2020

CALCULUS 30: UNIT 1 DAY 1 - FACTORING

To factor using a GCF that has negative and rational exponents. To factor the sum and difference of cubes.

REVIEW: Types of Factoring

1) Greatest Common Factor (GCF): (REVIEW)

Always take out a Greatest Common Factor first. To do this see if all numbers can be divided by the same number. If there are the same variable in all of the terms, take out the lowest exponent:

a) $-2x^2 + 12x - 4$

$\frac{-2x^2}{-2} + \frac{12x}{-2} \frac{-4}{-2}$ ← Divide all terms by GCF

$= -2(x^2 - 6x + 2)$

b) $\frac{5}{3}x + \frac{3}{2}x^3$

$\frac{5 \cdot 2}{3 \cdot 2}x + \frac{3 \cdot 3}{2 \cdot 3}x^3$

★ create a common denominator ★

$= \frac{\frac{10}{6}x}{\frac{1}{6}x} + \frac{\frac{9}{6}x^3}{\frac{1}{6}x}$

You must factor out all denominators in every coefficient, so by the common denominator

$= \frac{10}{6}\left(\frac{6}{1}\right) + \frac{9}{6}\left(\frac{6}{1}\right)x^2$

$= \frac{1}{6}x(10 + 9x^2)$

Remember when ÷ by fractions it is the same as multiplying by the reciprocal

or Factor out the common denominator

$= \frac{\frac{5}{3}x^1}{\frac{1}{6}x^1} + \frac{\frac{3}{2}x^3}{\frac{1}{6}x}$

$\left(\frac{5}{3} \cdot \frac{6}{1}x^0 + \frac{3}{2} \cdot \frac{6}{1}x^2\right)$

$= \frac{1}{6}x(10 + 9x^2)$

c) $12xyz - 24x^2y^3 + 3xy + 15x^5z^3$

$\frac{12xyz}{3x} - \frac{24x^2y^3}{3x} + \frac{3xy}{3x} + \frac{15x^5z^3}{3x}$

$= 3x(4yz - 8xy^3 + y + 5x^4z^3)$

2) Polynomials of the form x²+bx+c and ax²+bx+c (REVIEW)

- Take out GCF
- **x^2+bx+c** - Find 2 numbers to multiply to give you the "c" value and add together to give you the "b" value
- **ax^2+bx+c** - Use the window/box method: https://goo.gl/dMqSeB or decomposition https://goo.gl/jg9P7e or guess and check

a) $x^2 - 5x - 14$

$\boxed{= (x-7)(x+2)}$

b) $-3x^2+15x-18$

$= -3(x^2+5x-6)$

$\boxed{= -3(x+6)(x-1)}$

c) $\frac{5}{6}x^2 + \frac{11}{12}x - \frac{1}{2}$

$\frac{\frac{5}{6}}{\frac{1}{12}} \quad \frac{\frac{11}{12}}{\frac{1}{12}} \quad \frac{\frac{1}{2}}{\frac{1}{12}}$

Factor out the lowest common denominator ★ so no fractional coefficients are left ★

$= \frac{1}{12}\left[\frac{5}{6}\left(\frac{12}{1}\right)x^2 + \frac{11}{12}\left(\frac{12}{1}\right)x - \frac{1}{2}\left(\frac{12}{1}\right)\right]$

$= \frac{1}{12}(10x^2 + 11x - 6)$

$\boxed{= \frac{1}{12}(5x-2)(2x+3)}$

d) $2x^2 - 4x - 10$

$= 2(x^2 - 2x - 5)$

Prime

3) Difference of Squares (REVIEW)

Ex. Factor

a) $2x^4 - 18x^2$

$= 2x^2(x^2-9)$

$\boxed{= 2x^2(x-3)(x+3)}$

b) $x^4 - 16y^4$

$= (x^2-4y^2)(x^2+4y^2)$

$\boxed{= (x-2y)(x+2y)(x^2+4y^2)}$

4) Factoring 4 or more terms (REVIEW)

- Take out GCF

METHOD 1: Use synthetic division to factor

Ex. Fully factor the following:

a) $2x^3 - 5x^2 - 4x + 3$ → Try factors of constant and each factor divided by factors of "a" (largest degree coefficient)

Step 1 Use the factor theorem to find the integral zero.

Try $\pm 1, \pm 3,$ or $\pm \frac{3}{2}$

Step 2 When $x=-1$; $2(-1)^3-5(-1)^2-4(-1)+3$

$= -2-5+4+3$

$= 0$

$\therefore (x+1)$ is a factor and $x=-1$ is a solution

Step 3

$$\begin{array}{r|rrrr} -1 & 2 & -5 & -4 & 3 \\ & \downarrow & -2 & 7 & -3 \\ \hline & 2 & -7 & 3 & \boxed{0} \end{array}$$

← subtract

$= (x+1)(2x^2-7x+3)$

$\boxed{= (x+1)(2x-1)(x-3)}$

Method 2: Factor by Grouping

b) $2x^3 - x^2 + 6x - 3$

Group two terms take GCF of each term

$= (2x^3-x^2) + (6x-3)$

*GCF is a binomial

$= (2x-1)\left(x^2\frac{(2x-1)}{(2x-1)} + 3\frac{(2x-1)}{(2x-1)}\right)$

$= (2x-1)(x^2+3)$

5) Factoring with Rational or Negative Exponents (NEW)

To take out the GCF when the exponents are fractions, take out the smallest exponents. - negative exponent in the factors only GCF.

Ex. To fully factor don't leave

a) $2x^{\frac{3}{2}} + 4x^{\frac{1}{2}} - 6x^{-\frac{1}{2}}$

$= 2x^{-1/2}\left(\frac{2x^{3/2}}{2x^{-1/2}} + \frac{4x^{1/2}}{2x^{-1/2}} - \frac{6x^{-1/2}}{2x^{-1/2}}\right)$

$= 2x^{-1/2}\left(x^{3/2-(-\frac{1}{2})} + 2x^{\frac{1}{2}-(-\frac{1}{2})} - 3x^{-1/2-(-1/2)}\right)$

$= 2x^{-1/2}(x^2+2x-3)$

$\boxed{= 2x^{-1/2}(x+3)(x-1)}$

b) $12x^{\frac{1}{4}} + 6x^{\frac{-2}{3}} - \frac{1}{4}x^{\frac{-1}{2}}$

$= \frac{1}{4}x^{-\frac{2}{3}}\left(\frac{12x^{\frac{1}{4}}}{\frac{1}{4}x^{-2/3}} + \frac{6x^{-\frac{2}{3}}}{\frac{1}{4}x^{-2/3}} - \frac{\frac{1}{4}x^{-1/2}}{\frac{1}{4}x^{-2/3}}\right)$

$= \frac{1}{4}x^{-\frac{2}{3}}\left(12\cdot\frac{4}{1}x^{\frac{1}{4}\cdot\frac{3}{3}-(-\frac{2}{3})\cdot\frac{4}{4}} + 6\cdot\frac{4}{1}x^{-\frac{2}{3}-(-\frac{2}{3})} - \frac{1}{4}\cdot\frac{4}{1}x^{-\frac{1}{2}\cdot\frac{3}{3}-(-\frac{2}{3})\cdot\frac{2}{2}}\right)$

$= \frac{1}{4}x^{-\frac{2}{3}}\left(48x^{\frac{3}{12}+\frac{8}{12}} + 24x^0 - x^{-\frac{3}{6}+\frac{4}{6}}\right)$

$\boxed{= \frac{1}{4}x^{-\frac{2}{3}}\left(48x^{11/12} + 24 - x^{1/6}\right)}$

Ex #4a)

Synthetic Division

$$2x^3 - 5x^2 - 4x + 3$$

$\boxed{\text{Step 1}}$ List possible integral zeros or (try factors of the constant and each of those factors divided by factors of "a")

$\pm 3, \pm 1, \pm \frac{3}{2}, \pm \frac{1}{2}$

Possible factors.
$(2x \pm 3)$ $(2x \pm 1)$
$(1x \pm 3)$ $(1x \pm 1)$

$\boxed{\text{Step 2}}$ Use factor theorem to find one of the integral zero

When $x = -1$
substitute into expression, if it equals 0 then that value is a zero

$2(-1)^3 - 5(-1)^2 - 4(-1) + 3$
$= -2 + 5 + 4 + 3$
$= -7 + 7$
$= 0$

$\therefore (x+1)$ is a factor and $x = -1$ is a zero

$\boxed{\text{Step 3}}$ Use synthetic division to factor

Since $(x+1)$ is a factor use

coefficients in descending degree order

then subtract each column.

$$\begin{array}{r|rrrr} +1 & 2 & -5 & -4 & 3 \\ & \downarrow & 2 & -7 & 3 \\ \hline & 2 & -7 & 3 & 0 \end{array}$$

$= (x+1)(2x^2 - 7x + 3)$
$= (x+1)(2x-1)(x-3)$

or

Since $x = -1$ is a zero use and add each column

check $(x+1)$

$$\begin{array}{r|rrrr} -1 & 2 & -5 & -4 & 3 \\ & \downarrow & -2 & 7 & -3 \\ \hline & 2 & -7 & 3 & 0 \end{array}$$

