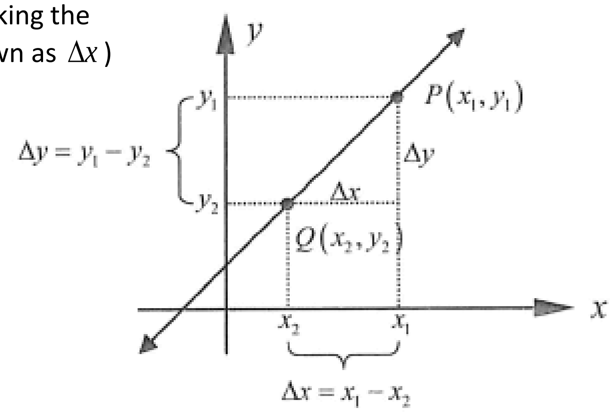
To review the concepts of slope and rate of change.

SLOPE OF A LINE:

- Is a measure of the steepness of a line and is found by taking the ratio of the rise (also known as) Δy to the run (also known as Δx)
- Steep lines have large slopes while lines that are almost horizontal have a small slope
- Horizontal lines have a slope of zero while vertical lines have undefined (infinite) slopes
- The formula for slope is: $m = \frac{\Delta y}{\Delta x} = \frac{y - y_1}{x - x_1}$
- Point Slope Formula for a line: $y - y_1 = m(x - x_1)$
- Slope Intercept Formula for a line: y = mx + b
- Parallel lines have the same slope
- Perpendicular lines have slopes that are negative reciprocals
- When slope is given with units attached it is called a **RATE OF CHANGE**.



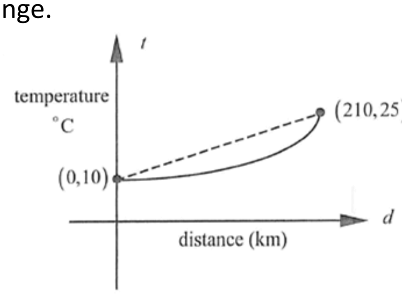
Ex #1:

a) Determine the slope of the line passing through the points (3, -7) and (-24, -28). What is the value of Δy and Δx ?

b) Find the equation of the line.

Ex #2:

Anna's car displays the outside temperature. When she leaves her house the temp is 10°C. If the following graph shows the temperature as a function of the distance travelled, find the average rate of change.



Ex #3: For a given function f(x), $\frac{\Delta y}{\Delta x} = \frac{-4}{3}$

a) If x increased by 3, how much does y change?

b) If x decreases by 18, how much does y change by?

Ex #4: A linear function is given by $y = 6 - 5x$. If x increases by 2, how does y change?

$$slope = \frac{\Delta y}{\Delta x} = -5, \text{ so...}$$

Ex #5: Find the equation of the line passing through (1, -2) with a slope of $2/3$.

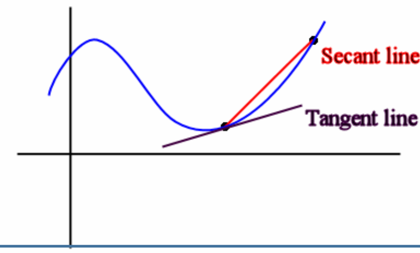
Ex #6: Find the equation of a secant line that passes through the points (-3, 5) and (-6, 7). Express your answer in slope intercept form.

UNIT 2: DAY 1 ASSIGNMENT #1
Textbook: Page P9 #1 - 6, 12

To introduce the tangent line and to estimate the slope at the tangent line

TANGENT LINE – A line that touches a curve in one place

SECANT LINE – A line touches a curve in two or more places



[Link to Desmos Graph for a Tangent line] [Link to Desmos graph for a Secant line]

USING THE SLOPE OF A SERIES OF SECANT LINES TO FIND THE SLOPE OF THE TANGENT LINE:

1. If we are given the following diagram where line l is tangent to the given curve at point P

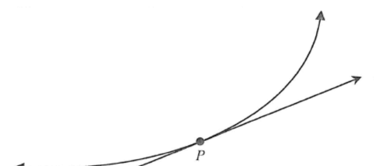


Figure 4

2. Suppose we find an infinite sequence of points Q_1, Q_2, Q_3…. that lie on the curve to the right of P, each one closer to P than it's neighbour.

