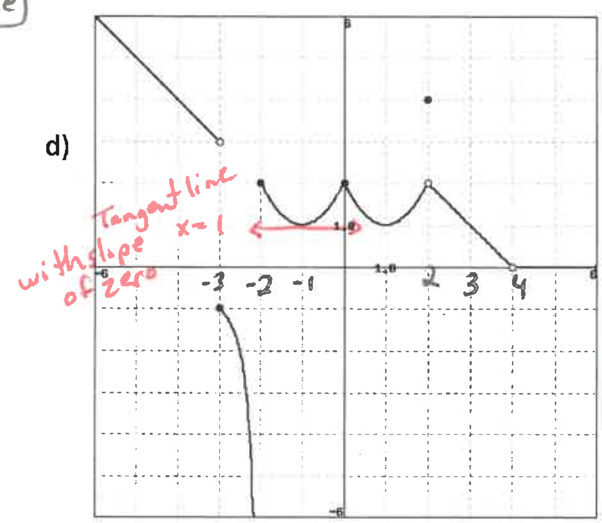
No tangent line exists at $x=-2$ and $x=2$ because the slope of from the left of each point $\neq$ slope from the right

c)



At $x=0$ it does have a tangent line but the tangent line has an undefined slope.

d)



Tangent lines Do not exist at points of discontinuity $x=-3$, $x=2$, $x=4$

Also at a cusp $x=-2$ and $x=0$

Note: Tangent lines will have a slope of zero at a vertexs (max or min)

Ex #2: a) Find the slope of the tangent line by taking the limit of the slope of the secant line. This can be expressed as: $= \frac{\Delta y}{\Delta x} =$

$$\lim_{Q\to P} m_{PQ} = m \quad \text{and} \quad \lim_{x\to p}\frac{f(x)-f(1)}{x-p} = \lim_{x\to 1}\frac{x^2-1}{x-1} = m \text{ (slope of the tangent line)}$$

y-coordinate of tangent point

slope of the tangent line

x-coordinate of tangent point.

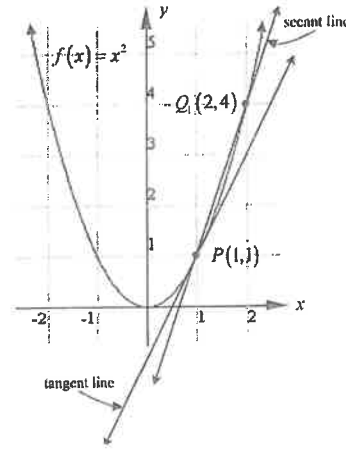
★Use your limit strategies to evaluate★

$$m = \lim_{x\to 1}\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}$$

$$m = \lim_{x\to 1}(x+1)$$

$$m = 1+1$$

$$\boxed{m = 2} \leftarrow \text{slope of the tangent line.}$$



a) Now that we know that the tangent line passes through P(1,1) and has slope 2, what is the equation of the tangent line?

(x_1, y_1) $m=2$

$(1, 1)$ $m = 2$

x_1 y_1

$$y - y_1 = m(x - x_1)$$

Equation in point-slope form. $\boxed{y - 1 = 2(x-1)}$

Slope-intercept form.

$$y - 1 = 2(x-1)$$

$$y - 1 = 2x - 2 \quad (+1)$$

$$\boxed{y = 2x - 1}$$

The equation of the tangent line on the function $y = x^2$ at $x = 1$.

