7.1 Exploring Quadratic Relations

mmy bears and Frogs Activity or Desmos "Will it hit the hoop?" https://teacher.desmos.com/activitybuilder/custom/56e0b6af0133822106a0bed1

Review

Functions: A specific type of relation that occurs when each element in the domain is only associated with one element in the range. Graphically we can decide if a relation is a function by using the vertical line test.

Some Types of Relations:

Linear Relation (line)

y = mx + b (slope-intercept form) OR **ax + by = c** (standard form) OR **ax+by+c=0** (general form)

- The exponent on the x variable is __1__ (1^{st} degree polynomial)

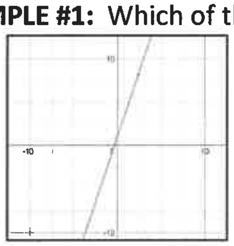
Quadratic Relation (parabola)

$y = ax^2 + bx + c$, where $a \neq 0$ (standard form) **$y = a(x-p)^2+q$, where $a\neq 0$** (vertex form)

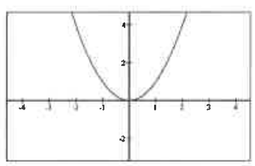
- The largest power of x is __2__ (__2nd__ degree polynomial)
- Why can $a \neq 0$? $y = 0x^2+bx+C$

$y = bx+C$ → The equation would change to linear

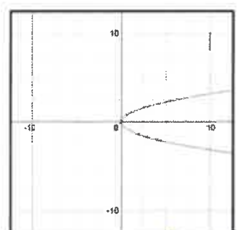
EXAMPLE #1: Which of the following relations are functions?

a) 

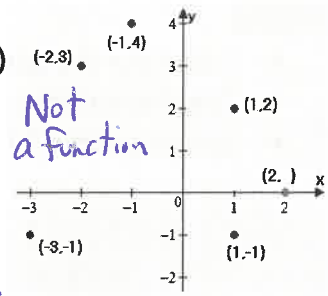
Function

b) 

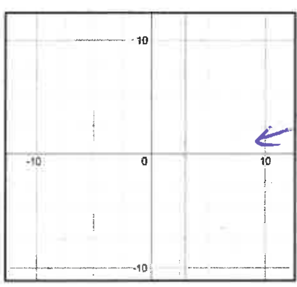
Function

c) 

Not a function

d) 

Not a function

e) 

← Not a function

g) {(6, 3), (1, 7), (12, 10), (5, 3)}

Function

h) {(4, 2), (5, 7), (4, 6), (9, 15)}

Not a Function.

EXAMPLE #2: Which of the following are quadratic? Why or why not??

a) $f(x) = 7x^2$

Yes, it has a degree of 2

b) $f(x) = 5x + 8$

No, it has a degree of 1. It is linear.

c) $y = (x+2)(x-1)$ $y = x^2 + 1x - 2$

Yes, once you expand, you see it has a degree of 2.

d) $f(x) = 2x(x-3)$

Yes, expanded form is $f(x) = 2x^2 - 6x$

e) $f(x) = 3x^2 + 7x + 8$

Yes degree of 2.

f) $y = 4x^3 - 3x^2$

No, it has degree 3.

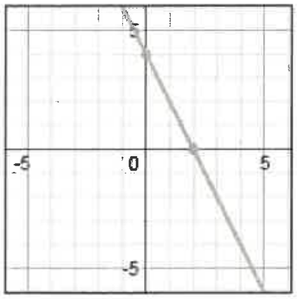
g) $y = x(x - 6)^2$

No, it has degree 3

h) $g(x) = 6(x + 3)^2 + 8$

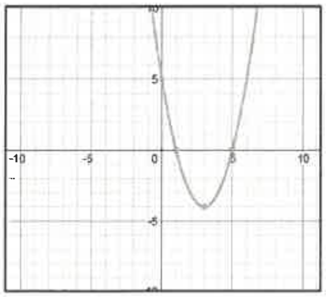
Yes, it has degree of 2.

EXAMPLE #3: Which of the following are quadratic functions?



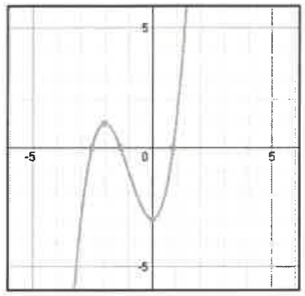
No

linear



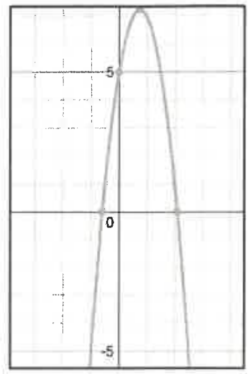
Yes

Quadratic



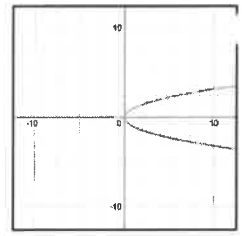
No

Polynomial Function



Yes

Quadratic



No

Radical function.

7.1/2 Properties of Graphs of Quadratic Functions (Day1)- Concept #5/6

PART 1:

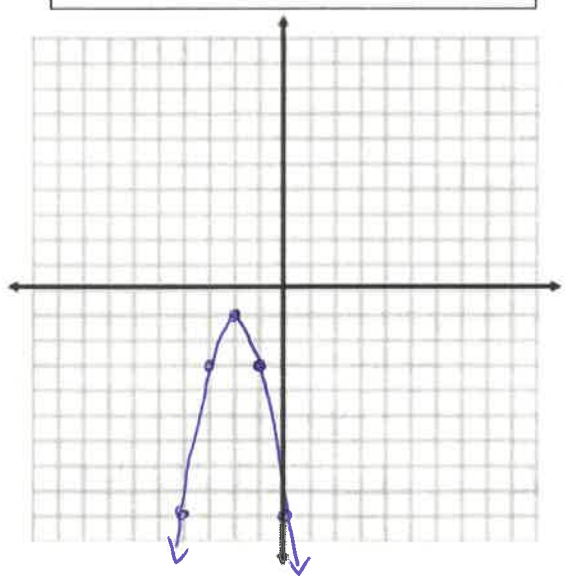
- Complete the table in column 1
- Graph the ordered pairs from the table onto the graph in column two. Join the points in a smooth curve
- Ignore column 3 for now........

