7.3,4,5,7,8: Applications of Quadratics Functions in Standard Form $y = ax^2+bx+c$ – Concept #10

(Day 7)

Example 1: A ball is thrown into the air and follows the path given by $h(t) = -5t^2 + 20t + 1$, where *h* represents height in meters and *t* represents time in seconds.

a) Determine ***the initial height*** of the ball. The initial height will occur when time is 0 secs.

$h(0) = -5(0)^2 + 20(0) + 1$

$h(0) = 1$ meter The initial height is 1m, which is also the y-int. (h-intercept in this case) of the graph.

b) Determine the ***vertex***.

Step 1: Find t-intercepts (x-int.) by factoring or quadratic formula
Step 2: Find the equation of the axis of symmetry, which is the x-value (t-value) of the vertex.
Step 3: Substitute the x-value (t-value) into the function to find the y-value (h-value) of the vertex.

$\boxed{\text{Step 1}}$ $0 = -5t^2 + 20t + 1$ ← Not easily factorable so use the quadratic formula

$t = \dfrac{-20 \pm \sqrt{20^2 - 4(-5)(1)}}{2(-5)}$

$t = \dfrac{-20 \pm \sqrt{400 + 20}}{-10}$

$t = \dfrac{-20 \pm \sqrt{420}}{-10}$

$t = \dfrac{-20 + \sqrt{420}}{-10}$ ← Sto in graphing calc "A"

$t = \dfrac{-20 - \sqrt{420}}{-10}$ ← Sto in graphing calc "B"

$\boxed{\text{Step 2:}}$ $t = \dfrac{A + B}{2}$

$t = 2$ ← equation of axis of symmetry and t-value of vertex

$\boxed{\text{Step 3:}}$ h-coordinate:

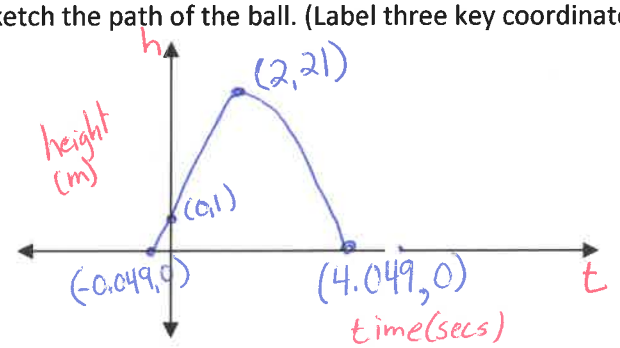
$h(2) = -5(2)^2 + 20(2) + 1$

$h(2) = -20 + 40 + 1$

$h(2) = 21$

$\boxed{\text{vertex} = (2, 21)}$

c) Sketch the path of the ball. (Label three key coordinates and the axes.)



d) What is the ball's ***maximum height***? 21m

e) How long does it take for the ball to reach its ***maximum height***? 2 seconds ~~4.049 secs~~ $\approx$ ~~$\dfrac{-20-\sqrt{420}}{-10}$~~

f) What is the height of the ball after 3 seconds?

$h(3) = -5(3)^2 + 20(3) + 1$

$h(3) = -45 + 60 + 1$

$h(3) = 16$m

g) What are the domain and range of this function?

$D = \{t \mid 0 \le t \le 4.049, t \in \mathbb{R}\}$

$R = \{h \mid 0 \le h \le 21, h \in \mathbb{R}\}$

Example 2 (Pg 407 #14)

Samuel is hiking along the top of the First Canyon on the South Nahanni River in the Northwest Territories. When he knocks a rock over the edge, it falls into the river, 1260m below. The height of the rock, h(t), at t seconds can be modelled by the following function: $h(t) = -25t^2 - 5t + 1260$

a) How long will it take the rock to reach the water? Note: height of rock equals 0m when it reaches the water