SLOPE OF A TANGENT LINE AT A SPECIFIC POINT (Formula 1):

- The slope of a line tangent to a curve at a point (a, f(a)) is defined as follows:

$$m = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

Ex #3: Find the slope and the equation of the tangent line to the curve $y = 4x^2 - 3x - 1$ at the point (2, 9) by using the above slope formula. https://www.desmos.com/calculator/asojyl0oho

f(x) a f(a)

$$m = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

Substitute in $\begin{cases} a = 2 \quad f(a) = 9 \\ f(x) = 4x^2 - 3x - 1 \end{cases}$

$$m = \lim_{x\to 2}\frac{4x^2-3x-1-9}{x-2}$$ ★Simplify★

$$m = \lim_{x\to 2}\frac{4x^2-3x-10}{x-2}$$ ★ Use limit Strategies ★ (Factor & Reduce)

$$m = \lim_{x\to 2}\frac{(4x+5)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$m = 4(2)+5$$

$$m = 8+5$$

$$\boxed{m = 13}$$

Equation of the tangent line

m = 13 point (2,9)

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 9 = 13(x-2)}$$

UNIT 2: DAY 2 ASSIGNMENT #2

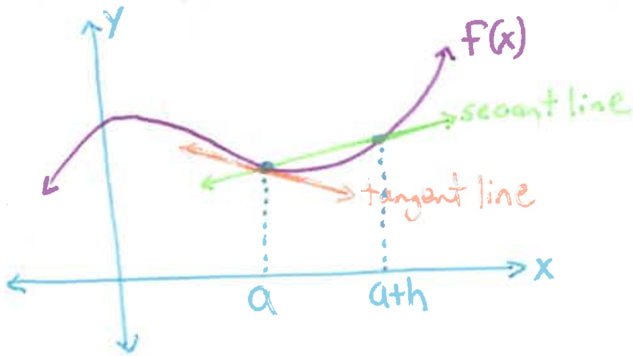
Duo Tang Assignment #2 questions # 1,2, Textbook: Pg 35 #1aib, 2aib , 8abcd (Use Formula 1)

CALCULUS 30: UNIT 2 DAY 3 – FINDING SLOPES OF TANGENT LINES USING LIMITS

Using Limits to find slopes of tangent lines at Specific Points

Today we are going to get a little more specific in how we can use limits to find the slope of a tangent line. Last day we used a table to estimate the slope of the tangent line, and then we used a new limit formula to find the slope without using a table. Today we are going to adapt that formula in such a way that it will help us transition to the next concept in Calculus 30. When we used the table to estimate the limit, we kept moving the position of Q closer and closer to the value of P. Today, instead of actually giving different values for Q, we are instead going to focus in on the horizontal distance that exists between Q and P, and see what happens when we take the limit of that horizontal distance (which we are going to call h) as it approaches zero.

Demonstration Ex #1:



① Given two points, how do you find the slope of a line?

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

② How would you find the slope of the secant line? point 1 $(a, f(a))$ Point 2 $(a+h, f(a+h))$

$$m = \frac{f(a+h) - f(a)}{(a+h) - a}$$

simplify →

$$m = \frac{f(a+h) - f(a)}{h}$$

③ As the second point $(a, f(a))$ approaches the first point $(a+h, f(a+h))$ what will "h" be approaching, for the secant line to become the tangent line?

As h approaches zero, "a" will eventually equal a+h

Slope of the tangent line $m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ $\boxed{\text{Formula 2}}$

$\boxed{\text{Note:}}$ The Slope of the tangent line is also known as the first derivative $f'(x)$ "f prime of x"

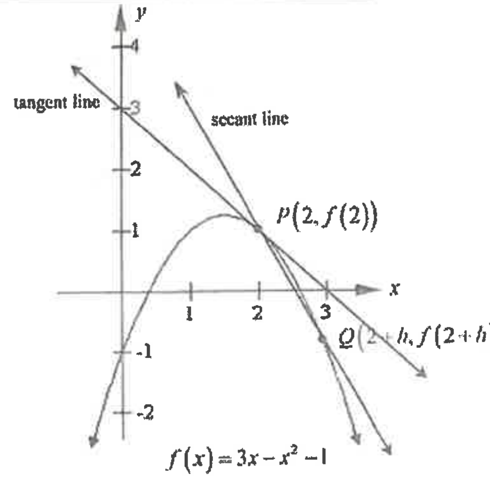
SLOPE OF A TANGENT LINE AT A SPECIFIC POINT (Formula 2):

- The slope of a line tangent to a curve at a point (a, f(a+h)) is defined as follows:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{(a+h)-a} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

Ex #2:

a) Find the slope of the tangent line drawn to the function $f(x) = 3x - x^2 - 1$ at the point P(2, f(2)).