THE FUNCTION USING A TABLE

x	$y=-2x^2-8x-9$	y
-4	$Y=-2(-4)^2-8(-4)-9$	-9
-3		-3
-2		-1
-1		-3
0		-9
1		
2		

THE GRAPH

a = -2 b = -8 c = -9



THE CHARACTERISTICS

X Intercept: None

Y Intercept: (0,-9)

Vertex: (-2,-1)

Is the Vertex a Maximum or Minimum? Max

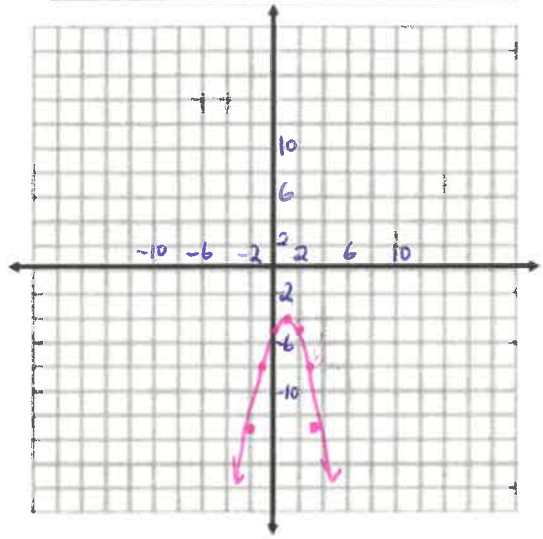
Axis of Symmetry: x=-2

Domain: $(-\infty,\infty)$

Range: $(-\infty,-1]$

x	$y = -x^2 + 2x - 5$	y
-2	$y=-(-2)^2+2(-2)-5$	-13
-1	$y=-(-1)^2+2(-1)-5$	-8
0	$y=0^2+2(0)-5$	-5
1	$y=-(1)^2+2(1)-5$	-4
2	$y=-(2)^2+2(2)-5$	-5
3	$y=-(3)^2+2(3)-5$	-8
4	$y=-(4)^2+2(4)-5$	-13

a = -1 b = 2 c = -5



X Intercept: None

Y Intercept: (0, -5)

Vertex: (1, -4)

Is the Vertex a Maximum or Minimum? Max

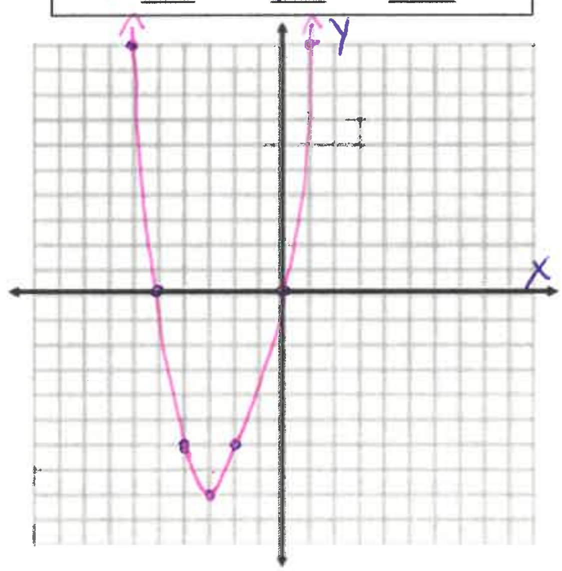
Axis of Symmetry: $x = 1$

Domain: $(-\infty, \infty)$

Range: $(-\infty, -4]$

x	$y = 2x^2 + 12x + 10$	y
-6	$y=2(-6)^2+12(-6)+10$	10
-5		0
-4		-6
-3		-8
-2		-6
-1		0
0		10

a = 2 b = 12 c = 10



X Intercept: (-5, 0) & (-1, 0)

Y Intercept: (0, 9)

Vertex: (-3, -8)

Is the Vertex a Maximum or Minimum? Min

Axis of Symmetry: $x = -3$

Domain: $(-\infty, \infty)$

Range: $[-8, \infty)$

PART 2:

Q: What pattern do you see in the above graphs compared to their equations? How does the value of 'a' affect the graph? What does the value of 'c' tell you about the graph?

The value of "a" determines if the graph opens up or down

If $a > 0$ the parabola is concave up

If $a < 0$ the parabola is concave down.

The value of "c" is equal to the y-intercept of the graph.

7.1/2 Day1 Assignment Pg 360 #1,2 Pg 370 # 7

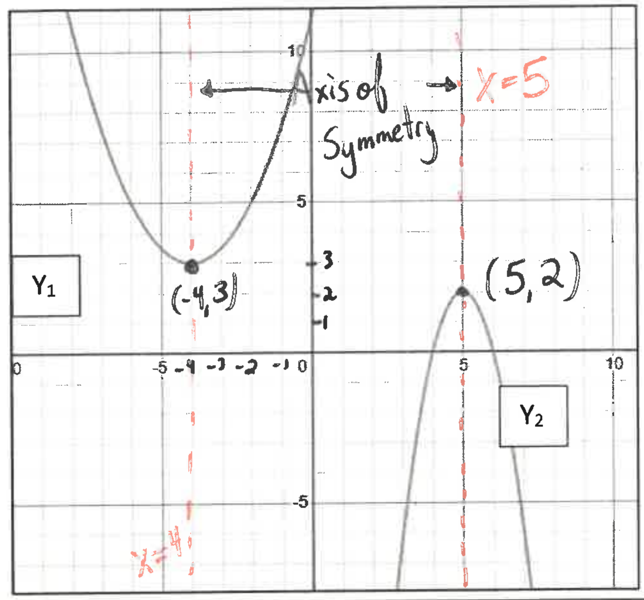
7.1/2 Properties of Graphs of Quadratic Functions (Day2)- Concept #5/6

1. VERTEX:

- An ordered pair (x, y) located at the top or bottom of the curve of a parabola
- A vertex at the top of the parabola is called a MAXIMUM
- A vertex at the bottom of the curve is called a MINIMUM
- What are the coordinates of the vertices for Y_1 and Y_2?

$\boxed{Y_1}$ Vertex: (-4,3)

$\boxed{Y_2}$ Vertex: (5,2)



Note: If I (or a textbook) gives you a grid where the graph goes right to the edge, it is implied that arrowheads exist at that point. **If YOU draw the graph you must include the arrowheads**

2. AXIS OF SYMMETRY:

- An "imaginary" vertical line that goes through the vertex of a parabola and cuts it into two symmetrical halves
- It is always written in equation form as x = The # that is the x coordinate of the vertex
- Draw the axis of symmetry in for both graphs. What is the equation of the axis of symmetry for both Y_1 and Y_2?

$\boxed{Y_1}$ $x=-4$ $\boxed{Y_2}$ $x=5$

3. DOMAIN:

- Describes the complete list of x values that the graph will cover/use when the entire graph is considered
- Describes how far left and right the graph will spread
  - NOTE: When drawing a graph of a parabola that goes on forever, you must draw arrowheads. When given a textbook question or test question of a digitally drawn image of a parabola that extends to the edges of the graph you need to assume that there are arrowheads at the end (most computer programs will not allow them to be added on).
- What is the domain for Y_1 and Y_2?