$= (x+1)(2x^2 - 7x + 3)$
$(x+1)(2x-1)(x-3)$

b) $-\frac{5}{3}x^{\frac{1}{2}}+\frac{2}{9}x^{-\frac{3}{2}}$

$= -\frac{1}{9}x^{-3/2}\left(-\frac{5}{3}x^{1/2}+\frac{2}{9}x^{-3/2}\right)$

$\quad -\frac{1}{9}x^{-3/2} \qquad -\frac{1}{9}x^{-3/2}$

$= -\frac{1}{9}x^{-3/2}\left(-\frac{5}{\cancel{3}_1}\cdot -\frac{\cancel{9}^3}{1}x^{1/2-(-3/2)} - \frac{2}{\cancel{9}}\cdot -\frac{\cancel{9}}{1}x^{-3/2-(-3/2)}\right)$

$= -\frac{1}{9}x^{-3/2}\left(15x^2-2x^0\right)$

$\boxed{= -\frac{1}{9}x^{-3/2}\left(15x^2-2\right)}$

$= \frac{-(15x^2-2)}{9x^{3/2}}$

NEW: SUM & DIFFERENCE OF CUBES

CHARACTERISTICS OF A SUM OR DIFFERENCE OF TERMS

- Two Terms
- The terms are separated by a + or a – sign
- Each term is a perfect cube

FORMULA FOR FACTORING A SUM OR DIFFERENCE OF CUBS

- $x^3+y^3=(x+y)(x^2-xy+y^2)$
- $x^3-y^3=(x-y)(x^2+xy+y^2)$

Ex #1: Factor the following

a) $8x^3 - 27$ "Difference of cubes"

$= (2x)^3 - (3)^3$

same sign; opposite sign; Always "+"

$= (2x-3)\left((2x)^2+(2x)(3)+(3)^2\right)$

Cube root of each term

$\boxed{= (2x-3)(4x^2+6x+9)}$

b) $1000x^{12} + 343y^6$ "Sum of cubes"

$= \left[(10x^4)^3 + (7y^2)^3\right]$

same sign; opposite sign

$= (10x^4+7y^2)\left((10x^4)^2-(10x^4)(7y^2)+(7y^2)^2\right)$

$\boxed{= (10x^4+7y^2)(100x^8-70x^4y^2+49y^4)}$

c) $\frac{2}{3}x^3-18$

$= \frac{1}{3}\left(\frac{2}{3}x^3 - 18\right)$

$\quad \frac{1}{3} \qquad \frac{1}{3}$

$= \frac{1}{3}\left(\frac{2}{\cancel{3}}\cdot\frac{\cancel{3}}{1}x^3 - 18\cdot\frac{3}{1}\right)$

$= \frac{1}{3}(2x^3-54)$

$= \frac{1}{3}\left(2(x^3-27)\right)$

$\boxed{= \frac{2}{3}(x-3)(x^2+3x+9)}$

d) $(x+6)^3 - y^3$

→ Do not leave answers of brackets within brackets

→ Do not leave a set of bracket unsimplified

$= \left((x+6)-y\right)\left((x+6)^2+(x+6)(y)+y^2\right)$

$= (x+6-y)\left((x+6)(x+6)+xy+6y+y^2\right)$

$= (x+6-y)(x^2+12x+36+xy+6y+y^2)$

$\boxed{= (x+6-y)(x^2+12x+36+xy+6y+y^2)}$

e) $27 + (2a-5)^3$

"Sum of cubes"

$= \left((3)^3 + (2a-5)^3\right)$

$= (3+(2a-5))\left(3^2 - (3)(2a-5) + (2a-5)^2\right)$

$= (3+2a-5)(9-6a+15+(2a-5)(2a-5))$

$= (2a-2)(24-6a+4a^2-20a+25)$

$\boxed{= 2(a-1)(4a^2-26a+49)}$

★To see if the final trinomial is factorable

b^2-4ac

$= (-26)^2-4(4)(70)$

$= 676-1120$

$= -$ ∴ Not factorable.

f) $2a^{\frac{7}{2}}b^{\frac{-1}{2}} - \frac{1}{4}a^{\frac{1}{2}}b^{\frac{5}{2}}$

$= \frac{1}{4}a^{1/2}b^{-1/2}\left(\frac{2a^{7/2}b^{-1/2}}{1/4\,a^{1/2}b^{-1/2}} - \frac{\frac{1}{4}a^{1/2}b^{5/2}}{1/4\,a^{1/2}b^{-1/2}}\right)$

$= 1/4\,a^{1/2}b^{-1/2}(8a^3-b^3)$

$= 1/4\,a^{1/2}b^{-1/2}((2a)-b)((2a)^2+(2a)(b)+b^2)$

$\boxed{= 1/4\,a^{1/2}b^{-1/2}(2a-b)(4a^2+2ab+b^2)}$

Summary

- ALWAYS, ALWAYS look for a GCF first , no matter how many terms
- If there are three terms you should try window/box method or decomposition
- If there are more than three terms with a degree larger than 2, try synthetic division or grouping.
- If there are two terms, you should see if you could factor as DIFFERENCE OF SQUARES or SUM/DIFFERENCE OF CUBES
- If co- efficients are rational exponents always take out a GCF that is the smallest rational exponent
- If the first term is negative, YOU MUST FACTOR OUT a negative coefficient.
- Leave all factors FULLY SIMPLIFIED.

NEW: CRAZY GCF (This will be used a LOT this year

Ex #2: Factor the following (Complete on looseleaf)

a) $(x^3+2)^{1/3} + (x^3+2)^{-5/3}$

b) $-12x^3(3x+5)^3 + 3x^2(3x+5)^4$

c) $6x(x^2+1)^2(2-3x)^4 - 12(2-3x)^3(x^2+1)^3$

d) $2x^3(x-2)^{-1}(x+1)^{\frac{3}{4}} - 4x^2(x-2)(x+1)^{-\frac{1}{4}}$

e) $\frac{5}{2}(2x^2+3)^2(5x-1)^{-\frac{1}{2}} + 8x(5x-1)^{\frac{1}{2}}(2x^2+3)$

f) $\frac{3}{10}(x-1)^{-2}(2x+1)^{-\frac{3}{4}} - \frac{9}{10}(x-1)^{-2}(2x+1)^{\frac{1}{4}}$

Unit 1 : Assignment #1

Textbook: Pg 4 #1,2

Duo-Tang: Assignment #1 Pg 1 #1-6 **NOTE: Question6 will be handed in**

New Crazy GCF

Examples

Factor the following

Ex#2 a)

a) $(x^3+2)^{1/3} + (x^3+2)^{-5/3}$

$= (x^3+2)^{-5/3}\left[\dfrac{(x^3+2)^{1/3}}{(x^3+2)^{-5/3}} + \dfrac{(x^3+2)^{-5/3}}{(x^3+2)^{-5/3}}\right]$

$= (x^3+2)^{-\frac{5}{3}}\left[(x^3+2)^2 + 1\right]$

$= (x^3+2)^{-\frac{5}{3}}\left[(x^3+2)(x^3+2)+1\right]$

$= (x^3+2)^{-\frac{5}{3}}\left[x^6+4x^3+4+1\right]$

$\boxed{= (x^3+2)^{-\frac{5}{3}}(x^6+4x^3+5)}$

★ Divide each term by GCF. Remember the smallest exponent is your GCF with the same bases ★

★ subtract exponents when dividing powers with the same base ★

★ multiply out and simplify what you can ★

b) $-12x^3(3x+5)^3 + 3x^2(3x+5)^4$

$= \underbrace{-3x^2(3x+5)^3}_{\boxed{GCF}}\left[\dfrac{-12x^3(3x+5)^3}{-3x^2(3x+5)^3} + \dfrac{3x^2(3x+5)^4}{-3x^2(3x+5)^3}\right]$

$= -3x^2(3x+5)^3\left[4x - (3x+5)\right]$

$= -3x^2(3x+5)^3(4x-3x-5)$

$= \boxed{-3x^2(3x+5)^3(x-5)}$

watch out because often there is another GCF to take out here

c) $6x(x^2+1)^2(2-3x)^4-12(2-3x)^3(x^2+1)^3$

$$= 6(x^2+1)^2(2-3x)^3\left[\frac{\cancel{6}x(x^2+1)^2(2-3x)^4}{\cancel{6}(x^2+1)^2(2-3x)^3} - \frac{12(2-3x)^3(x^2+1)^3}{6(2-3x)^3(x^2+1)^2}\right]$$

$$= 6(x^2+1)^2(2-3x)^3\left[x(2-3x) - 2(x^2+1)\right]$$

$$= 6(x^2+1)^2(2-3x)^3\left[2x-3x^2-2x^2-2\right]$$

$$= 6(x^2+1)^2(2-3x)^3(\underbrace{-5x^2+2x-2}_{\text{GCF of } -1})$$

$$= -1\cdot 6(x^2+1)^2(2-3x)^3(5x^2-2x+2)$$ ← check to see if factorable

$$= \boxed{-6(x^2+1)^2(2-3x)^3(5x^2-2x+2)}$$

$(-2)^2-4(5)(2)$

$4-40$

$=-36$ Not factorable

d) $2x^3(x-2)^{-1}(x+1)^{3/4}-4x^2(x-2)(x+1)^{-1/4}$

$$= 2x^2(x-2)^{-1}(x+1)^{-1/4}\left[\frac{2x^3(x-2)^{-1}(x+1)^{3/4}}{2x^2(x-2)^{-1}(x+1)^{-1/4}} - \frac{4x^2(x-2)(x+1)^{-1/4}}{2x^2(x-2)^{-1}(x+1)^{-1/4}}\right]$$