$0 = -25t^2 - 5t + 1260$

$0 = -5(5t^2 + 1 + 252)$ ← solve by factoring

$0 = -5(5t + 36)(t - 7)$

$5t + 36 = 0$ $t - 7 = 0$

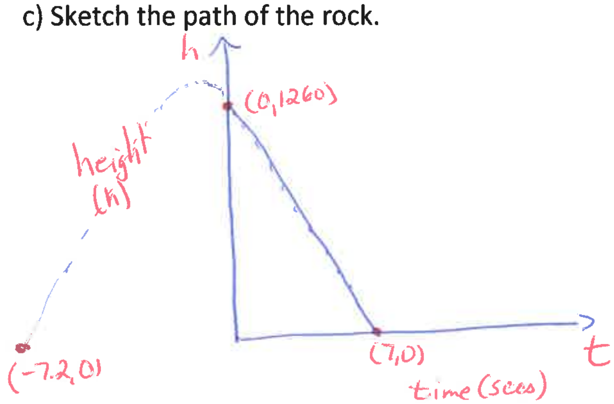
$5t = -36$ $t = 7$

$t = \frac{-36}{5} = -7.2$ $t = 7$

Inadmissible solution as time is not negative

The rock will take 7 seconds to reach the water

c) Sketch the path of the rock.



d) What is the domain and range of the function?

$D = \{t \mid 0 \leq t \leq 7, t \in \mathbb{R}\}$

$R = \{h \mid 0 \leq h \leq 1260, h \in \mathbb{R}\}$

d) Demonstrate the solution using your graphing calculator

Example 3- Pg 374 Ex.1

The flight time for a long-distance water ski jumper depends on the initial velocity of the jump and the angle of the ramp. For one particular jump, the ramp has a vertical height of 5 m above water level. The height of the ski jumper in flight, $h(t)$, in metres, over time, t, in seconds, can be modelled by the following function:

$$h(t) = 5.0 + 24.46t - 4.9t^2$$

a) How long does this water ski jumper hold his flight pose?

The skier holds his flight pose until he is 4.0 m above the water.

b) What is the highest height the ski jumper reaches? Use technology to help you answer these question

a) $4 = 5 + 24.46t - 4.9t^2$ (−4 on both sides)

$0 = -4.9t^2 + 24.46t + 1$ ← graph on graphing calculator and find zeros.

$t = 5.0323$ seconds

b) on graphing calculator calculate the maximum value

$h = 31.525$ m

7.3/4/5/7/8 (Day 7)
(Concept #10)

Assignment: Pg 371 #12-14 Pg 407 #13(Solve by factoring) Pg 428 #8, 10 Pg 380 #7,9 (Using Graphing Calc)

7.6 Vertex Form of the Quadratic Function $y = a(x-h)^2 + k$ or $y = a(x-p)^2 + q$ Concept #11, 12

VERTEX-GRAPHING FORM

Foundations 20 textbook uses: $y = a(x-h)^2 + k$ **or** Pre- Calculus 20 textbook uses: $y = a(x-p)^2 + q$

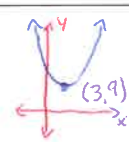
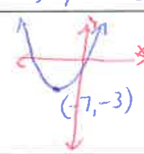
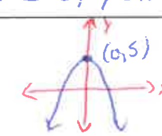
- the vertex is at (p,q) or (h,q)
- if a > 0 (is positive), the parabola opens up
- if a < 0 (is negative), the parabola opens down
- the equation of the axis of symmetry is x=h or x=p

Note: a ≠0

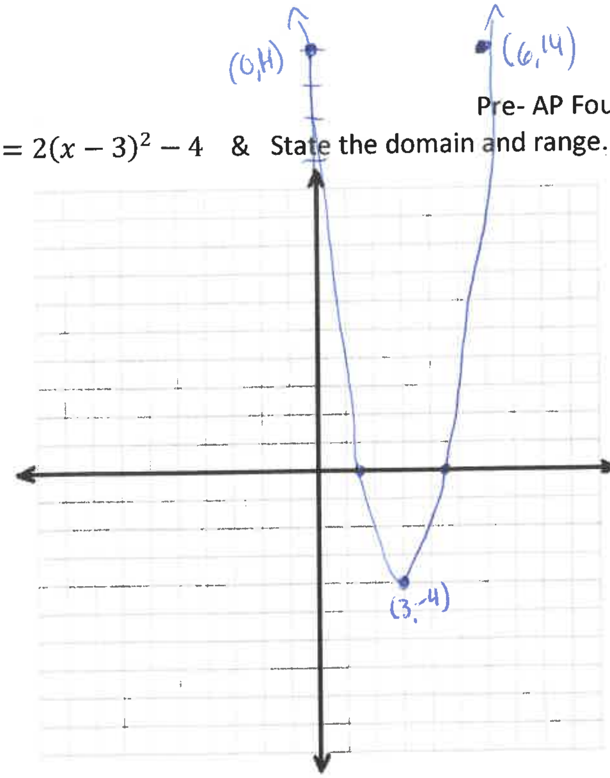
The graph of the function can be sketched more easily using this form.