Domain for both $(-\infty,\infty)$ or $\{x \mid x \in \mathbb{R}\}$

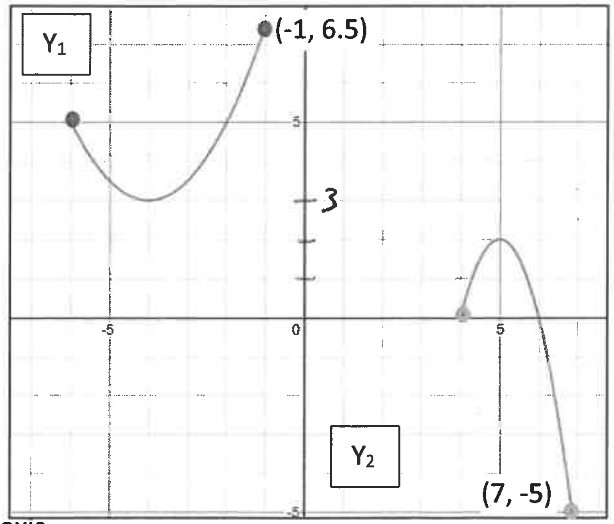
4. RANGE:

- Describes the complete list of y values that the graph will cover/use when the entire graph is considered
- Describes how down and up the graph will spread
  NOTE: Use the same arrowhead rule as described above in the domain.
- What is the range for Y_1 and Y_2?

$\boxed{Y_1}$ Range: $[3,\infty)$ or $\{y \mid 3 \le y, y \in \mathbb{R}\}$ $\boxed{Y_2}$ Range: $(-\infty, 2]$ or $\{y \mid y \le 2, y \in \mathbb{R}\}$

- When a real life problem is modeled by a quadratic function, the domain and range may need to be restricted to values that have meaning in the context of the problem.

If the graph was changed to the following, what would the domain and range be?



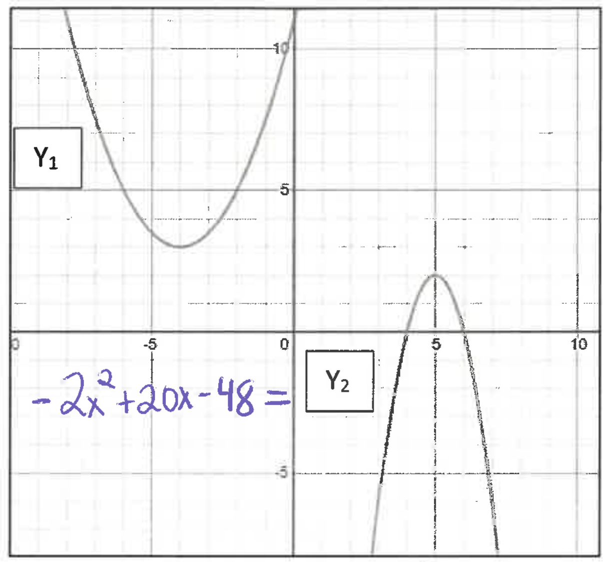
Y₁ D: [-6, -1] R: [3, 6.5]

Remember left, right low, high

Y₂ D: [4, 7] R: [-5, 2]

5. Y INTERCEPT:

- Describes the coordinate where the graph crosses the y axis
- When a parabola is in standard form $y = ax^2 + bx + c$, the value of "c" will be the y intercept – the coordinates of the y intercept will then be (0, c)



Y₁ Y-intercept: (0, 11)

Y₂ Y-int.: You can't tell by the graph, but know it will be lower than -10.

By the equation given (0, -48)

6. X INTERCEPTS:

- Describes the coordinate(s) where the graph of the parabola crosses the x axis
- It is possible to have one, two or no x intercepts
- In this course, the following terms also mean the same thing as x intercepts (and I may use these words interchangeably)

X intercepts = Roots = Zeros = Solutions

Y₁ - Has no x-intercepts (As it does not cross the x-axis)

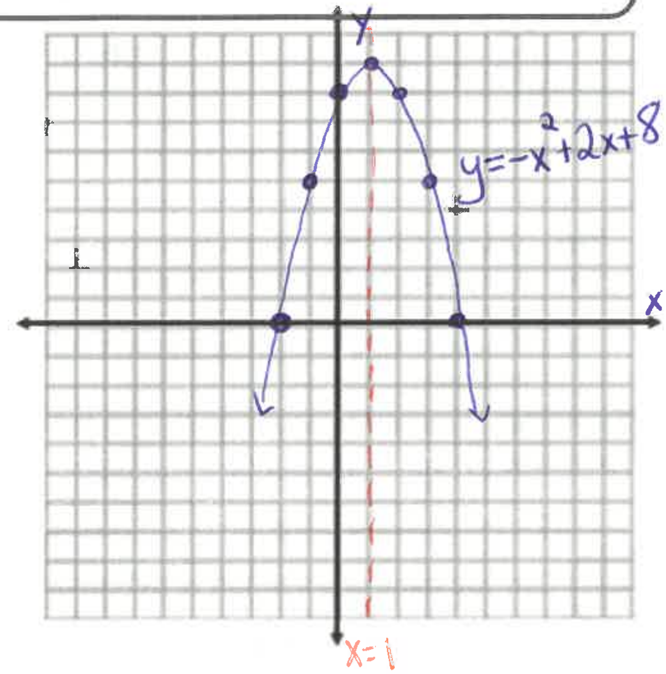
Y₂ - Has x-intercepts of (4, 0) and (6, 0)

EXAMPLE #1: Determine the vertex, the y intercept, the x intercept(s), the equation of the axis of symmetry, max. or min. value, domain, range and sketch the following function: $y = -x^2+2x+8$

Method 1: Create a table of values, sketch the parabola and "read" the necessary information off of the graph. If you are not given values of x to use, choose a reasonable list and keep adding until your graph is a parabola in shape!

x	$y = -x^2+2x+8$	y
-2	$y=-(-2)^2+2(-2)+8$ $=-4-4+8$	0
-1	$y=-(-1)^2+2(-1)+8$ $=-1-2+8$	5
0	$y=-(0)^2+2(0)+8$	8
1	$y=-(1)^2+2(1)+8$	9
2	$y=-(2)^2+2(2)+8$ $=-4+4+8$	8
3		5
4		0

(x,y)
(-2,0)
(-1,5)
(0,8)
(1,9)
(2,8)
(3,5)
(4,0)



- VERTEX: (1,9)
- Y Intercept: (0,8)
- X Intercept(s): (-2,0) & (4,0)
- Domain: $(-\infty,\infty)$ or $x=\{x \mid x\in\mathbb{R}\}$

- Equation of Axis of Symmetry: $x=1$
- Maximum or minimum value: Max of 9 when x=1
- Range: $(-\infty,9]$ or $y=\{y \mid y\le 9, y\in\mathbb{R}\}$

GO BACK TO THE NOTES FROM YESTERDAY AND FILL IN COLUMN THREE!