$$= 2x^2(x-2)^{-1}(x+1)^{-1/4}\left[x(x+1) - 2(x-2)^2\right]$$

$$= 2x^2(x-2)^{-1}(x+1)^{-1/4}\left[x^2+x-2(x-2)(x-2)\right]$$

$$= 2x^2(x-2)^{-1}(x+1)^{-1/4}\left[x^2+x-2(x^2-4x+4)\right]$$

$$= 2x^2(x-2)^{-1}(x+1)^{-1/4}(x^2+x-2x^2+8x-8)$$

$$= 2x^2(x-2)^{-1}(x+1)^{-1/4}(\underbrace{-x^2+9x-8}_{\text{GCF of } -1})$$

$$= 2x^2(x-2)^{-1}(x+1)^{-1/4}-1(x^2-9x+8)$$

$$= -2x^2(x-2)^{-1}(x+1)^{-1/4}(x^2-9x+8) = -2x^2(x-2)^{-1}(x+1)^{-1/4}(x-1)(x-8)$$

e) $\frac{5}{2}(2x^2+3)^2(5x-1)^{-\frac{1}{2}} + 8x(5x-1)^{1/2}(2x^2+3)$

$= \frac{1}{2}(2x^2+3)(5x-1)^{-1/2}\left[\frac{\frac{5}{2}(2x^2+3)^2(5x-1)^{-1/2}}{1/2(2x^2+3)(5x-1)^{-1/2}} + \frac{8x(5x-1)^{1/2}(2x^2+3)}{1/2(2x^2+3)(5x-1)^{-1/2}}\right]$

$= \frac{1}{2}(2x^2+3)(5x-1)^{-1/2}\left[5(2x^2+3) + 16x(5x-1)\right]$

$= \frac{1}{2}(2x^2+3)(5x-1)^{-1/2}\left[10x^2+15+80x^2-16x\right]$

$\boxed{= \frac{1}{2}(2x^2+3)(5x-1)^{-1/2}(90x^2-16x+15)}$

→ check to see if factorable b^2-4ac

$(-16)^2-4(90)(15)$
$= 256 - 5400$
$= -5144$ NOT Factorable

f) $\frac{3}{10}(x-1)^{-2}(2x+1)^{-3/4} - \frac{9}{10}(x-1)^{-2}(2x+1)^{1/4}$

$= \frac{3}{10}(x-1)^{-2}(2x+1)^{-3/4}\left[\frac{\frac{3}{10}(x-1)^{-2}(2x+1)^{-3/4}}{3/10(x-1)^{-2}(2x+1)^{-3/4}} - \frac{\frac{9}{10}(x-1)^{-2}(2x+1)^{1/4}}{3/10(x-1)^{-2}(2x+1)^{-3/4}}\right]$

$= \frac{3}{10}(x-1)^{-2}(2x+1)^{-3/4}\left[1 - 3(2x+1)\right]$

$= \frac{3}{10}(x-1)^{-2}(2x+1)^{-3/4}\left[1-6x-3\right]$

$= \frac{3}{10}(x-1)^{-2}(2x+1)^{-3/4}(\underbrace{-6x-2}_{\text{GCF of }-2})$

$= -\cancel{2}^{1}\cdot\frac{3}{\cancel{10}_{5}}(x-1)^{-2}(2x+1)^{-3/4}(3x+1)$

$\boxed{= -\frac{3}{5}(x-1)^{-2}(2x+1)^{-3/4}(3x+1)}$

* For now we will leave answers that have expressions with negative exponents like this. In the future we will often change to this

$$\frac{-3(3x+1)}{5(x-1)^2(2x+1)^{3/4}}$$

CALCULUS 30: UNIT 1 DAY 2 - RATIONALIZING

To rationalize numerators or denominators of a given expression.

RATIONALIZAING A NUMERATOR OR DENOMINATOR

- Will turn that numerator or denominator into a RATIONAL expression (will remove the roots)
- To rationalize the numerator or the denominator, multiply both the numerator and the denominator by the conjugate of the numerator or denominator that you are rationalizing
  - REMEMBER: The **CONJUGATE** of a binomial is a binomial that is identical to the original binomial but containing the opposite middle sign

Ex #1: State the conjugate of each of the following:

a) $\sqrt{a}-\sqrt{b}$

conjugate is $\sqrt{a}+\sqrt{b}$

b) $\sqrt{x+4}+2$

conjugate is $\sqrt{x+4}-2$

Ex #2:

a) Rationalize the numerator of

$$\frac{\sqrt{x+4}-2}{x}\cdot\frac{(\sqrt{x+4}+2)}{(\sqrt{x+4}+2)}$$

To rationalize the numerator multiply by the conjugate. To keep the value the same multiply by a form of 1.

$$=\frac{(x+4)+2\sqrt{x+4}-2\sqrt{x+4}-4}{x\sqrt{x+4}+2x}$$

$$=\frac{x+4-4}{x\sqrt{x+4}+2x}$$

$$=\frac{x}{x\sqrt{x+4}+2x}$$ ← Factor out GCF = x

$$=\frac{\cancel{x}}{\cancel{x}(\sqrt{x+4}+2)}=\frac{1}{(\sqrt{x+4}+2)}$$

b) Rationalize the denominator of

$$\frac{5}{\sqrt{x+3}+\sqrt{x}}\cdot\frac{(\sqrt{x+3}-\sqrt{x})}{\sqrt{x+3}-\sqrt{x}}$$

$$=\frac{5\sqrt{x+3}-5\sqrt{x}}{(\sqrt{x+3})^2-\sqrt{x+3}(\sqrt{x})+\sqrt{x+3}(\sqrt{x})-(\sqrt{x})^2}$$

$$=\frac{5\sqrt{x+3}-5\sqrt{x}}{(x+3)-x}$$

$$=\frac{5(\sqrt{x+3}-\sqrt{x})}{3}$$ or $$\frac{5\sqrt{x+3}-5\sqrt{x}}{3}$$

or

$$=\frac{5}{3}(\sqrt{x+3}-\sqrt{x})$$

Unit 1: DAY 2 ASSIGNMENT #2

Text: P4 #1, 2

CALCULUS 30: UNIT 1 DAY 3 - THE LIMIT OF A FUNCTION (VISUAL PERSPECTIVE)

To be able to understand graphically what a limit is, to find the limit graphically, to learn graphically when a limit doesn't exisit and to learn the proper notation to writing limits (Textbook Section 1.2).

Most of us understand what the word limit means outside of the world of mathematics
- the speed that you are allowed to drive on a highway
- the amount of weight you can bench press in the gym
- how far you can argue with your parents
- how high a ball will bounce if you let it drop from your hand

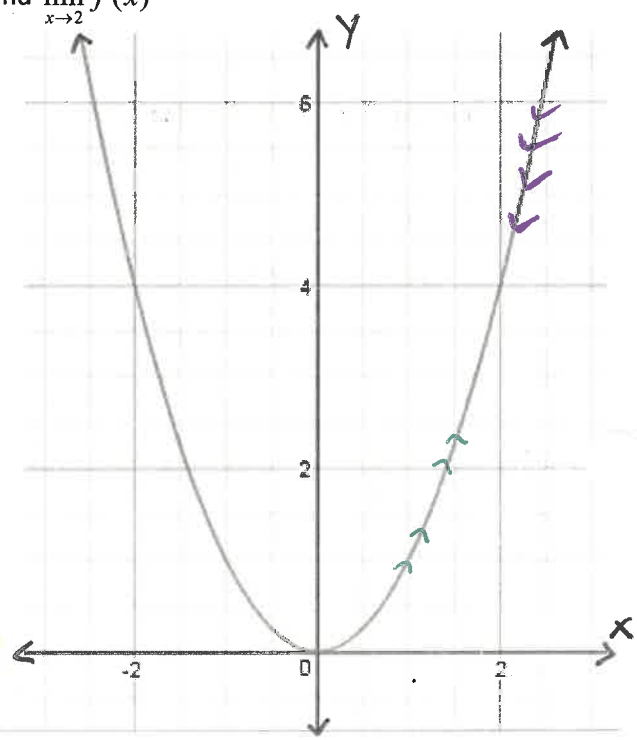
Ex #1: Given the function $f(x) = x^2$, determine the limit as x approaches 2.

- Mathematically this would be written as follows: If $f(x) = x^2$, find $\lim_{x\to 2} f(x)$
- To find this answer, you must try approaching the indicated x value of two from BOTH the left and the right side of the graph. This means you must complete the following two calculations:
  - $\lim_{x\to 2^-} f(x) = \boxed{4}$

    (this means to "drive" from the **left** towards x = 2 ON the graph and see how high you are on the y axis)

    And
  - $\lim_{x\to 2^+} f(x) = \boxed{4}$

    (this means to "drive" from the **right** towards a value of x = 2 ON the graph and see how high you are on the y axis
- If both of your answers in the above step were the same, you have found the limit! Therefore $\lim_{x\to 2} f(x) = \boxed{4}$



NOTE: the function does not actually have to exist at the value of the limit – there can be a hole or an asymptote at the actual location!