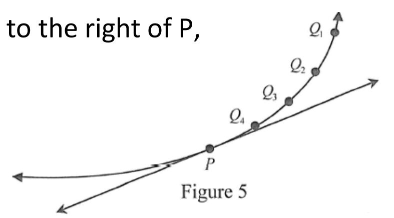


Figure 5

3. Now draw a sequence of secant lines $\overleftrightarrow{PQ_1}$, $\overleftrightarrow{PQ_2}$, $\overleftrightarrow{PQ_3}$, …, as shown in figure 6.

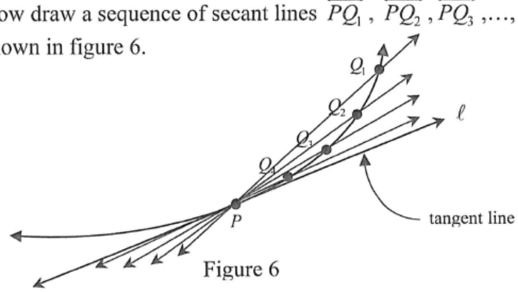


Figure 6

Notice in figure 7 that if we had taken our infinite sequence of points Q_n to the left of P and drawn the corresponding secant lines, they would also have a limiting position that would coincide with that of the tangent line.

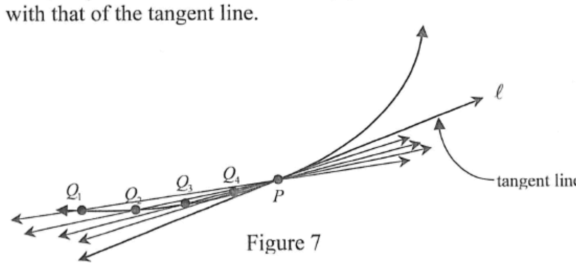


Figure 7

NOTE: Finding each individual slope between PQ_1, PQ_x, etc is finding an AVERAGE RATE OF CHANGE.

A Tangent line EXISTS if :

- the secant lines $\overleftrightarrow{PQ_n}$ approach the same unique line regardless as to whether the points Q_n are on the left or the right side of P. The unique line that is formed is called the TANGENT line to the curve at point P, and we say that the tangent line exists.

Animation of secant line approaching tangent line

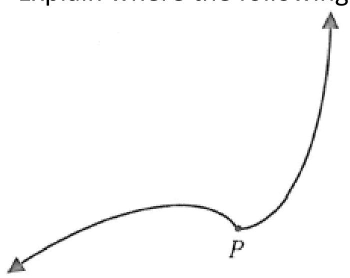
https://www.youtube.com/watch?v=9aWZGbBSwas https://www.desmos.com/calculator/8ubngtz3ei

A Tangent line DO NOT EXIST if:

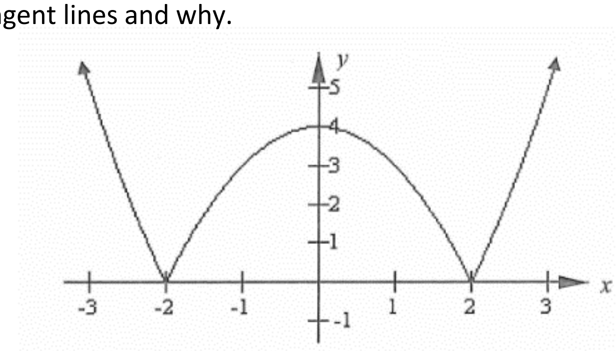
- At points where the slope of the tangent lines approached from the left do not equal the slope of the tangent lines approached from the right. This occurs as at a ___________ or a _____________.
- At points of discontinuity (Jump, removable, infinite)

Ex #1: Explain where the following graphs do not have tangent lines and why.