EXAMPLE #2: **a)** State whether the parabola's will have a maximum or minimum? B) State the coordinates of the y intercept? C) Find an additional ordered pair for each function.

i) $y = -5x^2+8x+3$

a) "a" is negative ∴ the parabola is concave down and will have a maximum value.

b) y-int: (0,3)

c) When x=1
$y=-5(1)^2+8(1)+3$
$y=-5+8+3$
$y=6$
(1,6)

ii) $y=7x^2+2x+5$

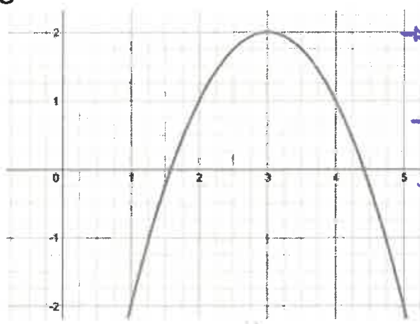
a) "a" is positive ∴ the parabola is concave up and will have a minimum.

b) y-int: (0,5)

c) When x=1
$y=7(1)^2+2(1)+5$
$y=7+2+5$
$y=14$
(1,14)

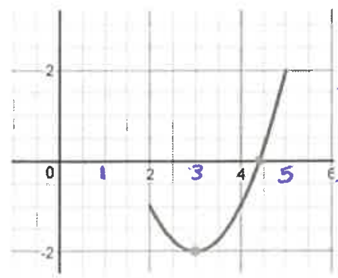
EXAMPLE #3: State whether the parabola has a maximum or minimum value. State its value. State the domain and range.

a)



→Max. of 2 when $x = 3$
→D: $(-\infty, \infty)$
→R: $(-\infty, 2]$

b)

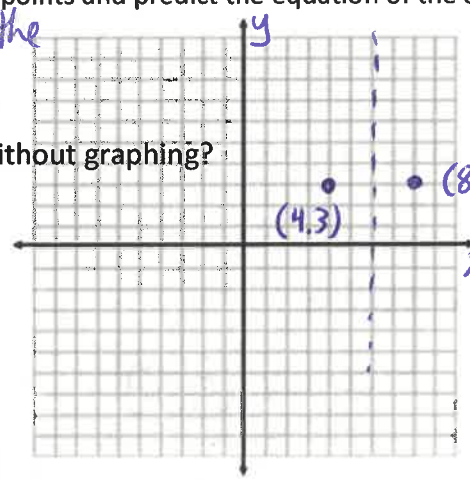


Min. of -2 when $x = 3$
→D: $[2, 5]$
→R: $[-2, 2]$

THE VALUE OF "a": Given a parabola in the form of $y = ax^2 + bx + c$

- If the value of "a" is positive, the parabola is concave up (opens up) and the vertex will be at the bottom and will be a maximum
- If the value of "a" is negative, the parabola is concave down (opens down), the vertex will be at the top and will be a minimum

EXAMPLE #4: The points (4, 3) and (8, 3) lie on the same parabola. Sketch the points and predict the equation of the axis of symmetry. →The axis of symmetry will be between the two points. The equation will be $x = 6$



Q: Can you think of a method to determine the axis of symmetry without graphing?

If two points have the same y-value, the axis of symmetry will be the average of the x-values

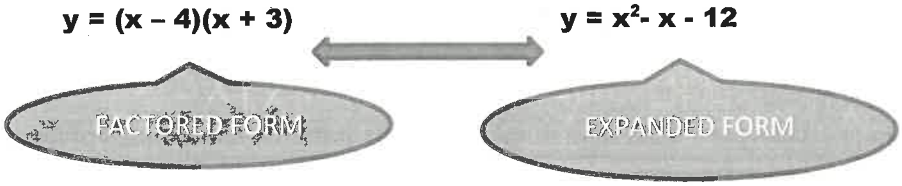
Given (x_1, y_1) & (x_2, y_2) where $y_1 = y_2$

axis of symmetry: $x = \frac{x_1 + x_2}{2}$

7.1/2 (Day 2) Assignment Pg 369 #2,3,5,6, 9ab,10, 11ab (Create a suitable table of values)

CH. 7 (Day 3) Review of Factoring

$y = (x - 4)(x + 3)$ **$y = x^2 - x - 12$**



Note: **Factoring quadratics is the inverse operation of expanding**

Greatest Common Factor

EXAMPLE #1: FACTOR THE FOLLOWING:

a) $f(x) = -2x^2 - 8x$

$f(x) = -2x(x + 4)$

b) $y = 6x^2 - 9x + 30$

$y = 3(2x^2 - 3x + 10)$

Note: These are examples of quadratic functions

Note: You can always check by expanding

c) $-15x^2y^6 - 10x^3y^9 + 15x^{11}$ Note: This is an example of a quadratic expression.

$= -5x^2\,(3y^6 + 2xy^9 - 3x^9)$

Trinomials Where a = 1

EXAMPLE #2: Factor the following:

a) $y = x^2 - 10x + 16$

window method

$$\begin{array}{ccl} x & -2 & -2x \\ x & -8 & -8x \\ & & \overline{-10x} \end{array}$$

middle term

$y = (x-2)(x-8)$

Decomposition

$\underline{-2} \times \underline{-8} = 16$

$\underline{-2} + \underline{-8} = -10$

$y = (x-2)(x-8)$

b) $y = x^2 + 2x - 8$

$$\begin{array}{ccl} x & 4 & 4x \\ x & -2 & -2x \\ & & \overline{2x}\ \checkmark \end{array}$$

$y = (x+4)(x-2)$

c) $f(x) = x^2 + 13x + 40$

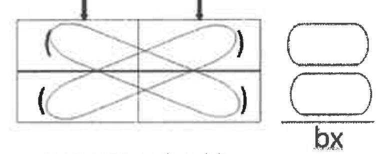
$\underline{8} \times \underline{5} = 40$

$\underline{8} + \underline{5} = 13$

$f(x) = (x+8)(x+5)$

FACTORING WHEN "A" IS A NUMBER OTHER THAN 1:

$y = ax^2 + bx + c$



ANSWER WILL BE:

y = (   )(   )

Trinomials Where a ≠ 1

EXAMPLE #3: Factor the following

a) $y = 2x^2 - 10x - 12$ ☆ Always look for GCF first

$$\begin{array}{ccl} 2x & 2 & 2x \\ x & -6 & -12x \\ & & \overline{-10x}\ \checkmark \end{array}$$