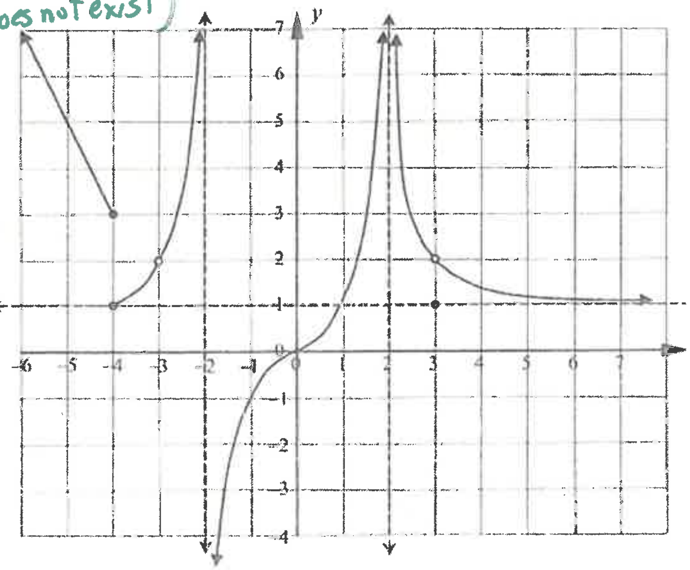
Limits are the "backbone" of understanding that connect algebra and geometry to the mathematics of calculus. In basic terms, a limit is just a statement that tells you what height a function *INTENDS TO REACH* as you get close to a specific *x*-value.

PROPER LIMIT NOTATIONS		
TYPE OF LIMIT	**PROPER NOTATION**	**VERBALLY:**
Right-hand limit	$\lim_{x\to c^+} f(x)$	The limit of the function f(x) as x approaches a certain value of c from it's right side
Left-hand limit	$\lim_{x\to c^-} f(x)$	" " " " ... it's left side
General limit	$\lim_{x\to c} f(x)$	The limit of the function f(x) as x approaches c from both directions

Ex #2: Using the given graph, calculate each limit or value:

(a) $f(-6)$ (b) $f(0)$ (c) $f(3)$ (d) $f(-4)$ (e) $f(-3)$ No equal sign. (Does not exist)

$= 7$ $= 0$ $= 1$ $= 3$ DNE



(f) $\lim\limits_{x\to 1} f(x)$ (g) $\lim\limits_{x\to -5} f(x)$ (h) $\lim\limits_{x\to 0} f(x)$ (i) $\lim\limits_{x\to 3} f(x)$

$= 1$ $= 5$ $= 0$ $= 2$

(j) $\lim\limits_{x\to -4^+} f(x)$ (k) $\lim\limits_{x\to -4^-} f(x)$ (l) $\lim\limits_{x\to -4} f(x)$ (m) $\lim\limits_{x\to 2^+} f(x)$

$= 1$ $= 3$ DNE $\infty \therefore$ DNE

(n) $\lim\limits_{x\to 2^-} f(x)$ (o) $\lim\limits_{x\to 2} f(x)$ (p) $\lim\limits_{x\to -2^+} f(x)$ (q) $\lim\limits_{x\to -2^-} f(x)$

$\infty \therefore$ DNE $\infty \therefore$ DNE $-\infty \therefore$ DNE $\infty \therefore$ DNE

(r) $\lim\limits_{x\to -2} f(x)$ (s) $\lim\limits_{x\to \infty} f(x)$ (t) $\lim\limits_{x\to -\infty} f(x)$

DNE $= 1$ $\infty \therefore$ DNE

- NOTE: If a limit goes to either $\pm\infty$, the BEST answer (and the one I expect) will be to first show that it goes to either $\pm\infty$ and THEN conclude that the limit DNE for that reason. IF you just say $\pm\infty$ or just say DNE you will not get full points. If the limit DNE because the limits on either side of an asymptote change between $\pm\infty$, for full marks you need to show that the limit from the left does not equal the limit from the right and then conclude the limit DNE.

x #3: Explain the difference between the limit of a function and the value of a function

The value of a function is the y-value for a certain x-value. (Ex. $f(3)=1$). It is the height the graph reaches for that x-value. The limit of a function is the height you would theoretically get to or approach as you "drive along the graph" from both directions towards the x-value. It is possible that there is an actual height at that value but there could be a hole or an asymptote, therefore the value of the fuction can be different than the limit at that x-value.

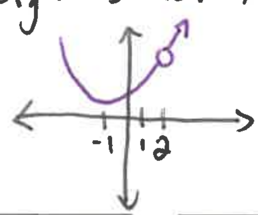
Ex #4: The following is a table of values for the function $y=\dfrac{x^3-8}{x-2}$. Use the table to predict $\lim\limits_{x\to 2}\dfrac{x^3-8}{x-2}$

https://www.desmos.com/calculator/smojrmbqu5

x	$y=\dfrac{x^3-8}{x-2}$
1.9	11.41
1.999	11.994001
1.9999	11.9994001
2	Undefined
2.0001	12.000600001
2.001	12.006001
2.01	12.0601

As we approach x=2 from the top & bottom of the table, the value of y seems to approach 12 (even though right at 2 our height is not known)

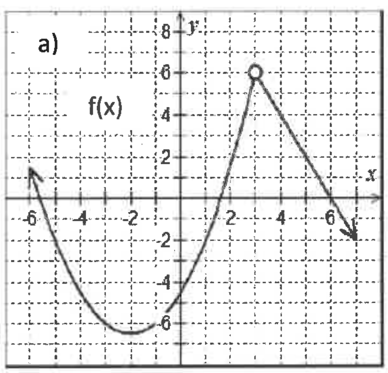
$$\lim_{x\to 2}\frac{x^3-8}{x-2} = 12$$



DEFINITION OF A LIMIT

- If, as x approaches b from both the right and the left, f(x) approaches the single real number L, then L is called the limit of the function f(x) as x approaches b and we write
  $\lim_{x\to b} f(x) = L$
- In order for $\lim_{x\to b} f(x)$ to exist, the limit as you approach b from either side must be the same number. That is that $\lim_{x\to b^+} f(x) = \lim_{x\to b-} f(x) = L$ where L is a real number
- If $\lim_{x\to b^+} f(x) \neq \lim_{x\to b-} f(x)$ then we say that "THE LIMIT DOES NOT EXIST" or DNE
- If the limit of the graph at a value b seems to approach infinity, we can say that $\lim_{x\to b} f(x) = \infty, \therefore DNE$. Some texts will just answer ∞, some texts will just answer DNE but the best answer (and the answer I would like is $\infty, \therefore DNE$. Just answering ∞ is a bit problematic in that ∞ is not a number as required. Just answering DNE is somewhat ambiguous.

Ex #5: Use the graph to find the limit, if it exists. If the limit does not exist, explain why.



A. $\lim_{x\to -3} f(x) = -6$

B. $\lim_{x\to -\infty} f(x) = \infty$
$\therefore$ Does not exist (DNE)

C. $\lim_{x\to 6} f(x) = 0$

D. $\lim_{x\to 1} f(x) = -2$

E. Does $\lim_{x\to 3} f(x)$ exist? Why or why not?

$\lim_{x\to 3^-} f(x) = 6$, $\lim_{x\to 3^+} = 6 \therefore \lim_{x\to 3} f(x) = 6$

Note: ∞ is not a real # it is a concept $\therefore$ DNE

Since the left & right limits of 3 are both 6, the limit at 3 is 6

Unit 1: DAY 3 ASSIGNMENT
Duo-Tang: Assignment #3 questions #3 – 14

CALCULUS 30:UNIT 1 DAY4 - STRATEGIES FOR EVALUATING LIMITS

To be use and choose from different methods to solve limits as x approaches a specific value (Textbook 1.2).

- When we don't have the graph of the function that we are finding the limit of, we need to use algebraic techniques in order to find the limit
- Today you will learn 5 different techniques. Sometimes only one of the five methods will work and sometimes more than one will work (in that case you want to work to try and find the most efficient method).

METHOD 1: SUBSITUTION

- This method involves directly substituting the value that the variable is approaching into the expression
- This method should always be the first thing you try (but you can't use it if the value that the variable is approaching is itself a non-permissible value of the expression)

Ex #1: Find the following limits: https://www.desmos.com/calculator/7abk3[illegible]xrn https://www.desmos.com/calculator/6lzuwjb[illegible]m3

a) $\lim_{x\to 3}\left(x^2-5x+4\right)$

$= 3^2-5(3)+4$

$= 9-15+4$

$\boxed{=-2}$

b) $\lim_{x\to -3}\frac{x+9}{x-3}$

$= \frac{-3+9}{-3-3}$

$= \frac{6}{-6} = -1$

NO CALCULATOR FOR C & D

c) $\lim_{x\to 9}\frac{\log_3 x}{\sin\left(\frac{\pi x}{18}\right)} = \frac{\log_3 9}{\sin\left(\frac{9\pi}{18}\right)} = \frac{\log_3 3^2}{\sin\left(\frac{\pi}{2}\right)}$

90°

$\sin 90^\circ$

$(0,1)$

$= \frac{2}{1}$

$\boxed{=2}$

Note: You must have your unit circle memorized

d) $\lim_{\theta\to \frac{2\pi}{3}}\frac{\cos\theta}{\theta} = \frac{\cos\frac{2\pi}{3}}{\frac{2\pi}{3}}$

$= \frac{-\frac{1}{2}}{\frac{2\pi}{3}} = -\frac{1}{2}\cdot\frac{3}{2\pi}$

$\boxed{=\frac{-3}{4\pi}}$

https://www.desmos.com/calculator/w[illegible]kamhkmoc https://www.desmos.com/calculator/c3[illegible]6[illegible]8[illegible]rwu

METHOD 2: FACTOR AND REDUCE

- If you have a rational function, you may be able to factor the numerator and denominator and reduce the function by cancelling. At that point you may be able to use the first method of SUBSTITUTION to find the limit.