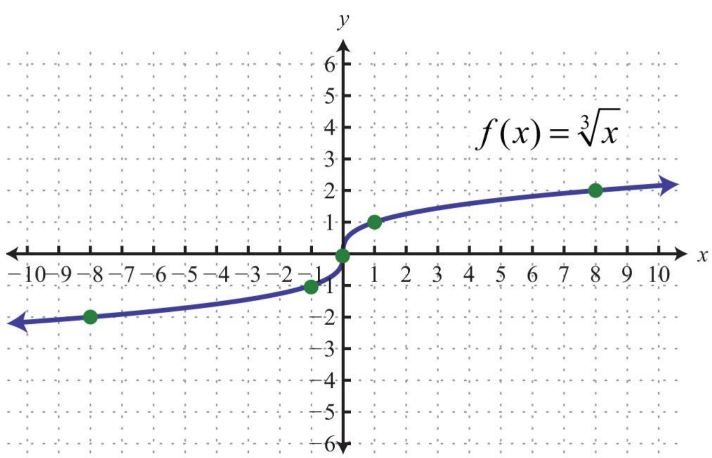
a)



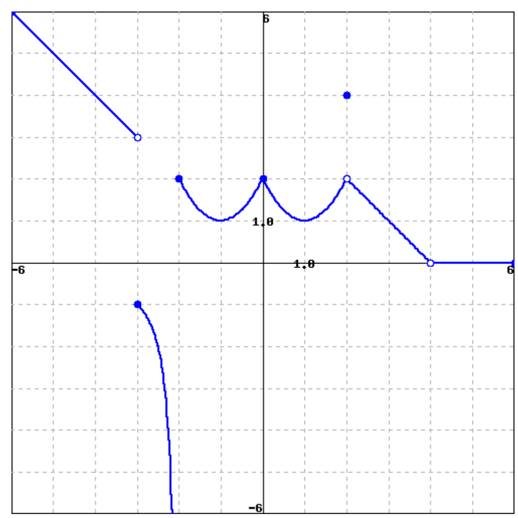
B)



c)



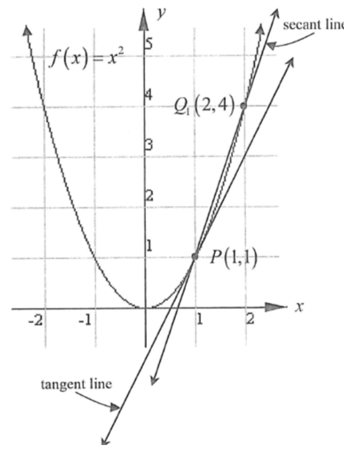
d)



Note: Tangent lines will have a slope of zero at ____________________

Ex #2: a) Find the slope of the tangent line by taking the limit of the slope of the secant line. This can be expressed as:

$$\lim_{Q \to P} m_{PQ} = m \quad \text{and} \quad \lim_{x \to p} \frac{f(x) - f(1)}{x - p} = \lim_{x \to 1} \frac{x^2 - 1}{x - 1} =$$



a) Now that we know that the tangent line passes through P(1,1) and has slope 2, what is the equation of the tangent line?

SLOPE OF A TANGENT LINE AT A SPECIFIC POINT (Formula 1):

- The slope of a line tangent to a curve at a point (a, f(a)) is defined as follows:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Ex #3: **Find** the slope and the equation of the tangent line to the curve $y = 4x^2 - 3x - 1$ at the point (2, 9) by using the above slope formula. https://www.desmos.com/calculator/asojyl0oho

UNIT 2: DAY 2 ASSIGNMENT #2

Duo Tang Assignment #2 questions # 1,2, Textbook: Pg 35 #1aib, 2aib , 8abcd (Use Formula 1)

Using Limits to find slopes of tangent lines at Specific Points

Today we are going to get a little more specific in how we can use limits to find the slope of a tangent line. Last day we used a table to estimate the slope of the tangent line, and then we used a new limit formula to find the slope without using a table. Today we are going to adapt that formula in such a way that it will help us transition to the next concept in Calculus 30. When we used the table to estimate the limit, we kept moving the position of Q closer and closer to the value of P. Today, instead of actually giving different values for Q, we are instead going to focus in on the horizontal distance that exists between Q and P, and see what happens when we take the limit of that horizontal distance (which we are going to call h) as it approaches zero.