Example1: For each quadratic function below, identify the following: **(CONCEPT #11)**

a) $f(x) = (x-3)^2 + 9$ b) $m(x) = 2(x+7)^2 - 3$ c) $r(x) = -2x^2 + 5$

	a)	b)	c)
i) Does the parabola open up or down? Max or Min?	up Min at y=9	up Min at y=-3	down Max y=5
ii)Coordinates of the vertex	(3,9)	(-7,-3)	(0,5)
iii)Equation of axis of symmetry	x=3	x=-7	x=0
iv)Domain ad Range	$D=\{x \mid x \in \mathbb{R}\}$ $R=\{y \mid y \geq 9, y \in \mathbb{R}\}$	$D=\{x \mid x \in \mathbb{R}\}$ $R=\{y \mid y \geq -3, y \in \mathbb{R}\}$	$D=\{x \mid x \in \mathbb{R}\}$ $R=\{y \mid y \leq 5, y \in \mathbb{R}\}$
v) How many zeros the function will have?	none 	Two 	Two 
vi) x and y–intercepts (Solve the x- int. using the square root property)	No x-intercepts y-int= set x=0 and solve $y=(0-3)^2+9$ $y=(9)+9$ $y=18$ $\boxed{y\text{-int}=(0,18)}$	x-int set y=0 and solve using the square root property $0 \overset{+3}{=} 2(x+7)^2 - 3^{+3}$ get $(x+7)^2$ by itself $\frac{3}{2} = \frac{2(x+7)^2}{2}$ $\pm\sqrt{\frac{3}{2}} = \sqrt{(x+7)^2}$ $\pm\sqrt{\frac{3}{2}}^{-7} = x+7^{-7}$ $\boxed{\pm\sqrt{\frac{3}{2}} - 7 = x}$ x intercepts y-int. $y=2(0+7)^2-3$ $y=2(49)-3$ $y=98-3$ $y=95$ $\boxed{y\text{-intercept }(0,95)}$	x-intercepts $0^{-5} = -2x^2+5^{-5}$ $\frac{-5}{-2} = \frac{-2x^2}{-2}$ $\sqrt{\frac{5}{2}} = \sqrt{x^2}$ $\boxed{\pm\sqrt{\frac{5}{2}} = x}$ xintercepts y-intercept $y=-2(0)^2+5$ $y=5$ $\boxed{y\text{-int}=(0,5)}$

Example 2: Sketch the graph of the following function: $f(x) = 2(x-3)^2 - 4$ & State the domain and range. **Concept #11**



- Which way does it open? up
- Where is the vertex? $(3,-4)$
- Where is the axis of symmetry? $x=3$
- What are the x- intercepts?

set $y=0$ or $f(x)=0$

"Get the squared term by itself"

$0^{+4} = 2(x-3)^2 - 4^{+4}$

$\frac{4}{2} = \frac{2(x-3)^2}{2}$

$\sqrt{2} = \sqrt{(x-3)^2}$

$\pm\sqrt{2}^{+3} = x - 3^{+3}$

$x = \{\pm\sqrt{2}+3\}$

- What is the y-intercept? $\approx 4.4, 1.585$
  - Reflect this point about the axis of symmetry to find the y intercepts reflection point

y-int = set $x=0$

$y = 2(0-3)^2 - 4$

$y = 2(-3)^2 - 4$

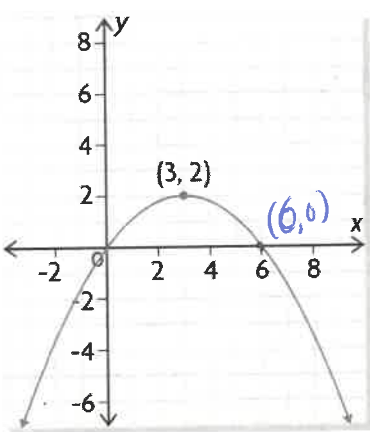
$y = 2(9) - 4$

$y = 18 - 4$

$y = 14$

$(0,14)$

Example 3: Determine the equation of the quadratic function in vertex form **(Concept #12)**