Show graph to get a visual note: Hole at $x=5$

Ex #2: Find the following limits: https://www.desmos.com/calculator/9chsbjbgvb

a) $\lim_{x\to 5}\frac{x^2-6x+5}{x-5}$

$\lim_{x\to 5}\frac{\cancel{(x-5)}(x-1)}{\cancel{(x-5)}}$

$\lim_{x\to 5}(x-1)$

$= 5-1$

$\boxed{=4}$

Note: You must write the $\lim_{x\to c}$ on every step until the substitution step.

If you try substitution first here you run into an issue where you have zero in the denominator making it undefined.

b) $\lim_{x\to 1}\frac{10x^2-10x}{x^3-1}$

$\lim_{x\to 1}\frac{10x\cancel{(x-1)}}{\cancel{(x-1)}(x^2+x+1)}$

$\lim_{x\to 1}\frac{10x}{x^2+x+1}$

$=\frac{10(1)}{1^2+1+1}$

$\boxed{=\frac{10}{3}}$

https://www.desmos.com/calculator/ulpykmbicw

METHOD 3: SIMPLIFYING

- If direct substitution leaves you with a zero in the denominator and you can't factor, apply your knowledge of adding/subtracting/multiplying/dividing fractions until you simplify the expression into one that substitution will work to find the limit

Ex #3: Find the following limits: https://www.desmos.com/calculator/8kytudylan

a) $\lim_{x\to 1}\frac{\frac{1}{x+1}-\frac{1}{2}}{x-1}$

$\lim_{x\to 1}\left[\frac{1(2)}{(x+1)(2)}-\frac{1(x+1)}{2(x+1)}\right]\div(x-1)$ * create a common denominator

$\lim_{x\to 1}\left[\frac{2}{2(x+1)}-\frac{(x+1)}{2(x+1)}\right]\cdot\frac{1}{x-1}$

$\lim_{x\to 1}\frac{2-x-1}{2(x+1)}\cdot\frac{1}{(x-1)}$

$\lim_{x\to 1}\frac{-x+1}{2(x+1)(x-1)}$ * Factor out GCF of -1

$\lim_{x\to 1}\frac{-1\cancel{(x-1)}}{2(x+1)\cancel{(x-1)}}$

$\lim_{x\to 1}\frac{-1}{2(x+1)}=\frac{-1}{2(1+1)}=\frac{-1}{2(2)}=\boxed{-\frac{1}{4}}$

*substitute.

b) $\lim_{h\to0}\frac{(-2+h)^3-2(-2+h)+4}{h}$ ← simplify the numerator

$= (-2+h)(-2+h)^2$
$= (-2+h)(-2+h)(-2+h)$
$= (4-4h+h^2)(-2+h)$
$= -8+8h-2h^2+4h-4h^2+h^3$
$= h^3-6h^2+12h-8$

$\lim_{h\to0}\frac{h^3-6h^2+12h-8+4-2h+4}{h}$

$\lim_{h\to0}\frac{h^3-6h^2+10h}{h}$ ← Factor out an "h"

$\lim_{h\to0}\frac{\cancel{h}(h^2-6h+10)}{\cancel{h}}$ ← got rid of the issue when we substituted we were getting zero in the denominator

$= 0^2-6(0)+10$ ← substitution

$\boxed{=10}$

https://www.desmos.com/calculator/zwn3noae95

METHOD 4: RATIONALIZING

- If your function has radicals in either its numerator or its denominator, rationalize to remove the radical by multiplying the numerator and denominator by the CONJUGATE

Ex #4: Find the following limits:

a) $\lim_{r\to6}\frac{\sqrt{3+r}-3}{r-6}\cdot\frac{(\sqrt{3+r}+3)}{(\sqrt{3+r}+3)}$ → expand (multiply) / → do not expand.

$\lim_{r\to6}\frac{(3+r)-9}{(r-6)(\sqrt{3+r}+3)}$

Must have $\lim$ until you substitute

$\lim_{r\to6}\frac{\cancel{r-6}}{\cancel{(r-6)}(\sqrt{3+r}+3)}$

$\lim_{r\to6}\frac{1}{\sqrt{3+r}+3}$

$=\frac{1}{\sqrt{3+6}+3}$

$=\frac{1}{\sqrt{9}+3}$

$=\frac{1}{3+3}$

$\boxed{=\frac{1}{6}}$

https://www.desmos.com/calculator/xn8w8um444

b) $\lim_{h\to6}\frac{6-h}{\sqrt{10-h}-\sqrt{h-2}}\cdot\frac{(\sqrt{10-h}+\sqrt{h-2})}{(\sqrt{10-h}+\sqrt{h-2})}$ → Do not expand. / → expand (multiply)

$\lim_{h\to6}\frac{(6-h)(\sqrt{10-h}+\sqrt{h-2})}{(10-h)-(h-2)}$ ← distribute the negative

$\lim_{h\to6}\frac{(6-h)(\sqrt{10-h}+\sqrt{h-2})}{10-h-h+2}$ ← combine like terms

$\lim_{h\to6}\frac{(6-h)(\sqrt{10-h}+\sqrt{h-2})}{-2h+12}$ ← Factor

$\lim_{h\to6}\frac{\cancel{(6-h)}(\sqrt{10-h}+\sqrt{h-2})}{2\cancel{(-h+6)}}$

$=\frac{\sqrt{10-6}+\sqrt{6-2}}{2}$

$=\frac{\sqrt{4}+\sqrt{4}}{-2}$

$=\frac{2+2}{-2}=\frac{4}{-2}\ \boxed{=-2}$

https://www.desmos.com/calculator/ldy20eorah

METHOD 5: SIGN ANALYSIS

- This method will work with rational functions **IF** the number that the variable is approaching is **ALSO THE LOCATION OF AN ASYMPTOTE** of the function – ie we are approaching some number *a* and there is a vertical asymptote at $x = a$.
- When you find $f(a)$ and get $\frac{k}{0}$ **OR** you end up with $\frac{k}{0}$ after factoring/canceling, then $\lim_{x\to a} f(x)$ does not exist. (This means that we wouldn't be able to use substitution in this situation because the value that the variable is approaching is also a non-permissible value and will produce a zero in the denominator)
- In order to find the limit, we need to find out how the graph is behaving on either side of the asymptote – we don't actually need any specific value to find the behavior, just the sign. We perform a sign analysis of the function f(x) to see if the graph is approaching $+\infty$ or $-\infty$ on either side of the asymptote
  - If the sign analysis shows that the function is approaching $+\infty$ on both sides of *a*, we can say that $\lim_{x\to a} f(x) = +\infty, \therefore$ Does Not Exist . **NOTE: We use this definition because it gives us a good image of how the graph looks, but technically $+\infty$ is not a defined limit because $+\infty$ is a concept, not a NUMBER (limits are defined to be a REAL NUMBER L)**
  - If the sign analysis shows that the function is approaching $-\infty$ on both sides of *a*, we can say that $\lim_{x\to a} f(x) = -\infty, \therefore$ Does Not Exist
  - If the sign analysis shows that one side is approaching $+\infty$ and one side approaching $-\infty$, the limit doesn't exist because $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$

Ex #5: Find the following limits: https://www.desmos.com/calculator/r8jd3qws4q

a) $\lim_{x\to 2} \frac{x}{x^2-4}$

$\lim_{x\to 2} \frac{x}{(x-2)(x+2)}$ } Can't substitute, Can't simplify, Can't rationalize, MUST Do Sign analysis.

Note there is a vertical asymptote at $x=2$ and $x=-2$. Since we are looking for the limit as $x\to 2$ we want to perform sign analysis at $x=2$

$\boxed{\text{Step 1}}$ Find all zeros of the numerators and denominators these will be your values on the # line

		-2		0		2		
	−	−	−	0	+	+	+	x
	−	DNE	+	+	+	+	+	$(x+2)$
	−	−	−	−	−	DNE	+	$(x-2)$
	−	DNE	+	0	−	DNE	+	$f(x)$
	moving down	asymptote	moving up	x-int.	moving down	asymptote	moving up	

} write all factors here. Even numerical factors.

Note: $f(x) = \frac{x}{x^2-4}$

} use these to visualize the graph.

Since we want $\lim_{x\to 2} f(x)$ we look at this portion from the left & right.

$\lim_{x\to 2^-} f(x) = -\infty \therefore$ DNE

$\lim_{x\to 2^+} f(x) = \infty \therefore$ DNE

This is my reason. → Since $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$ then $\lim_{x\to 2} f(x)$ DNE

Unit 1: DAY 4 ASSIGNMENT #4
Text: P19 #2, 3, 4, 5ac, 6

CALCULUS 30: UNIT 1 DAY5 - LIMITS AT INFINITY

To be able to use and choose from different methods to solve limits as x approaches infinity.