Demonstration Ex #1:

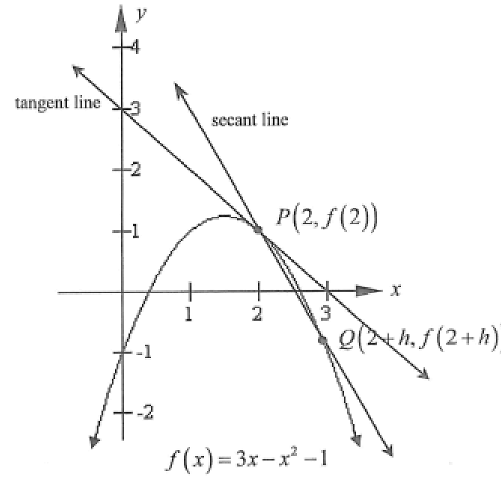
SLOPE OF A TANGENT LINE AT A SPECIFIC POINT (Formula 2):

- The slope of a line tangent to a curve at a point (a, f(a+h)) is defined as follows:

$$m == \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{(a+h) - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Ex #2:

a) Find the slope of the tangent line drawn to the function $f(x) = 3x - x^2 - 1$ at the point P(2, f(2)).



Begin by identifying the following

- a =
- (a+h) =
- f(a) =
- f(a+h) =

$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

b) Find the equation of the tangent line

Ex #3: Find the equation to the tangent line drawn to the function g(x) = $\frac{1}{x+1}$ at x=1

Ex #4: a) Find the slope of the tangent line of f(x) = x^2+x-2 at a general specific point where x=a
b)Find the slopes of the tangent line when the x- coordinate is: i) 3 ii) -1.

Unit 2 DAY 3 ASSIGNMENT #3

Textbook Pg 35 (ONLY USE FORMULA 2) #1a(ii)b, 6a (for i), 7ab for i, ii, v, 8cd, 9, 10

CALCULUS 30: UNIT 2 DAY 4 – VELOCITY AND LIMITS

Calculating Average Velocity and Using Limits to Calculate Instantaneous Velocity at a Point.

AVERAGE VELOCITY:

$$\text{average velocity} = \frac{\text{distance travelled}}{\text{time elapsed}} = \frac{\Delta s}{\Delta t} = \frac{displacement}{change-\text{in}-\text{t}\,ime} = \text{slope of the secant line}$$

Ex #1: You drive from Regina to Calgary. It takes 2.25 hours to reach Swift Current, which is 235km from Regina. Then you stop briefly in Medicine Hat to get gas. You notice that you have now travelled a total distance of 436 km and it has taken you 4.5 hours in all. By the time you reach Calgary, you have travelled 670 km in 7.5 hours.

a) What is your average velocity between Regina and Swift Current?

b) What is your average velocity between Swift Current and Medicine Hat?

c) What is your average velocity between Regina and Calgary? https://www.desmos.com/calculator/yuyqkt0fhz

AVERAGE VELOCITY: (also known as Average Rate of Change)

- We can expand upon our earlier definition of average velocity by saying that it is slope of a secant line between two points (x, y) and (x_1, y_1) on a distance time graph where y is distance and x is time.

$$\text{Average Velocity}=\frac{\text{distance travelled}}{\text{time elapsed}}=\frac{\text{change in distance}}{\text{change in time}}=\frac{\Delta y}{\Delta x}$$