$y = (2x+2)(x-6)$

$y = 2(x+1)(x-6)$

b) $f(x) = -x^2 + 12x - 35$ GCF of -1

$f(x) = -1(x^2 - 12x + 35)$

$f(x) = -1(x-7)(x-5)$

c) $g(x) = 2x^2 + x - 3$

$$\begin{array}{ccl} 2x & 3 & 3x \\ x & -1 & -2x \\ & & \overline{1x}\ \checkmark \end{array}$$

$g(x) = (2x+3)(x-1)$

Decomposition

$\underline{3} \times \underline{-2} = (a \times c)\ -6$

$\underline{3} + \underline{-2} = 1\ (b)$

$g(x) = 2x^2 + 3x - 2x - 3$

$g(x) = x(2x+3) - 1(2x+3)$

$g(x) = (x-1)(2x+3)$

d) $y = 3x^2 - 20x - 7$

$\underline{-21} \times \underline{1} = -21\ (a \times c)$

$\underline{-21} + \underline{1} = -20\ (b)$

$y = 3x^2 - 21x + 1x - 7$

$y = 3x(x-7) + 1(x-7)$

$\boxed{y = (3x+1)(x-7)}$

e) $y = 8x^2 - 18x - 5$

$4x \quad 1 \quad 2x$

$2x \quad -5 \quad -20x$

$-18x$ ✓

$y=(4x+1)(2x-5)$

f) $y = 5x^2 + 7x - 6$

$5x \quad -3 \quad -3x$

$1x \quad 2 \quad 10x$

$7x$ ✓

$y=(5x-3)(x+2)$

Decomp.

$-3 \times 10 = -30$

$-3 + 10 = 7$

$y=5x^2-3x+10x-6$

$y=x(5x-3)+2(5x-3)$

$y=(x+2)(5x-3)$

Difference of Squares

What do we need in order for a question to be a difference of squares?

- Binomial
- Terms are being subtracted
- Both terms are perfect squares

EXAMPLE #4: Factor the following

a) $y = x^2 - 4$

$y=(x-2)(x+2)$

b) $f(x) = 4x^2 - 25$

$f(x)=(2x-5)(2x+5)$

FACTORING FLOWCHART:

Putting it all Together

1. **ALWAYS CHECK FOR GCF FIRST, factor it out if there is one**
2. **Do you have ax^2+bx+c where a = 1? Do the "easy factoring"**
3. **Do you have ax^2+bx+c where a is NOT 1? Use the "Window" method of factoring or decomposition**
4. **Do you have ax^2- b where "a" and "b" are perfect squares? Use difference of squares**

EXAMPLE #5: Factor the following

a) $h(x) = 2x^2 - 10x - 12$

$h(x)=2(x^2-5x-6)$

$h(x)=2(x-6)(x+1)$

b) $y = 2x^2 + 18x + 28$

$y=2(x^2+9x+14)$

$y=2(x+7)(x+2)$

c) $f(x) = 4x^2- 100$

$f(x)= 4(x^2-25)$

$f(x)= 4(x-5)(x+5)$

d) $y = -9x^2 + 48x + 36$ GCF of -3

$y=-3(3x^2-16x-12)$

$y=-3(3x+2)(x-6)$

CH. 7 (Day 3) Review of Factoring Assignment

1. Factor the following questions:

a)	$y = 15x^2 - 65x + 20$	b)	$g(x) = 18x^2 + 15x - 18$
c)	$y = 12x^2 - 52x - 40$	d)	$f(x) = 24x^2 - 2x - 70$
e)	$y = 4x^2 + 4x - 48$	f)	$y = -5x^2 + 40x - 35$
g)	$h(x) = -3m^2 - 18m - 24$	h)	$f(x) = 10x^2 + 80x + 120$
i)	$y = 7x^2 - 35x + 42$	j)	$y = 18x^2 - 2$
k)	$f(x) = 16x^2 - 1$	l)	$g(x) = -x^2 + 1$
m)	$y = 16x^2 - 81$	n)	$h(x) = 2 - 8x^2$

2. Factor fully . Use the strategy that you prefer.

a)	$9k + 6$	b)	$3x^2 - 6x^4$	c)	$-3c^2 - 13c^4 - 12c^3$
d)	$x^2 + 12x - 28$	e)	$y^2 - 2y - 48$	f)	$8a^2 + 18a - 5$
g)	$15a^2 - 65a + 20$	h)	$s^2 + 11s + 30$	i)	$2x^2 + 14x + 6$
j)	$3x^2 + 15x - 42$	k)	$15a^3 - 3a^2b - 6ab^2$	l)	$w^2 + 10w - 24$
m)	$3c^2d - 10cd - 2d$	n)	$f^2 + 17f + 16$	o)	$4t^2 + 9t - 28$
p)	$h^2 - 25j^2$	q)	$6x^2 - 17xy + 5y^2$	r)	$28a^2 - 7a^3$
s)	$25t^2 + 20tu + 4u^2$	t)	$3x^2 - 3x - 60$	u)	$18m^2 - 2n^2$

SOLUTIONS:

1.

a)	$y = 5(x - 4)(3x - 1)$	b)	$g(x) = 3(2x + 3)(3x - 2)$	c)	$y = 4(3x + 2)(x - 5)$
d)	$f(x) = 2(3x + 5)(4x - 7)$	e)	$y = 4(x + 4)(x - 3)$	f)	$y = -5(x - 7)(x - 1)$
g)	$h(x) = -3(x + 4)(x + 2)$	h)	$f(x) = 10(x + 6)(x + 2)$	i)	$y = 7(x - 3)(x - 2)$
j)	$y = 2(3x - 1)(3x + 1)$	k)	$f(x) = (4x - 1)(4x + 1)$		

l) $g(x) = -(x - 1)(x + 1)$ or $g(x) = (1 - x)(1 + x)$ m) $y = (4x - 9)(4x + 9)$

n) $h(x) = 2(1 - 2x)(1 + 2x)$

2.

a)	$3(3K + 2)$	b)	$3x^2(1 - 2x^2)$	c)	$-c^2(3 + 13c^2 + 12c)$
d)	$(x + 14)(x - 2)$	e)	$(y - 8)(y + 6)$	f)	$(4a - 1)(2a + 5)$
g)	$5(3a - 1)(a - 4)$	h)	$(s + 5)(s + 6)$	i)	$2(x^2 + 7x + 3)$
j)	$3(x + 7)(x - 2)$	k)	$3a(5a^2 - ab - 2b^2)$	l)	$(w + 12)(w - 2)$
m)	$d(3c^2 - 10c - 2)$	n)	$(f + 16)(f + 1)$	o)	$(4t - 7)(t + 4)$
p)	$(h - 5j)(h + 5j)$	q)	$(3x - y)(2x - 5y)$	r)	$7a^2(4 - a)$
s)	$(5t + 2u)^2$	t)	$3(x - 5)(x + 4)$	u)	$2(3m - n)(3m + n)$

7.5 Solving Quadratic Equations by Factoring (Day 4) – Concept #7

Zeros: The value(s) which make an expression equal to zero **Roots:** The value(S) that are the solution(s) to a mathematical equation

X- Intercepts: Points on the graph of a relation where the relation crosses the x-axis. These are the points for which the y-value is 0.