The Symbol for infinity ∞ does not represent a real number. We use ∞ to describe the behavior of a function when the values in its domain or range outgrow all finite bounds.
For example, when we say **"the limit of *f* as x approaches infinity"** we mean the limit of *f* (or the height of the y value) as x moves increasingly far to the right on the number line.

When we say **"the limit of *f* as x approaches negative infinity (-∞)"** we mean the limit of *f* (or the height of the y value) as x moves increasingly far to the left on the number line.

Ex #1: Given that $f(x)=\dfrac{x+1}{x}$, use a graph and tables to find the following: https://www.desmos.com/calculator/qyhomijc9c

a) $\lim\limits_{x\to\infty} f(x) = 1$

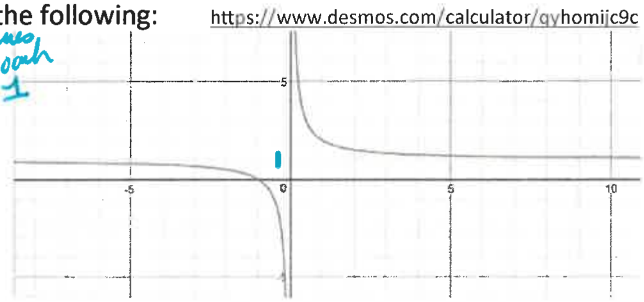
b) $\lim\limits_{x\to-\infty} f(x) = 1$

As the x values approach −∞ The y-values approach 1

X	Y1
-150	.99333
-100	.99
-50	.98
0	ERROR
50	1.02
100	1.01
150	1.0067

Y1=(X+1)/X

As the x-values approach ∞ The y-values approach 1



c) Identify all horizontal asymptotes

Horizontal Asymptote at y=0

QUESTION:

- What happens if I take a number such as 6, and continually try dividing it by larger and larger numbers? What will the answer eventually approach? $\dfrac{6}{2},\dfrac{6}{4},\dfrac{6}{6},\dfrac{6}{8},\ldots,\dfrac{6}{1000},\dfrac{6}{100\ 000\ 000},\ldots,\dfrac{6}{\infty}$
- What happens if I take the same question but square all of the numbers on the bottom? $\dfrac{6}{2^2},\dfrac{6}{4^2},\dfrac{6}{6^2},\dfrac{6}{8^2},\ldots,\dfrac{6}{1000^2},\dfrac{6}{100\ 000\ 000^2},\ldots,\dfrac{6}{\infty^2}$

DIVIDING BY INFINITY

- Any number that is divided by a very large number (like ∞) will get so close to zero that we may as well say it is equal to zero
- $\lim\limits_{x\to\infty}\dfrac{1}{x}=0$ (additionally this is true for higher powers of x: $\lim\limits_{x\to\infty}\dfrac{1}{x^2}=0,\lim\limits_{x\to\infty}\dfrac{1}{x^3}=0,\ldots$)

Ex #2: Find the following limits. Explain what the result of the limit means about the graph of each rational function

a) $\lim_{x\to\infty}\frac{2x-3}{x^2-1}$

$$\lim_{x\to\infty}\frac{2x-3}{x^2-1}\cdot\frac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)}$$

$$\lim_{x\to\infty}\frac{\frac{2x}{x^2}-\frac{3}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}}$$

$$\lim_{x\to\infty}\frac{\frac{2}{x}-\frac{3}{x^2}}{1-\frac{1}{x^2}}$$

$$=\frac{\frac{2}{\infty}-\frac{3}{\infty^2}}{1-\frac{1}{\infty}}$$

$$=\frac{0-0}{1-0}$$

$$=0$$

We can conclude there is a horizontal asymptote at $y=0$ on the $f(x)=\frac{2x-3}{x^2-1}$

b) $\lim_{x\to-\infty}\frac{2x^2+5}{3x^2-4x+1}$

$$\lim_{x\to-\infty}\frac{2x^2+5}{3x^2-4x+1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$\lim_{x\to\infty}\frac{\frac{2x^2}{x^2}+\frac{5}{x^2}}{\frac{3x^2}{x^2}-\frac{4x}{x^2}+\frac{1}{x^2}}$$

$$\lim_{x\to\infty}\frac{2+\frac{5}{x^2}}{3-\frac{4}{x}-\frac{1}{x^2}}$$

$$=\frac{2+0}{3-0-0}$$

$$=\frac{2}{3}$$

The graph of $f(x)=\frac{2x^2+5}{3x^2-4x+1}$ has a horizontal asymptote at $y=\frac{2}{3}$

c) $\lim_{x\to\infty}\frac{4x^3-2x^2}{6-5x}$

$$\lim_{x\to\infty}\frac{4x^3-2x^2}{6-5x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}$$

$$\lim_{x\to\infty}\frac{\frac{4x^3}{x}-\frac{2x^2}{x}}{\frac{6}{x}-\frac{5x}{x}}$$

$$\lim_{x\to\infty}\frac{4x^2-2x}{\frac{6}{x}-5}$$

$$=\frac{\infty}{0-5}$$ ← a very large # divided by -5

$$=-\infty$$

$\therefore$ DNE

This means that the graph $f(x)=\frac{4x^3-2x^2}{6-5x}$ does not have a horizontal asymptote and that as the function travels right, its height continually travels downward.

Lily Bella

FINDING THE LIMIT OF A RATIONAL EXPRESSION AS $x \to \infty$ or $x \to -\infty$

- Multiply the numerator and denominator by the reciprocal variable (or divide each term) with the highest degree in the denominator, simplify and apply the limit.
- If the degree of the numerator is the same as the degree of the denominator, the answer to the limit as $x \to \pm\infty$ will be equal to coefficient of largest power in numerator ÷ coefficient of largest power in denominator
- If the degree of the numerator is less than the degree of the denominator, the answer to the limit as $x \to \pm\infty$ will be equal to $x=0$
- If the degree of the numerator is greater than the degree of the denominator, the answer to the limit as $x \to \pm\infty$ will be equal to $\infty \therefore$ DNE or $-\infty \therefore$ DNE (No horizontal asymptote)

FINDING THE LIMIT OF A RADICAL EXPRESSION AS $x \to \infty$ or $x \to -\infty$

- NOTE: Because $\sqrt{x^2} = \pm x$, we can really say that $\sqrt{x^2} = |x|$ Ex./ $\sqrt{(3)^2} = 3$ or $\sqrt{(-3)^2} = 3$
- With a radical expression we need to factor out a GCF from under the root sign that you will be able to take the exact root of. If there is a non radical numerator or denominator, you need to take out a GCF of the highest power of its variable
- You need to consider whether the question is asking for $x \to \infty$ or $x \to -\infty$ as this will direct you as to whether you are using $\pm x$ when the question has a $\sqrt{x^2}$ or a $|x|$

Ex #2: Find the following limits:

a) $\lim\limits_{x \to -\infty} \frac{\sqrt{x^2 - 5x}}{3x+2}$ → factor out x^2 (in root) → factor out x

$\lim\limits_{x \to -\infty} \frac{\sqrt{x^2(1 - \frac{5}{x})}}{x(3 + \frac{2}{x})}$

$\lim\limits_{x \to -\infty} \frac{\sqrt{x^2}(\sqrt{1 - \frac{5}{x}})}{x(3 + \frac{2}{x})}$

$\lim\limits_{x \to -\infty} \frac{|x|\sqrt{1 - \frac{5}{x}}}{x(3 + \frac{2}{x})}$ positive negative

$|x| = -x$ in this case because our $x \to -\infty$

$\lim\limits_{x \to -\infty} \frac{-x\sqrt{1 - \frac{5}{x}}}{x(3 + \frac{2}{x})}$

x's cancel left w/ -1

$= \frac{-1\sqrt{1-0}}{3+0}$

$= \frac{-1\sqrt{1}}{3} = \boxed{\frac{-1}{3}}$

The equation of the horizontal asymptote is $y = -\frac{1}{3}$

b) $\lim\limits_{x \to \infty} \frac{x^2 + 4}{\sqrt{4x^4 + x^2 + 1}}$ → Factor out x^2 → Factor out x^4

$\lim\limits_{x \to \infty} \frac{x^2(1 + \frac{4}{x^2})}{\sqrt{x^4(4 + \frac{1}{x^2} + \frac{1}{x^4})}}$

$\lim\limits_{x \to \infty} \frac{x^2(1 + \frac{4}{x^2})}{\sqrt{x^4}(\sqrt{4 + \frac{1}{x^2} + \frac{1}{x^4}})}$

$\lim\limits_{x \to \infty} \frac{\cancel{x^2}(1 + \frac{4}{x^2})}{\cancel{x^2}(\sqrt{4 + \frac{1}{x^2} + \frac{1}{x^4}})}$

$\lim\limits_{x \to \infty} \frac{(1 + \frac{4}{x^2})}{(\sqrt{4 + \frac{1}{x^2} + \frac{1}{x^4}})}$

$= \frac{1+0}{\sqrt{4+0+0}} = \frac{1}{\sqrt{4}} = \boxed{\frac{1}{2}}$

The equation of the horizontal asymptote will be $y = \frac{1}{2}$

Topic 1: DAY 5 ASSIGNMENT

Duo-Tang: Assignment #5 #40-51

CALCULUS 30: UNIT 1 DAY 6: REVIEW

Interval Notation:

- CLOSED intervals contain their boundary points
- OPEN intervals contain NO boundary points.
- NOTE: IF you learned this in French Immersion you learned it slightly differently. Please take note of this method for University!