Often the variables used are not x and y. Instead of y they often use "s". Instead of x they often use "t". In that case we would say that

$$\text{Average Velocity}=\frac{\text{change in distance}}{\text{change in time}}=\frac{\Delta s}{\Delta t}$$

- Instantaneous velocity is slope of the TANGENT line at a certain point in time. This will be the limit of an average velocity as Δt approaches zero

$$\text{Instantaneous Velocity}=v(a)=\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}$$

To actually use this limit algebraically to find an answer to the instantaneous velocity of a function at time "a", we need to apply the formula we learned in the last section:

$$\boxed{\text{Instantaneous Velocity}=v(a)=\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}}$$

Ex #2: A ball is dropped from the top of a 400 foot tall building and falls such that its distance from the ground at *t* seconds is $s = -16t^2 + 400$ feet. (Note: Complete on Looseleaf)

a) What is the average velocity of the ball for the first 4 seconds?

b) What is the instantaneous velocity at 4 seconds?

c) Find the velocity after t seconds

d) When will the ball it the ground?

e) With what velocity will the ball hit the ground?

Unit 2 DAY 4 ASSIGNMENT #4
Textbook Page 43 # 1a (i & ii), 1b, 2a(i & iv), 2b, 3, 5a(i only), 9

Definition of a Derivative.

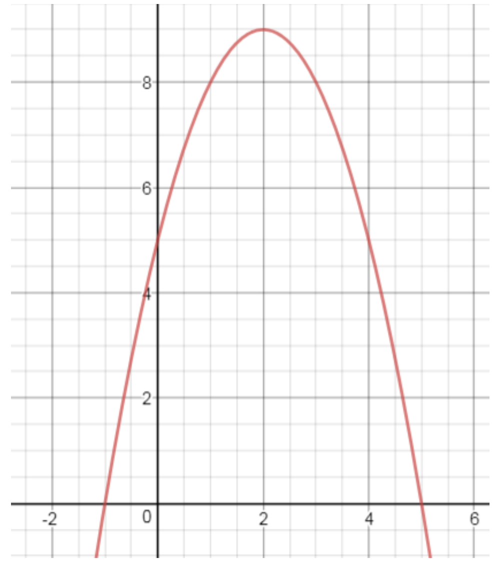
THE GENERAL DEFINITION OF A DERIVATIVE:

- The derivative of a function *f(x)*, is a formula that finds the slope of a tangent line to the curve f(x)at ANY POINT x = a to the curve of *f(x*)
- The derivative of the function *f(x)* is named *f'(x)* (which is read f prime of x) or can be known as $\frac{dy}{dx}$ (which is read as dee y by dee x)
- The process of finding the derivative is called **DIFFERENTIATION**
- To differentiate f(x) and find it's derivative f'(x), we can use the definition of the derivative:
$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Ex #1: Given $f(x) = 5 + 4x - x^2$

a) Find f '(x) *"Can you develop a formula that will find the slope of a tangent line at any x-value for this particular function?"(note: this question could also be worded as"Find the derivative of this function?"*

b) Use your answer in part "a" to find f'(4)



c) Find the coordinates of a point on the curve f(x) = 5 + 4x – x^2 at which the slope of the tangent line is 1. *(This could also be worded as "where the first derivative is 1")*

d) What is the difference between the answer to f'(2) and f(2)? Use the given graph to help demonstrate your answer.

e) Where on the graph is f'(x) = 0? How do you know?

f)Find the equation of the tangent line to the curve of F(x) at x=2

Ex#2: Given the function $f(x)=\dfrac{10}{x}$, determine:

https://www.desmos.com/calculator/e5prgzf0cy

a) f'(x)

b) f'(2)

b) The equation of the tangent line at the point (2, f(2)). Leave answer in slope-intercept form

c) The value of x at which the slope of the tangent line to the curve is –5.

d) Where or if the slope of the tangent line will ever be positive? (Graph on desmos)

Ex#3: Find the derivative of a) $f(x) = 8$ b) $f(x) = 2x+1$

Unit 2 DAY 5 ASSIGNMENT #5

DUO TANG Unit 2 : Assignment #5 : #1b-g, 2ab, 3 ,9 Extension Questions #4, 5

Definition of a Derivative.