Begin by identifying the following

- a = 2
- (a+h) = $2+h$
- f(a) = $f(2) = 3(2)-(2)^2-1 = 6-4-1 = 1$
- f(a+h) = $3(2+h)-(2+h)^2-1$

$$(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

$$m = \lim_{h \to 0} \frac{[3(2+h)-(2+h)^2-1] - 1}{h}$$

$$m = \lim_{h \to 0} \frac{[6+3h-(4+4h+h^2)-1] - 1}{h}$$

$$m = \lim_{h \to 0} \frac{\cancel{6}+3h-\cancel{4}-4h-h^2-\cancel{1}-\cancel{1}}{h}$$

$$m = \lim_{h \to 0} \frac{-h-h^2}{h}$$

$$m = \lim_{h \to 0} \frac{-\cancel{h}(1+h)}{\cancel{h}}$$

$$m = \lim_{h \to 0} -(1+h)$$

$$m = -(1+0)$$

$$\boxed{m = -1}$$ slope of the tangent line is -1

b) Find the equation of the tangent line

$$y - 1 = -1(x-2)$$

Ex #3: Find the equation to the tangent line drawn to the function $g(x) = \dfrac{1}{x+1}$ at x=1

$f(1) = \frac{1}{1+1} = \frac{1}{2}$ $a = 1$

$m = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$

$m = \lim_{h\to 0} \frac{\frac{1}{(1+h)+1} - \frac{1}{2}}{h}$

$m = \lim_{h\to 0} \left[\frac{1}{(2+h)}\frac{(2)}{2} - \frac{1}{2}\frac{(2+h)}{(2+h)}\right] \div h$

$m = \lim_{h\to 0} \left[\frac{2}{2(2+h)} - \frac{(2+h)}{2(2+h)}\right] \cdot \frac{1}{h}$

$m = \lim_{h\to 0} \frac{2-2-h}{2(2+h)h}$

$m = \lim_{h\to 0} \frac{-\cancel{h}}{2(2+h)\cancel{h}}$

$m = \lim_{h\to 0} \frac{-1}{2(2+h)}$

$m = \frac{-1}{2(2+0)}$

$\boxed{m = -\frac{1}{4}}$

Equation of the tangent line

$m = -\frac{1}{4}$ Point $(1, \frac{1}{2})$

$\boxed{y - \frac{1}{2} = -\frac{1}{4}(x-1)}$

Ex #4: a) Find the slope of the tangent line of f(x) = $x^2 + x - 2$ at a general specific point where x=a

b) Find the slopes of the tangent line when the x- coordinate is: i) 3 ii) -1.

a) $m = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

$m = \lim_{h\to 0} \frac{(a+h)^2 + a + h - 2 - (a^2 + a - 2)}{h}$

$m = \lim_{h\to 0} \frac{\cancel{a^2} + 2ah + h^2 \cancel{+a} + h \cancel{-2} \cancel{-a^2} \cancel{-a} \cancel{+2}}{h}$