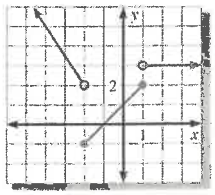
Open or closed?	Interval notation	Set notation	Graph of all points x
open, open	$(-1, 2)$	$\{x \mid -1 < x < 2, x \in \mathbb{R}\}$	
cl, cl	$[-1, 2]$	$\{x \mid -1 \le x \le 2, x \in \mathbb{R}\}$	
cl, open	$[-1, 2)$	$\{x \mid -1 \le x < 2, x \in \mathbb{R}\}$	
open, clo	$(-1, 2]$	$\{x \mid -1 < x \le 2, x \in \mathbb{R}\}$	
cl, op	$[-1, \infty)$	$\{x \mid -1 \le x, x \in \mathbb{R}\}$	
op, op	$(-1, \infty)$	$\{x \mid -1 < x, x \in \mathbb{R}\}$	
op, cl	$(-\infty, 2]$	$\{x \mid x \le 2, x \in \mathbb{R}\}$	
continues, open	$(-\infty, 2)$	$\{x \mid x < 2, x \in \mathbb{R}\}$	
continues, continues	$(-\infty, \infty)$	$\{x \mid x \in \mathbb{R}\}$	

- **Union** $(A \cup B)$ consists of all elements that are in A or in B or in both.
- **Intersection** $(A \cap B)$ consists of all elements that are found in both A and B.

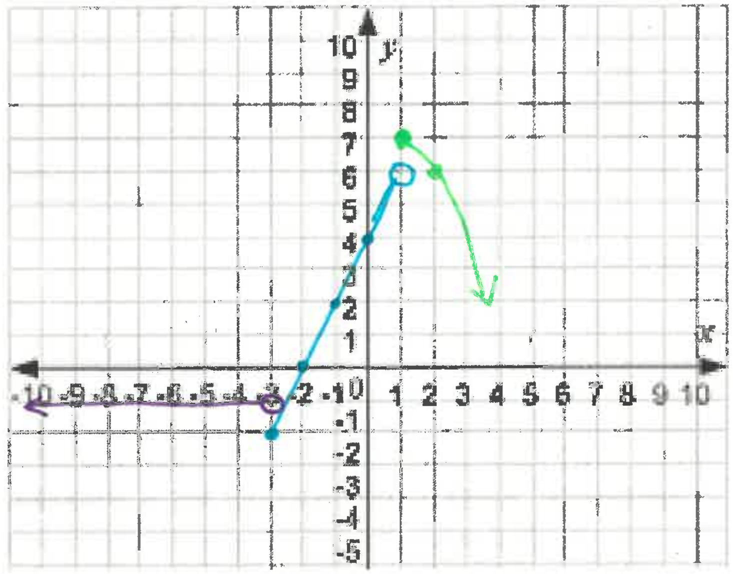
To find the limits of absolute value functions and piecewise functions.

A **piecewise function** is defined by more than one equation. Each equation corresponds to a different part of the domain of the function.

$$f(x)=\begin{cases} -\frac{3}{2}x-1, & \text{if } x<-2 \\ x+1, & \text{if } -2\le x\le 1 \\ 3, & \text{if } x>1 \end{cases}$$



Ex #1: Sketch the following piecewise function:



a)
$$f(x)=\begin{cases} -1, & x\in(-\infty,-3) \\ 2x+4, & x\in[-3,1) \\ -(x-1)^2+7, & x\in[1,\infty) \end{cases}$$

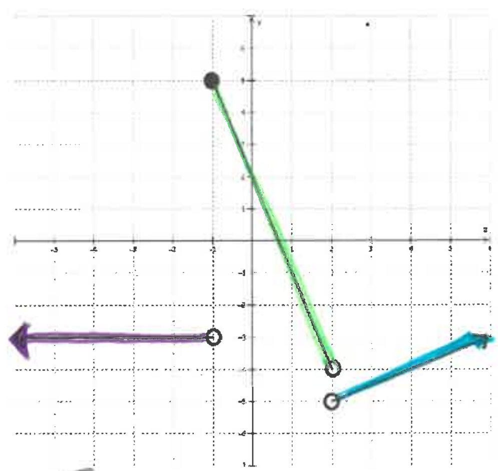
① $y=-1$, $(-\infty,-3)$ Horizontal line at $y=-1$

② $y=2x+4$ $[-3,1)$ Oblique line Slope $=2$ y-int $=4$

③ $y=-(x-1)^2+7$ Quadratic parabola vertex $(1,7)$ open down

When $x=2$ $y=-(2-1)^2+7$
$y=-1+7$
$y=6$

Ex #2: Find the equation of the following piecewise function.



$y=-3$, $(-\infty,-1)$

$y=-3x+2$, $[-1,2)$

$y=\frac{1}{2}x-6$ $(2,\infty)$

$$f(x)=\begin{cases} -3, & x\in(-\infty,-1) \\ -3x+2, & x\in[-1,2) \\ \frac{1}{2}x-6, & x\in(2,\infty) \end{cases}$$

or

$$f(x)=\begin{cases} -3, & \text{if } x<-1 \\ -3x+2, & \text{if } -1\le x<2 \\ \frac{1}{2}x-6, & \text{if } x>2 \end{cases}$$

Ex #3: Given the function $f(x)=\begin{cases}(x+3)^3, x\geq -2\\(x+1)^3, x<-2\end{cases}$, determine the following limits:

Since -2 is on the boundary between the two pieces. We need to take the limit from both sides

a) $\lim_{x\to 1} f(x)$

We are approaching $x\to 1$ which piece of the graph has $x=1$ in the domain?

$\lim_{x\to 1}(x+3)^3$ "substitution"

$=(1+3)^3$

$=4^3$

$=64$

b) $\lim_{x\to -3} f(x)$

Which piece of the graph has $x=-3$ in it's domain?

$\lim_{x\to -3}(x+1)^3$ "substitution"

$=(-3+1)^3$

$=(-2)^3$

$=-8$

c) $\lim_{x\to -2} f(x)$

$\lim_{x\to -2^-}(x+1)^3$

$=(-2+1)^3$

$=(-1)^3$

$=-1$

$\lim_{x\to -2^+}(x+3)^3$

$=(-2+3)^3$

$=1^3$

$=1$

Since $\lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x)$

$\therefore \lim_{x\to -2} f(x)$ Does Not Exist

REVIEW: ABSOLUTE VALUE AS A PIECEWISE FUNCTION

- Recall that absolute value graphs can also be written as piecewise functions. In general,

$y=|x|$ can also be written as $y=\begin{cases}x, x\geq 0\\-x, x<0\end{cases}$

Show graphically $y=\dfrac{|x+1|}{x+1}$

$y=\begin{cases}-(x+1), x\in(-\infty,-1)\\(x+1), x\in(-1,\infty)\end{cases}$

Ex #4: Evaluate the following limits:

a) $\lim_{x\to -1^+}\dfrac{|x^2-2x-3|}{x+1}$

$\lim_{x\to -1^+}\dfrac{|(x-3)(x+1)|}{x+1}$

$\lim_{x\to -1^+}\dfrac{|(x-3)|\,|(x+1)|}{x+1}$

$\boxed{\text{Note}}$ $|(x+1)|=(x+1)$ or $-(x+1)$ depending on what you are approaching.

Since we are approaching -1 from the right we are substituting in values of x that are slightly to the right of -1 Ex./ such as -0.9.

$-1 \quad 0 \quad 1$

If you put $x=-0.9$ into $|x+1|$ you will get $|-0.9+1|=|0.1|$ positive

$\therefore |x+1|=(x+1)$

$\lim_{x\to -1^+}\dfrac{|(x-3)|\cancel{(x+1)}}{\cancel{(x+1)}}$

$=|-2-3|=|-4|\boxed{=4}$

b) $\lim_{x\to -1^-}\dfrac{|x^2-2x-3|}{x+1}$

$\lim_{x\to -1^-}\dfrac{|(x-3)(x+1)|}{x+1}$

$\lim_{x\to -1^-}\dfrac{|(x-3)|\,|(x+1)|}{x+1}$

$\lim_{x\to -1^-}\dfrac{|(x-3)|\,-\cancel{(x+1)}}{\cancel{x+1}}$

Since we are approaching from the left of $x=-1$

$|-1.1+1|$

$=|-0.1|$ neg

$\lim_{x\to -1^-}|x-3|(-1)$

$=|-1-3|(-1)$

$=|-4|(-1)$

$=(4)(-1)$

$\boxed{=-4}$

You must check $\lim_{x\to 4^+} f(x)$ and $\lim_{x\to 4^-} f(x)$ to see if $\lim_{x\to 4} f(x)$ exists.

b) $\lim_{x\to 4}\frac{|x^2+x-20|}{(x-4)}$

$\lim_{x\to 4^-}\frac{|x+5||x-4|}{x-4}$

→ when $x\to 4^-$ ∴ $|x-4|$ will be $|-\#|$ ∴ $|x-4| = -(x-4)$

$\lim_{x\to 4^-}\frac{|x+5|\cdot -\cancel{(x-4)}}{\cancel{x-4}}$

$\lim_{x\to 4^-} |x+5|(-1)$

$= -1|4+5|$

$= -1|9|$

$\boxed{= -9}$

$\lim_{x\to 4^+}\frac{|x+5||x-4|}{x-4}$

When $x\to 4^+$ ∴ $|+\#|$

∴ $|x-4| = (x-4)$

$\lim_{x\to 4^+}\frac{|x+5|\cancel{(x-4)}}{\cancel{(x-4)}}$

$\lim_{x\to 4^+} |x+5|$

$= |4+5|$

$\boxed{= 9}$

Since

$\lim_{x\to 4^-}\frac{|x^2+x-20|}{(x-4)} \neq \lim_{x\to 4^+}\frac{|x^2+x-20|}{(x-4)}$

$\lim_{x\to 4}\frac{|x^2+x-20|}{(x-4)}$ Does Not exist.