THE DERIVATIVE:

$$\frac{dy}{dx} = f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

NOTE: Sometimes instead of asking for f'(x) they merely say f'

There are many ways to denote the derivative of a function $y = f(x)$.

Derivative Notation	Words describing derivative notation	Important points about the derivative notation
$f'(x)$		
$f'(a)$		
y'		
$y'(a)$		
$\frac{d}{dx}y = \frac{dy}{dx}$		
$\left.\frac{dy}{dx}\right\rvert_{x=a}$		
$\frac{df}{dx} = \frac{d}{dx}f(x)$		

$$\therefore f'(x) = y' = y'(x) = \frac{d}{dx}y = \frac{dy}{dx} = \frac{d}{dx}f(x) = \frac{df}{dx}$$

__Note:__ Any notation using $\frac{d}{dx}$ should NOT be thought of as a fraction or as itself a derivative; it should be thought of as an operator that instructs you to take the derivative and treat x as the variable.

Ex #1: If $f(x)=\sqrt{x-2}$, find $f'(x)$ and state the domains of f and $f'(x)$ .https://www.desmos.com/calculator/rf4xj1l27b

Ex #2: If given the following limit of a function *f* at some value of *a*, state the value of *a* and the equation of f(x)

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{(4+h)^2-16}{h}$$

Ex #3: Find f' if $f(x)=\dfrac{x+1}{3x-2}$. https://www.desmos.com/calculator/7brln65nth

Ex #4: If $f(x) = -5x^2 + 4x$, find $f'(2)$

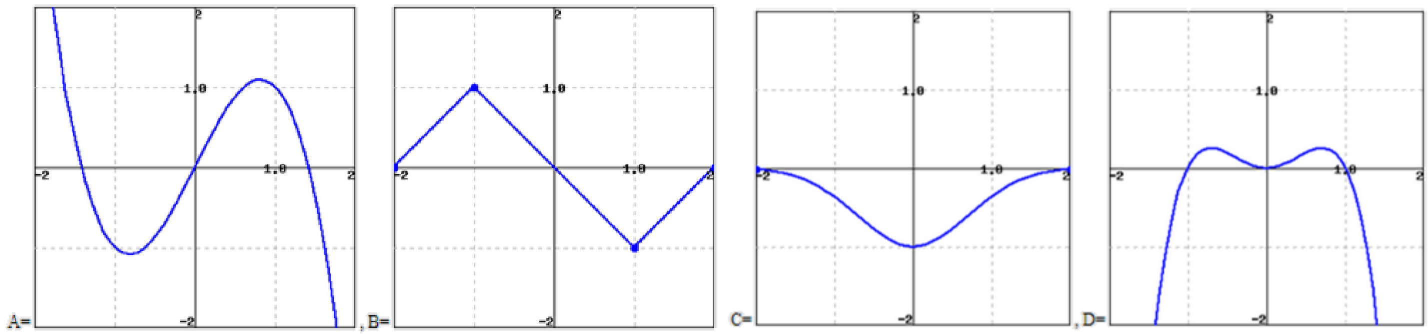
Ex #5: The first set of graphs represent a set of original functions f(x) and the second set of graphs represent the graphs of the derivatives f'(x) of the original functions. Match the original graph f(x) with its derivative f'(x)

https://www.intmath.com/differentiation/derivative-graphs.php

https://www.maa.org/sites/default/files/images/upload_library/4/vol4/kaskosz/derapp.html