$m = \lim_{h\to 0} \frac{h^2 + 2ah + h}{h}$

$m = \lim_{h\to 0} \frac{\cancel{h}(h + 2a + 1)}{\cancel{h}}$

$m = \lim_{h\to 0} h + 2a + 1$

$m = 0 + 2a + 1$

$\boxed{m = 2a + 1}$ general formal to find the slope of any tangent line at x=a on $f(x) = x^2 + x - 2$

b) i) $a = 3$

$m = 2(3) + 1$

$\boxed{m = 7}$

ii) $a = -1$

$m = 2(-1) + 1$

$m = -2 + 1$

$\boxed{m = -1}$

Unit 2 DAY 3 ASSIGNMENT #3

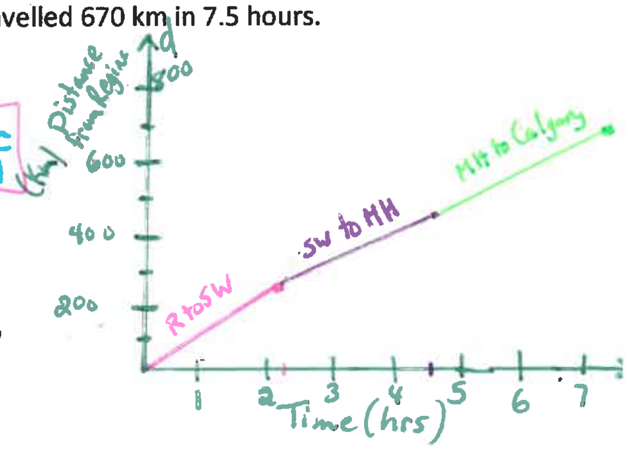
Textbook Pg 35 (ONLY USE FORMULA 2) #1a(ii)b, 6a (for i), 7ab for i, ii, v, 8cd, 9, 10

Calculating Average Velocity and Using Limits to Calculate Instantaneous Velocity at a Point.

AVERAGE VELOCITY:

$$\text{average velocity}=\frac{\text{distance travelled}}{\text{time elapsed}}=\frac{\Delta s}{\Delta t}=\frac{\textit{displacement}}{\textit{change}-\text{in}-\textit{time}}=\text{slope of the secant line}$$

Ex #1: You drive from Regina to Calgary. It takes 2.25 hours to reach Swift Current, which is 235km from Regina. Then you stop briefly in Medicine Hat to get gas. You notice that you have now travelled a total distance of 436 km and it has taken you 4.5 hours in all. By the time you reach Calgary, you have travelled 670 km in 7.5 hours.



a) What is your average velocity between Regina and Swift Current?

$$\text{Avg velocity} = \frac{235-0\,Km}{2.25-0\,hr} = \frac{235\,Km}{2.25\,hr} = \boxed{104.4\,\frac{Km}{h}}$$

b) What is your average velocity between Swift Current and Medicine Hat?

$$\text{Avg velocity} = \frac{\Delta s}{\Delta t} = \frac{436-235\,Km}{4.5-2.25\,h} = \frac{201\,Km}{2.25\,h}$$

$$\boxed{= 89.33\,Km/h}$$

t(hrs)	d(Km)
0	0
2.25	235
4.5	436
7.5	670

c) What is your average velocity between Regina and Calgary? https://www.desmos.com/calculator/yuyqkt0fhz

$$\text{avg velocity} = \frac{\Delta s}{\Delta t} = \frac{670\,Km - 0\,Km}{7.5\,h - 0\,hr} = \boxed{89.53\,Km/h}$$

AVERAGE VELOCITY: (also known as Average Rate of Change)

- We can expand upon our earlier definition of average velocity by saying that it is slope of a secant line between two points (x, y) and (x_1, y_1) on a distance time graph where y is distance and x is time.

$$\text{Average Velocity} = \frac{\text{distance travelled}}{\text{time elapsed}} = \frac{\text{change in distance}}{\text{change in time}} = \frac{\Delta y}{\Delta x}$$

Often the variables used are not x and y. Instead of y they often use "s". Instead of x they often use "t". In that case we would say that

$$\text{Average Velocity} = \frac{\text{change in distance}}{\text{change in time}} = \frac{\Delta s}{\Delta t}$$