Point $(6,0)$ x y

$y = a(x-p)^2 + q$

$y = a(x-3)^2 + 2$

$0 = a(6-3)^2 + 2$

$0^{-2} = a(3)^2 + 2^{-2}$

$\frac{-2}{9} = \frac{9a}{9}$

$-\frac{2}{9} = a$

$\boxed{y = -\frac{2}{9}(x-3)^2 + 2}$

Assignment: pg 417 #2(Pick 3 to sketch) 3,4, 5, 7, 11, 14

7.6 Application of Quadratics using Vertex Form $y=a(x-h)^2+k$ or $y=a(x-p)^2+q$

Example 1: Given the equation $y=x^2+4$. If the graph is shifted down 2 units, which equation describes the new graph?

the y-coordinate of the vertex will go down 2.

a) $y=x^2+6$ b) $y=(x-2)^2+2$ c) $y=(x-2)^2+4$ d) $\boxed{y=x^2+2}$

vertex $(0,4)\downarrow 2$ $(0,2) \leftarrow$ New vertex

Example 2: Given $y=-2(x-3)^2$ If the function is shifted 8 units to the right and 3 units up, write an equation that describes the new function.

x-coordinate will increase by 8

y-coordinate will increase by 3

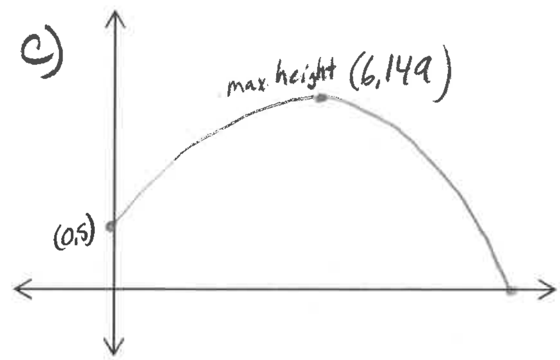
$y=-2(x-\underset{+8}{3})^2+\underset{+3}{0}$

$y=-2(x-11)^2+3$

EXAMPLE 3: A toy rocket is shot up in the air from a hill. Its height in meters above ground, *h*, is recorded after *t* seconds. The path the rocket follows is given by the following equation: $h(t)=-4(t-6)^2+149$

a) Write the function in $y=ax^2+bx+c$ form.
b) What is the initial height of the rocket?
c) Sketch the path of the rocket. (Label your sketch)
d) When will the rocket reach its maximum height?
e) What is the maximum height of the rocket?
f) How long does the toy rocket remain in the air for?

c)



a) $h(t)=-4(t-6)^2+149$

$h(t)=-4(t-6)(t-6)+149$

$h(t)=(-4t+24)(t-6)+149$

$h(t)=-4t^2+24t+24t-144+149$

$h(t)=-4t^2+48t+5$

b) Initial height is the y-intercept 5m

d) Vertex (6, 149)

The rocket will reach its maximum height at 6secs

e) The maximum height the rocket reaches is 149m

f) Need to calculate x-intercepts.

$x=\dfrac{-48\pm\sqrt{(48)^2-4(-4)(5)}}{2(-4)}$

$x=\dfrac{-48\pm\sqrt{2304+80}}{-8}$

$x=\dfrac{-48\pm\sqrt{2384}}{-8}$

$x=-0.103$ $x=12.103$

The rocket is in the air for 12.103secs.

EXAMPLE 4:

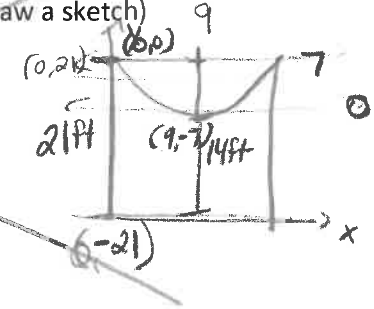
A cable that hangs between two telephone poles makes a parabola shape that has the equation $y = \frac{1}{10}(x-9)^2 - 7$ where x and y are measured in feet. If the cable is attached to both poles at a height of 21 feet and the lowest point of the cable is 14 feet above the ground, how far away are the poles from each another? (Draw a sketch)