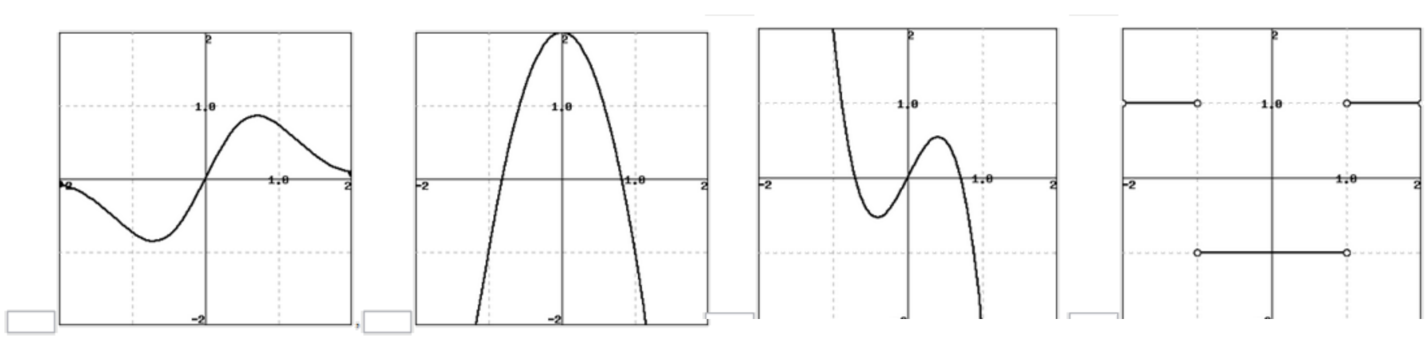
f(x) graphs:

A **B** **C** **D**

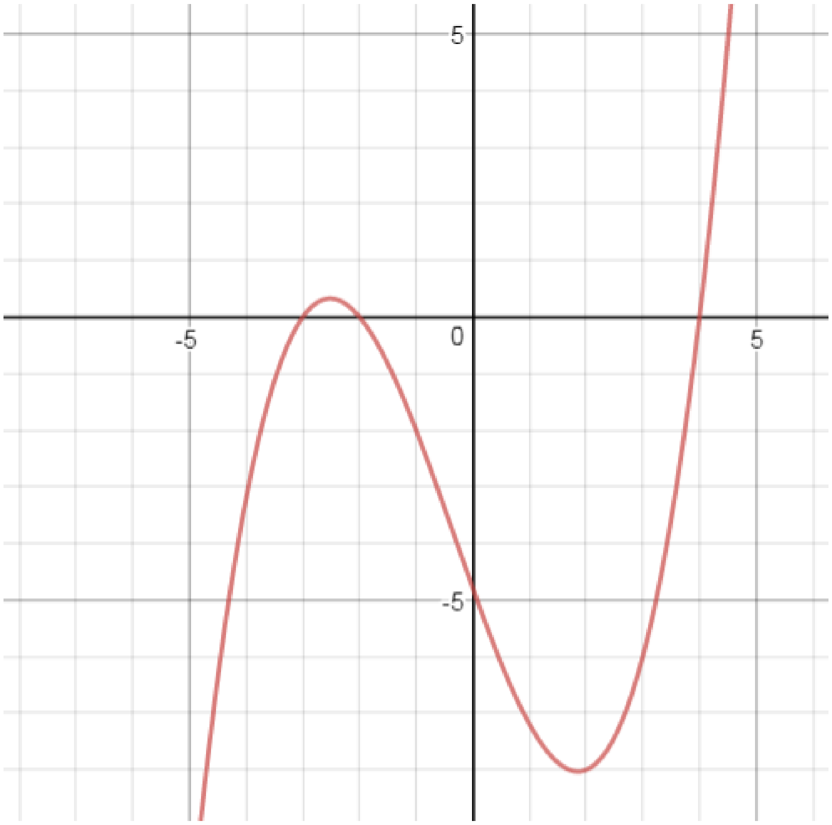


f'(x) graphs

______ ______ ______ ______



Ex #6: Given the following graph of f(x), sketch the graph of f'(x)



https://www.desmos.com/calculator/rmzuqwiyho

UNIT 2 DAY 6 ASSIGNMENT #6

Textbook Page 75 #1b-d, 2, 3, 5, 6, 8bc, 10b, 11a, 12cd, 13,14