- Remember when taking the limit of absolute value and piecewise functions you need to approach the limit from both the positive and negative sides of the x value. If the limit from the left does not equal the limit from the right, the limit at that value does not exist.

Unit 1 : DAY 6 ASSIGNMENT

Duo- Tang: Assignment #6 #52-63

CALCULUS 30: UNIT 1 DAY 7 - CONTINUITY AND LIMITS

To learn what is meant by a continuous function and to learn about the three different types of discontinuities..:

CONTINUOUS FUNCTIONS: Informal Definition

A function is considered to be continuous on an interval if in that interval you could trace the graph of the function with your pencil and not have to lift it from the page

CONTINUOUS FUNCTIONS: Formal Definition

- A function f(x) is considered to be continuous at a specific x value of "a" if all of the following conditions are satisfied:

1 $f(a)$ exists (Note: it is a real #)

2 $\lim_{x\to a} f(x)$ exists (Note: In order for a limit to exist $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$)

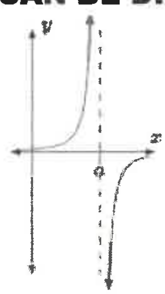
3 $\lim_{x\to a} f(x) = f(a)$

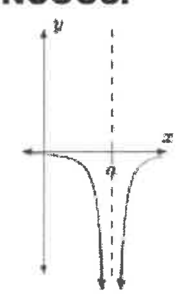
"The right limit at a = The left limit at a = The actual height of the function at a"

THERE ARE THREE DIFFERENT WAYS THAT A FUNCTION CAN BE DISCONTINUOUS:

1. INFINITE DISCONTINUITY

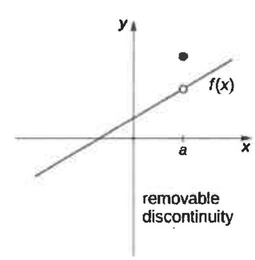
- This is where the graph has a vertical asymptote and where the limit of a graph approaches ∞ or $-\infty$
- They can be found algebraically by finding where the values of x where the denominator of a rational function will be zero
- At least one of the one sided limits does not exist





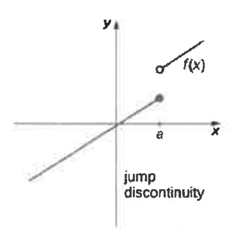
2. REMOVABLE DISCONTINUITY

- On the graph there will be a hole
- This happens in equations where a factor in the numerator cancels with a factor in the denominator
- The ordered pair of the hole can be found algebraically by cancelling the common factor and then substituting the x value of that factor into the resulting equation
- The two sided limit exists at the hole but does not equal the functions value

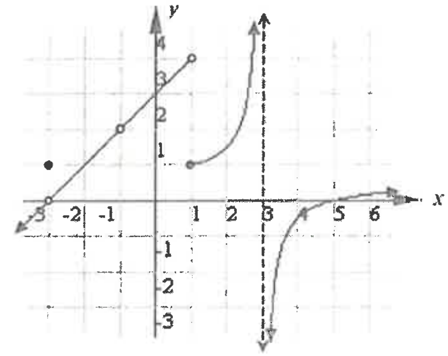


3. JUMP DISCONTINUITY

- On the graph this will look like the graph changes or jumps from one part of a graph to another at a specific x value
- These will usually be piecewise or absolute value functions
- The left sided limit does not equal the right sided limit



Ex #1: Identify and classify the discontinuities in m(x). Use Calculus to explain why the discontinuity exists.



$m(x)$

① $x=-3$

$m(-3)=1$; $\lim_{x\to 3^+} m(x)=0$; $\lim_{x\to -3^-} m(x)=0$

It is removable discontinuity at $x=-3$ because $\lim_{x\to -3} m(x) \neq m(-3)$

② $x=-1$ $m(-1)=$ DNE ; $\lim_{x\to -1} m(x)=2$

It is removable discontinuity at $x=-1$ because $m(-1) \neq \lim_{x\to -1} m(x)$

④ $x=3$

③ $x=1$ $m(1)=1$; $\lim_{x\to 1^+} m(x)=1$; $\lim_{x\to 1^-} m(x)=4$

It is Jump discontinuity at $x=1$ because $\lim_{x\to 1^+} m(x) \neq \lim_{x\to 1^-} m(x)$

Ex #3: Use CALCULUS to determine whether or not the following functions are continuous (using PC 30 methods will not earn you any credit). If it is not continuous, identify the x value or the point where the discontinuity occurs and classify the discontinuity.

a) $f(x)=\dfrac{x^2-4}{x-2}$

We can see there is a hole at $x=2$. Use Calculus to verify.

$f(x)=\dfrac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$

$f(x)=x+2$; $x \neq 2$

• $f(2)$ DNE

• $\lim_{x\to 2} x+2$
$=2+2$
$=4$

Since $f(2) \neq \lim_{x\to 2} f(x)$ there is a removable discontinuity at $x=2$ or the point $(2,4)$

$y=2+2$
$y=4$

b) $f(x)=\begin{cases} x^2, x\neq 2 \\ 1, x=2 \end{cases}$

Possible discontinuity at $x=2$
Looks like a jump or removable

$f(2)=1$

$\lim_{x\to 2} f(x) = \lim_{x\to 2} x^2$
$=2^2$
$=4$

③ Since the $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) \neq f(2)$

there is removable discontinuity at $x=2$, or the point $(2,4)$

Substitute $x=2$ into function to find y

$y=x^2$
$y=2^2$
$y=4$

c) $f(x)=\dfrac{|x^2-3x+2|}{x-2}$ > possible problem at $x=2$. Need to check $\lim_{x\to2^-} f(x)$ and $\lim_{x\to2^+} f(x)$

① $\lim_{x\to2^-} \dfrac{|(x-2)(x-1)|}{x-2}$

★ Since we are approaching from the left $|x-2| = -(x-2)$

$\lim_{x\to2^-} \dfrac{-\cancel{(x-2)}\,|(x-1)|}{\cancel{x-2}}$

$\lim_{x\to2^-} -1\,|(x-1)|$

$= -1\,(2-1)$

$= -1$

② $\lim_{x\to2^+} \dfrac{\cancel{(x-2)}\,|(x-1)|}{\cancel{x-2}}$

★ since we are approaching from the right $|x-2| = (x-2)$

$= |2-1|$

$= 1$

③ Since $\lim_{x\to2^-} f(x) \neq \lim_{x\to2^+} f(x)$ there is a jump discontinuity at $x=2$

d) $f(x)=\begin{cases} x+1, x\le 2 \\ \dfrac{x-5}{x-3}, x>2 \end{cases}$ > Could be a problem at $x=2$ need to check $f(2)$, $\lim_{x\to2} f(x)$

Asymptote at $x=3$ → Check $\lim_{x\to3^-} f(x)$ or $\lim_{x\to3^+} f(x)$

Part A

① $f(2) = 2+1$
$= 3$

② $\lim_{x\to2^+} \dfrac{x-5}{x-3}$
$= \dfrac{2-5}{2-3}$
$= \dfrac{-3}{-1}$
$= 3$

③ $\lim_{x\to2^-} x+1$
$= 2+1$
$= 3$

③ Since $f(2) = \lim_{x\to2} f(x)$

$f(x)$ is continuous at $x=2$

Part B

① $f(3)$ DNE

② $\lim_{x\to3^-} \dfrac{x-5}{x-3}$

		$x=3$		$x=5$	
$x-5$	−		−	0	+
$x-3$	−	DNE	+		+
	+	DNE	−	0	+

$\lim_{x\to3^-} f(x) = \infty \therefore$ DNE

③ Since $\lim_{x\to3^-} f(x) = \infty \therefore$ DNE, there is an infinite discontinuity at $x=3$

e) $f(x) = \sqrt{x-25}$

Domain: $X - 25 \geq 0$

$X \geq 25$

f(x) is continuous on its domain

f) $f(x) = \sqrt[3]{x-5}$

cuberoot so no restriction on domain.

f(x) is continuous on $X \in \mathbb{R}$

e) $f(x) = \tan x$

Infinite discontinuities at $X = \frac{\pi}{2}, \frac{3\pi}{2}$ or $x = \frac{\pi}{2} + \pi n$ such that n is an integer

Since $\tan x = \frac{\sin x}{\cos x}$ whenever $\cos x = 0$ $\tan x$ is undefined. ∴ has an asymptote and infinite discontinuities.

Unit 1: DAY 7 ASSIGNMENT

Duo-Tang: Assignment #7 #1-5 , 10bcegkmo and 11