- Instantaneous velocity is slope of the TANGENT line at a certain point in time. This will be the limit of an average velocity as Δt approaches zero

$$\text{Instantaneous Velocity} = v(a) = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

To actually use this limit algebraically to find an answer to the instantaneous velocity of a function at time "a", we need to apply the formula we learned in the last section:

$$\boxed{\text{Instantaneous Velocity} = v(a) = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

Ex #2: A ball is dropped from the top of a 400 foot tall building and falls such that its distance from the ground at t seconds is $s = -16t^2 + 400$ feet. (Note: Complete on Looseleaf)

a) What is the average velocity of the ball for the first 4 seconds?

b) What is the instantaneous velocity at 4 seconds?

c) Find the velocity after t seconds

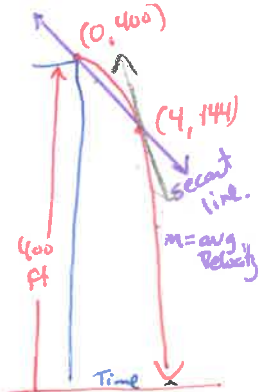
d) When will the ball it the ground?

e) With what velocity will the ball hit the ground?

Unit 2 DAY 4 ASSIGNMENT #4

Textbook Page 43 # 1a (i & ii), 1b, 2a(i & iv), 2b, 3, 5a(i only), 9

Ex #3: A ball is dropped from the top of a 400 foot tall building and falls such that its distance from the ground at t seconds is $s = -16t^2 + 400$ feet.



a) What is the average velocity of the ball for the first 4 seconds?

At 4 secs the height is:

$f(4) = -16(4)^2 + 400$

$f(4) = -256 + 400$

$f(4) = 144$

$\therefore$ At 4 secs the height is 144 ft.

$\text{Avg. Velocity} = \dfrac{\Delta s}{\Delta t} = \dfrac{144 - 400}{4 - 0} = \dfrac{-256}{4}$

$\text{Avg. Velocity} = -\dfrac{64\text{ ft}}{\text{sec}}$

b) What is the instantaneous velocity at 4 seconds?

$v(a) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$ ← slope of the tangent line at $a = 4$

$v(4) = \lim\limits_{h \to 0} \dfrac{-16(4+h)^2 + 400 - [-16(4)^2 + 400]}{h}$

$v(4) = \lim\limits_{h \to 0} \dfrac{-16(16 + 8h + h^2) + 400 - [-256 + 400]}{h}$

$v(4) = \lim\limits_{h \to 0} \dfrac{\cancel{-256} - 128h - 16h^2 + \cancel{400} + \cancel{256} - \cancel{400}}{h}$

$v(4) = \lim\limits_{h \to 0} \dfrac{-16h^2 - 128h}{h}$

$v(4) = \lim\limits_{h \to 0} \dfrac{-\cancel{h}(16h + 128)}{\cancel{h}}$

$v(4) = -(16(0) + 128)$

$v(4) = -128$

$\therefore$ The instantaneous velocity of the ball after 4 secs is 128 ft/sec.

Q: Why is it negative? Can speed be negative? It is negative because of the displacement. It is traveling in a negative direction (down). No speed can't be negative. Speed is defined as |velocity| ← absolute value

c) Find the velocity after t seconds

$v(t) = \lim\limits_{h \to 0} \dfrac{f(t+h) - f(t)}{h}$

$v(t) = \lim\limits_{h \to 0} \dfrac{-16(t+h)^2 + 400 - [-16t^2 + 400]}{h}$

$v(t) = \lim\limits_{h \to 0} \dfrac{-16(t^2 + 2th + h^2) + 400 + 16t^2 - 400}{h}$

$v(t) \lim\limits_{h \to 0} = \dfrac{\cancel{-16t^2} - 32th - 16h^2 + \cancel{400} + \cancel{16t^2} - \cancel{400}}{h}$

$v(t) \lim\limits_{h \to 0} = \dfrac{-16h^2 - 32th}{h}$

$v(t) \lim\limits_{h \to 0} \dfrac{-16\cancel{h}(h + 2t)}{\cancel{h}}$

$v(t) = (0 + 2t) - 16$

$v(t) = (2t) - 16$

$v(t) = -32t$

d) When will the ball hit the ground?