The x-intercepts are also called the **'zeros'** of the function, or the **'roots'** of the function.

Zero Product Property: If $a \times b = 0$ then a = 0 or b= 0 (ex./ (ax-n)(bx-m) =0 then ax-n=0 or bx-m=0)

Note: Any factors with an "x" variable set = 0 and solve.

Example #1 a) Solve. $75p^2 - 192 = 0$

GCF of 3

$3(25p^2-64)=0$

$3(5p-8)(5p+8)=0$

$5p-8=0$ (+8 +8) $5p+8=0$ (−8 −8)

$\frac{5p}{5}=\frac{8}{5}$ $\frac{5p}{5}=\frac{-8}{5}$ $p=\frac{-8}{5}$

$\boxed{p=\frac{8}{5} \quad p=-\frac{8}{5}}$ or Solution set $\boxed{p=\left\{\frac{8}{5},-\frac{8}{5}\right\}}$

b) Determine the zeros of $y=-2(2x-7)(4x-1)$

$0=-2(2x-7)(4x-1)$

$2x-7=0$ (+7 +7) $4x-1=0$ (+1 +1)

$\frac{2x}{2}=\frac{7}{2}$ $\frac{4x}{4}=\frac{1}{4}$

$\boxed{x=\frac{7}{2} \quad x=\frac{1}{4}}$ (Zero's are written as x=#, x=#)

c) Determine the roots of $4x^2+28x+49=y$

$0=4x^2+28x+49$

$0=(2x+7)(2x+7)$

$2x \quad 7 \mid 14x$
$2x \quad 7 \mid 14x$
$28x$ ✓

Since both factors are the same you only need to solve for one. This was a perfect square trinomial

$2x+7=0$ (−7 −7)

$\frac{2x}{2}=\frac{-7}{2}$

$\boxed{x=-\frac{7}{2}}$

d) Solve. $1.4t^2+5.6t=16.8$ −16.8

$1.4t^2+5.6t-16.8=0$ (10) (10) (10) (10)

$14t^2+56t-168=0$

$7(2t^2+8t-24)=0$

$7(2)(t^2+4t-12)=0$

$14(t+6)(t-2)=0$

$t+6=0$ (−6 −6) $t-2=0$ (+2 +2)

$\boxed{t=-6 \quad t=2}$ or Solution Set $\boxed{t=\{-6,2\}}$

QUESTION: *What is the difference between the answers for the following questions:*

a) Find the x intercept of y = x² + 5x + 4

- starts w/ y=
- answer will be ordered pairs

To find x-int. set y=0 and solve for x.

$0=x^2+5x+4$

$0=(x+4)(x+1)$

$x=-4,-1$

x-ints are (-4,0) and (-1,0)

b) Solve 0 =x² + 5x + 4

- starts w/ 0=
- answers will be x=#, x=#

$0=(x+1)(x+4)$

$x=-1,\ x=-4$

Would it make sense to switch these questions so that they looked like this instead:

a) Solve y = x² + 5x + 4 b) Find the x intercepts of x² + 5x + 4= 0

NO it does not make sense.

RULE: A question will ask you to "solve" if it has 0= instead of y= or f(x)=

A question will ask you to find the x intercepts/the zero's/the roots if it has y= or f(x)=

EXAMPLE #2: **a)** Find the zeros of the following and sketch.

Step 1: An x-intercept/root/solution/zero is when y = 0, so set the function equal to 0.
Step 2: Factor your trinomial into two sets of brackets using your preferred method
Step 3: Solve for the x-intercepts.
Step 4: Find the vertex (algebraically) and the y intercept.
Step 5: Graph by sketching all the above **points**.

$y = x^2 - 10x + 16$

Factor
$0 = (x-8)(x-2)$

Find x-int
$x - 8 = 0$ $\quad +8 \; +8$ $\qquad x - 2 = 0$ $\quad +2 \; +2$

$x = 8 \qquad x = 2$

x-intercepts (8,0) & (2,0)

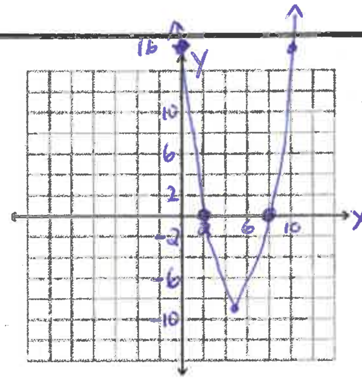
y-int (0,16)

axis of symmetry $x = \frac{8+2}{2}$

Find vertex
x-coordinate of the vertex → $x = 5$

$y = 5^2 - 10(5) + 16$
$y = 25 - 50 + 16 \qquad y = -9$

Vertex (5,-9)



b) $f(x) = -2x^2 - 8x$

$0 = -2x(x+4)$

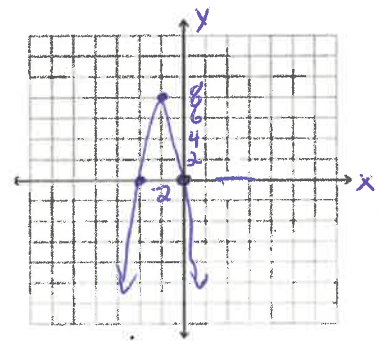
$-2x = 0$ $\quad \div -2 \; \div -2$ $\qquad x + 4 = 0$ $\quad -4 \; -4$

$x = 0 \qquad x = -4$

x-int's (0,0) & (-4,0)

y-int (0,0)

Vertex.
$x = \frac{0 + -4}{2}$
$x = -2$
$y = -2(-2)^2 - 8(-2)$
$y = -8 + 16$
$y = 8$
(-2,8)



c) Verify your zeros algebraically to (b)

When x = 0
$0 = -2(0)^2 - 8(0)$
$0 = 0$ ✓

When x = -4
$0 = -2(-4)^2 - 8(-4)$
$0 = -2(16) + 32$
$0 = -32 + 32$
$0 = 0$ ✓

Example #3 Suppose your best friend solved the quadratic equation as shown:

$4x^2 = 9x$ ← They did not set equation equal to zero

$\frac{4x^2}{x} = \frac{9x}{x}$ Divided both sides by x

$4x = 9$

$x = \frac{9}{4}$ or 2.25

$4x^2 - 9x = 0$

$x(4x-9) = 0$

$x = 0$ $\quad 4x - 9 = 0$ $\quad +9 \; +9$

$\frac{4x}{4} = \frac{9}{4}$

$x = \frac{9}{4}$

Is this solution correct? If not, identify the error. Then, solve the quadratic correctly.