$+7 = \frac{1}{10}(x-9)^2$

$\sqrt{70} = (x-9)^2$

$\pm\sqrt{70}+9 = x = 17.36$

18

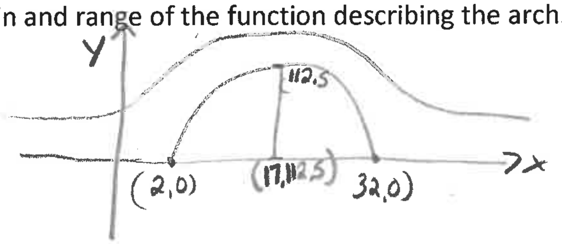


Example 5:

A bridge with a parabolic archway has zeros located at (2, 0) and (32, 0).The arch has a maximum height of 112.5 ft.

a) Determine the equation of the archway in vertex form.

b) State the domain and range of the function describing the arch.



$y = a(x-h)^2+k$ $\qquad y = ax^2+bx+c$

$y = a(x-17)^2+112.5$

$0 = a(2-17)^2+112.5$

$0 = a(225)+112.5-112.5$

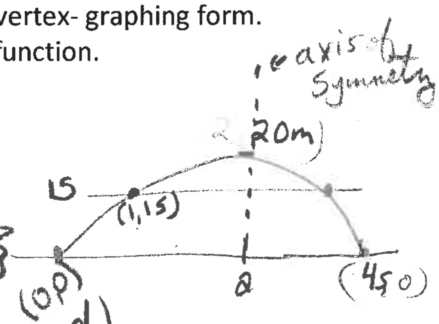
$\frac{-112.5}{225} = \frac{225a}{225}$

$-0.5 = a$

$\boxed{y = -0.5(x-17)^2+112.5}$

Example6: A soccer ball is kicked from the ground. After 2 seconds, the ball reaches its maximum height of 20 m. It lands on the ground at 4s.

a) Determine the quadratic function that models the height of the kick. Write it in vertex- graphing form.
b) Determine any restrictions that must be placed on the domain and range of the function.
c) What is the height of the ball after 1 s?
d) When was the ball at the same height on the way down?



a) $y = a(x-k)^2+q$

$y = a(x-2)^2+20$

$0 = a(4-2)^2+20$

$0 = a(4)+20-20$ $\quad -20$

$-20 = 4a$

$\frac{-20}{4} = \frac{4a}{4}$

$-5 = a$

$y = -5(x-2)^2+20$

b) $\{x \mid 0 \le x \le 4, x \in \mathbb{R}\}$

$\{y \mid 0 \le y \le 20, y \in \mathbb{R}\}$

c) $y = -5(1-2)^2+20$

$y = -5(-1)^2+20$

$y = -5(1)+20$

$y = 15$

The ball is 15m above the ground at 1second.

d) at 3secs

Assignment

1. Given $y - 1 = 2(x+1)^2$. If the equation is shifted left 5 units, which equation describes the new graph?
vertex (-1,0) x-coordinate -5

a. $y - 6 = 2(x-4)^2$
b. $y - 1 = 2(x-4)^2$
c. $\boxed{y - 1 = 2(x+6)^2}$
d. $y - 1 = 2(x+5)^2$
e. $y - 1 = 2(x-6)^2$

2. If the given function $y = (x-1)^2 + 3$ is shifted up 3 units and left 4 units, which equation describes the new graph?
vertex = (1,3) +3 to y -4 to x New vertex (-3,6)

a. $y = (x-4)^2 + 3$
b. $y - 3 = (x+4)^2$
c. $y - 7 = (x+2)^2$
d. $y = (x-5)^2$
e. $\boxed{y = (x+3)^2 + 6}$

3. If the given function $y = (x+3)^2 + 4$ is shifted down 5 units, which equation describes the new function?
vertex (-3,4) y value -5

a. $y = (x+3)^2 + 9$
b. $\boxed{y = (x+3)^2 - 1}$
c. $y = (x+8)^2 + 4$
d. $y = (x-2)^2 + 4$
e. $y = -5(x+3)^2 + 4$

4. Given $y = x^2$. If the function is shifted 4 units to the left, write an equation that describes the new function.
x-coordinate -4

$y = (x+4)^2$

Pg 420 #13, 15, 16, 17 – 19 (Choose 2 questions from 17-19)