When the height is zero $s = 0$

$0 = -16t^2 + 400$

$0 = -16(t^2 - 25)$

$0 = -16(t - 5)(t + 5)$

$t - 5 = 0 \quad t + 5 = 0$

$t = 5 \quad t = -5$

* Note: If you can't factor use quadratic formula, or solve by square root.

The ball will hit the ground after 5 secs.

e) With what velocity will the ball hit the ground?

$v(5) = -32(5)$

$v(5) = -160\text{ ft/sec.}$

The ball will hit the ground at 160 ft/sec.

OUTCOME 4A DAY 4 ASSIGNMENT

Textbook Page 43 # 1a (i & ii), 1b, 2a(i & iv), 2b, 3, 5a(i only), 9

Definition of a Derivative.

VIDEO LINKS:

a) https://goo.gl/E3dkvY b) https://goo.gl/xYh5Li c) https://goo.gl/DM3kio

THE DEFINITION OF A DERIVATIVE at a value "*a*":

- The derivative of a function *f*, at the point (a, f(a)), is the slope of the line tangent to the curve at the point (a, f(a)
- The derivative of the function f(x) at the point (a, f(a)), is denoted by f'(a) – this is read as" f prime of a". To calculate f'(a), we use the limit formula:

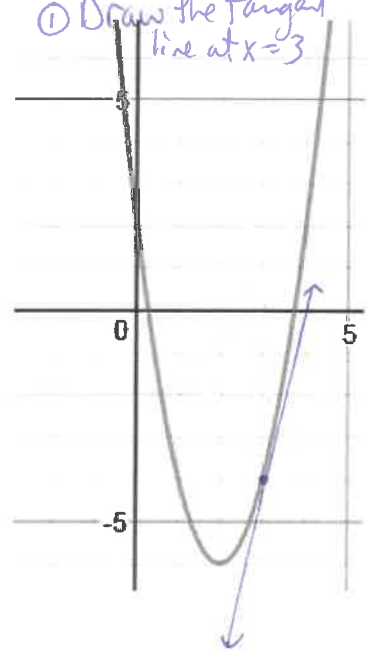
$$\boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

$$f'(a) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- An alternative way to write the definition of the derivative is $\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$

Ex #1: If f(x) = 2x² – 8x + 2, find f'(3) using the ~~first formula above~~ Definition of the derivative. This means that you are actually finding the slope of the tangent line drawn to the curve of f(x) at the x value of 3 or the derivative of *f* at 3 (a graph has been provided so that you can approximate the slope)

① Draw the tangent line at x = 3



$$f(3) = 2(3)^2 - 8(3) + 2$$
$$f(3) = 18 - 24 + 2$$
$$f(3) = -4$$

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$f'(3) = \lim_{h \to 0} \frac{2(3+h)^2 - 8(3+h) + 2 - (-4)}{h}$$

$$f'(3) = \lim_{h \to 0} \frac{2(9+6h+h^2) - 24 - 8h + 2 + 4}{h}$$

$$f'(3) = \lim_{h \to 0} \frac{\cancel{18} + 12h + 2h^2 - \cancel{24} - 8h + \cancel{6}}{h}$$

$$f'(3) = \lim_{h \to 0} \frac{2h^2 + 4h}{h}$$

$$f'(3) = \lim_{h \to 0} \frac{\cancel{h}(2h+4)}{\cancel{h}}$$

$$f'(3) = 2(0) + 4$$

$$f'(3) = 4$$

This means the slope of the tangent line is 4 at the point where x = 3 on f(x) = 2x² – 8x + 2