No, You cannot divide by a variable because x could = 0 and you can't divide by zero

$x = \{0, \frac{9}{4}\}$

7.5 Assignment: page 405 #1 -3 4ad, 6(acd), 9-11

7.4 Factored Form of a Quadratic Function (Day 5) - Concept #8

RULE: When a quadratic in the form $y = ax^2 + bx + c$ is factored and written in the form $Y = a(qx - r)(sx - t)$, then the following is true:

- The x intercepts are at $x = \boxed{\frac{r}{q}}$ and $x = \boxed{\frac{t}{s}}$
- The axis of symmetry will always be the average of the above two x intercepts
- The y intercept is the value of c of the equation in standard form. To find it in factored form you need to substitute in x=0 & simplify
- The value of a affects the following characteristics of a parabola by:
  - ➢ Direction of opening:
    - → when $a > 0$ (positive) the parabola is concave up ∪
    - → when $a < 0$ (negative) the parabola is concave down ∩
  - ➢ Vertical Stretch affecting width:
    - Ex./ $y = 1x^2$ → when $a = 1$ or $a = -1$ (or $|a| = 1$) the parabola is a normal width
    - $y = 3x^2$ → when $a > 1$ or $a < -1$ (or $|a| > 1$) the parabola is narrower than normal width
    - $y = \frac{1}{3}x^2$ → when $0 < a < 1$ or $-1 < a < 0$ or ($|a| < 1$) the parabola is wider than normal width

EXAMPLE #1:

a) A parabola has roots of x = 7 and x = -6. What factors did those roots come from?

$(x-7)$ $(x+6)$

b) A parabola has roots of x = 0 and x = 1. What factors did those roots come from?

$x(x-1)$

d) A parabola has roots of $x = \frac{2}{3}$ and $x = -5$. What factors did those roots come from? (Factors do not contain fractions!)

Not $\cancel{(x - \frac{2}{3})}$

$x = -5$

$(x+5)$

$3x = \frac{2}{3}(3)$

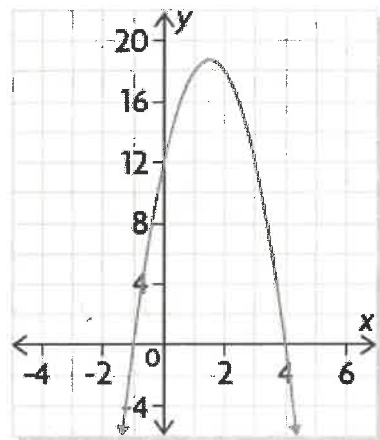
$3x = 2$ -2 -2

$3x - 2 = 0$

Factor $= (3x-2)$

Factors are $(3x-2)(x+5)$

Example #2 - **Determine the function that defines the parabola. Write the function in standard form $y = ax^2 + bx + c$ (Concept #8)**



Step 1: Find the x-intercepts

$(-1, 0)$ $(4, 0)$

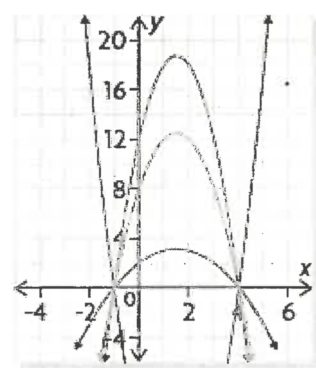
Step 2: Write the factored form of the quadratic function.

$x = -1$ $x = 4$

$x + 1 = 0$ $x - 4 = 0$

$y = a(x+1)(x-4)$

Step 3: Since there are infinitely many parabolas that have these two zeros (ours is the tallest parabola facing downward), we need to find the value of a.



- Choose any other point that you know **lies on** the parabola.
  This point will serve as an (x,y) point that satisfies the quadratic equation.

  Since the y-intercept is 12, we'll use the point
  $(0, 12)$
  x y

$y = a(x+1)(x-4)$

$12 = a(0+1)(0-4)$

$12 = a(1)(-4)$

$\frac{12}{-4} = \frac{-4a}{-4}$

$\boxed{-3 = a}$

In factored form, the quadratic function is:

$y = -3(x+1)(x-4)$

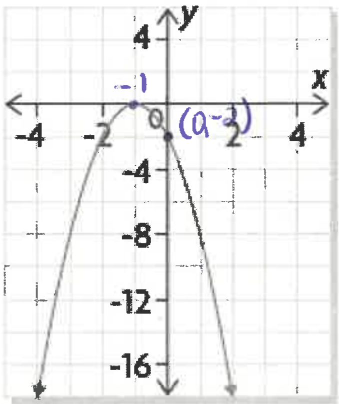
In standard form, the quadratic function is:

$y = -3(x^2 - 4x + 1x - 4)$

$y = -3(x^2 - 3x - 4)$

$\boxed{y = -3x^2 + 9x + 12}$

Example #3- Determine the quadratic function of the graph below written in factored form? and standard form



x-intercept $(-1,0)$ Only one x-intercept.

$y=a(x+1)(x+1)$

Substitute y-int or another point that is on the parabola in for x & y to solve for the a-value.

y-int $(0,-2)$

$y=a(x+1)(x+1)$

$-2=a(0+1)(0+1)$

$-2=a$

$\boxed{y=-2(x+1)^2}$

$y=-2(x^2+2x+1)$

$\boxed{y=-2x^2-4x-2}$

EXAMPLE #4: Find **AN** equation of a parabola that has roots at $-\frac{4}{3}$ and 5.

To find factors

$(3)x=-\frac{4}{3}(3)$ $\quad$ $x\overset{-5}{=}5^{-5}$

$3x\overset{+4}{=}-4^{+4}$ $\quad$ $x-5=0$

$3x+4=0$ $\quad$ $(x-5)$

$(3x+4)$

$y=a(3x+4)(x-5)$

Since it asks to find AN equation "a" can equal any value

$\boxed{y=-\frac{1}{2}(3x+4)(x-5)}$

EXAMPLE #5: Find **THE** equation of the parabola in factored form that has roots at $-\frac{4}{3}$ and 5 and passes through the point $(\overset{x}{-1}, \overset{y}{30})$

$y=a(3x+4)(x-5)$

$30=a(3(-1)+4)(-1-5)$

$30=a(-3+4)(-6)$

$30=a(1)(-6)$

$\frac{30}{-6}=\frac{-6a}{-6}$

$-5=a$

$\boxed{y=-5(3x+4)(x-5)}$

7.4 Assignment: p. 391 #1,3, 4acdf, 5acdf, 11,13

7.7 Solving Quadratic Equations using the Quadratic Formula- Concept #9

What we know so far:

- **SOLVING** a quadratic equation means to find the x intercepts of it. We also call this finding the roots, the zero's or the solutions.
- We can graph the quadratic to find the x intercepts using table of values or key points.
- We can solve the quadratic equation (Set y=o) then factor and solve to find the x intercepts

PROBLEMS WITH THESE METHODS:

- Graphing by hand using a table of values can be time consuming – we don't always know which values of x to start with in the table
- Some quadratics won't factor because their zeros are not rational numbers.

https://www.youtube.com/watch?v=O8ezDEk3qCg (Quadratic Formula Song)

The roots of a quadratic equation in the form **$ax^2 + bx + c = 0$**, where $a \neq 0$, can be determined by using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The **Quadratic Formula** allows us to find the 'roots', 'zeros' or x-intercepts of **any** quadratic equation, regardless if we can fully factor it or not. However, we often try solving for the x-intercepts by factoring first, as this method is usually quicker.

Example #1: a)Solve the equation using the **quadratic formula**. Leave your answer in exact form and round to the nearest hundredth. (3 decimals places) **Remember to set the equation equal to zero first!**

$x^2 + 5x = -8-x$ a= 1 b= 6 c= 8

Step1 Set equation equal to 0

$x^2+5x \overset{+8+x}{=} -8 \overset{+6}{-}x \;{+x}$

$x^2+6x+8=0$

↑ a ↑ b ↑ c

$x = \frac{-6 \pm \sqrt{6^2-4(1)(8)}}{2(1)}$

$x = \frac{-6 \pm \sqrt{36-32}}{2}$

$x = \frac{-6 \pm \sqrt{4}}{2}$

$x = \frac{-6+2}{2}$ $x = \frac{-6-2}{2}$

$x = \frac{-4}{2}$ $x = \frac{-8}{2}$

$\boxed{x = -2 \quad x = -4}$ or $\boxed{x = \{-2, -4\}}$

b) Could you have solved by factoring?

Yes

Example #2: Given $2x^2 + 8x - 5 = 0$, find the zero's using the quadratic formula. State your answer as an exact value (leave as a reduced radical). and approx value to the nearest thousandth.

$a=2 \quad b=8 \quad c=-5$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(2)(-5)}}{2(2)}$$

$$x = \frac{-8 \pm \sqrt{64+40}}{4}$$

$$x = \frac{-8 \pm \boxed{\sqrt{104}}}{4}$$

→ simplify the radical if it is not a perfect square under the radicand.

$$x = \frac{-8 \pm \sqrt{4 \cdot 26}}{4}$$

$$x = \frac{-8^{\div 2} \pm 2^{\div 2}\sqrt{26}}{4^{\div 2}}$$

$$x = \frac{-4 \pm \sqrt{26}}{2}$$

$$x = \left\{ \frac{-4+\sqrt{26}}{2}, \frac{-4-\sqrt{26}}{2} \right\}$$

$$x \approx \{0.550, -4.550\}$$

b) Could you have found the zeros by factoring?

NO

Example #3: Find the roots for the quadratic $y = x^2 + 9x + 23$, using the quadratic formula.

$0 = x^2 + 9x + 23$

$a=1 \quad b=9 \quad c=23$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(1)(23)}}{2(1)}$$

$$x = \frac{-9 \pm \sqrt{81-92}}{2}$$

$$x = \frac{-9 \pm \sqrt{-11}}{2}$$

← Can't take the square root of a negative number so there is no real root

If asking for the solution

$x = \{\varnothing\}$ no solution

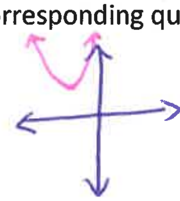
$x = \{\ \}$ empty set

Referring to the real #'s.

No real roots

What does this mean about the graph of the corresponding quadratic?

There are no x-intercepts



Example #4: a)Find the x- intercepts of $y - 13x - 5 = 6x^2$ using the quadratic formula. b) Could you have solved the x-intercepts by factoring?

$\boxed{\text{Step 1}}$ Set $y=0$ and then set equation equal to 0.

$0 - 13x - 5 = 6x^2$ $\quad +13x \quad +5 \quad$ $+13x + 5$

$0 = 6x^2 + 13x + 5$

$a=6 \quad b=13 \quad c=5$

$$x = \frac{-13 \pm \sqrt{13^2 - 4(6)(5)}}{2(6)}$$

$$x = \frac{-13 \pm \sqrt{169-120}}{12}$$

$$x = \frac{-13 \pm \sqrt{49}}{12}$$

$$x = \frac{-13+7}{12} \qquad x = \frac{-13-7}{12}$$

$$\boxed{x = \frac{-6^{\div 6}}{12^{\div 6}} = \frac{-1}{2}} \qquad x = \frac{-20^{\div 4}}{12^{\div 4}} = \frac{-5}{3}$$

The x-intercepts are

$\left(-\frac{1}{2}, 0\right)$ & $\left(-\frac{5}{3}, 0\right)$

b) Yes

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Discriminant

NOTE • $b^2 - 4ac$ (Found in the quadratic Formula) is called the discriminant

- If the discriminant simplifies to a **perfect square**, then the quadratic equation could also have been solved by factoring.
- If the discriminant simplifies to a **negative** number, then there is no real roots / no solution / no x-intercepts (the parabola will not cross or touch the x-axis).
- If the discriminant simplifies to **zero**, there is 1 solution (the vertex lies on the x-axis).

7.7 Assignment: page 428 # 1 (solve only), 2,4, 6

Extra Questions: 1) Determine x –intercept(s) and vertex for each of the following quadratic functions:

a) $y = -x^2 + 6x - 5$ b) $y = \frac{1}{3}x^2 - 2x + 3$

2) Use the quadratic formula to solve each of the following quadratic equations.

a) $x^2 - 2\sqrt{2}x + 2 = 0$ b) $\sqrt{3}x^2 - 7x = -2\sqrt{3}$ ← If time permits try question together.

Solutions #1a) x = 1 and x = 5; vertex: (3, 4) b) x = 3; vertex: (3, 0) hmm... ☺ **#2a)** x = 1.414 OR $x = \sqrt{2}$

b) x = 3.464 and x = 0.577 OR $x = 2\sqrt{3}$ *and* $\frac{\sqrt{3}}{3}$