Topic 6 (Day1) - 5.1 Representing Relations

Concept #24: 5.1/5.2 Be able to express relationships in a variety of ways. Correctly identify whether a relationship is a function or not with justification

SET: A collection/list of distinct objects.

EX: A List of Teachers: {Ms. Carignan, Mr. Foreman, Mr. Adams, Ms. Moroz, Ms. Sebastian}

EX: A List of Subjects: {English, Math, Science, Wellness}

ELEMENT: What you individually call each item in the SET.

EX: Mr. Foreman is one element in the Random List of Teachers

NOTE: *The list of elements in the set are usually enclosed in braces (curly brackets). The order in which the elements are listed does not matter.*

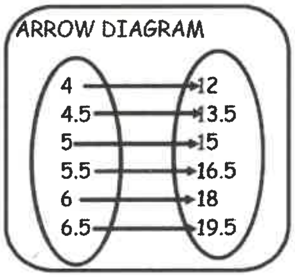
RELATION: Something that associates the elements of one set to the elements of another set

A relation can be presented in a variety of ways. For example,

WORDS
Three times the length of your ear, e, is equal to the length of your face, f (from chin to hairline)

EQUATION
$f = 3e$

ORDERED PAIRS
(4, 12), (4.5, 13.5), (5, 15), (5.5, 16.5), (6, 18), (6.5, 19.5)



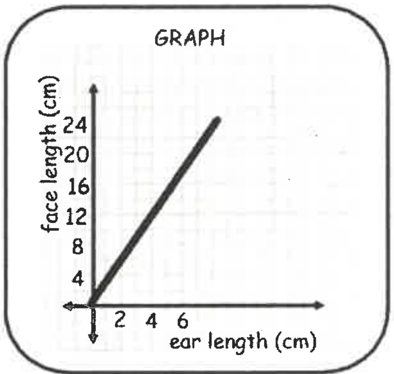


TABLE OF VALUES

Ear Length (cm)	Face Length (cm)
4	12
4.5	13.5
5	15
5.5	16.5
6	18
6.5	19.5

Example #1: Represent the relationship between the set of Teachers to the Set of Subjects in the following ways:

EX: A List of Teachers: {Ms. Carignan, Mr. Foreman, Mr. Adams, Ms. Moroz, Ms. Sebastian}

EX: A List of Subjects: {English, Math, Science, Wellness}

a) In a Table

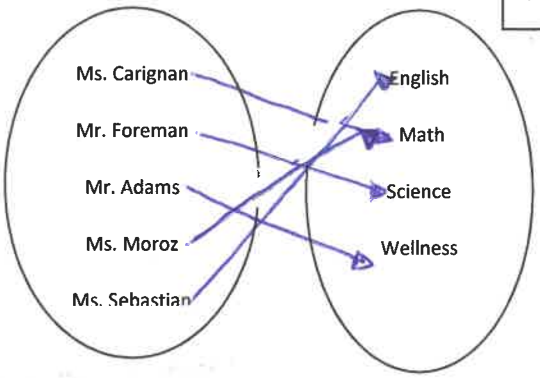
Teacher Name	Subject
Ms. Carignan	Math
Mr. Foreman	Science
Mr. Adams	Wellness
Ms. Moroz	Math
Ms. Sebastian	English

b) As a list of Ordered Pairs

{(Ms.Carignan, Math), (Mr. Foreman, Science), (Ms.Moroz, Math), (Mr. Adams, Wellness), (Ms.Sebastian, English)}

***Note:** ordered pairs are normally associated with graphing points on a graph – ex the ordered pair (4, 5) tells me to start at the middle of the graph, go over 4, up 5, stop and draw a dot. Ordered pairs can also be used to describe words (element from first set, element from second set)*

c) As an arrow diagram

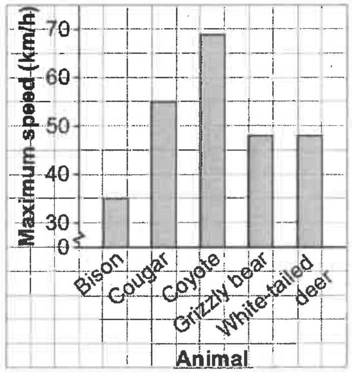


When the elements of one or both sets are numbers, the relation can be represented as a bar graph.

Example #2: Using the information represented by the following bar graph, re-represent the relation as:

a) A table

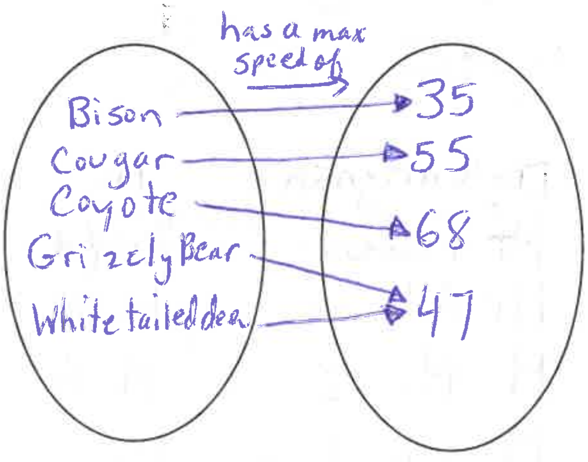
Maximum Speeds of Different Animals



Animal	Max.Speed (Km/h)
Bison	33 Km/h
Cougar	55 Km/h
Coyote	68 Km/h
Grizzly Bear	47 Km/h
White-tailed deer	47 Km/h

b) A list of ordered pairs

c) An arrow diagram



d) In words

The relation shows "Max speed of" from a set of animals to a set of speeds in Km/h

Topic 6 Day 1 Assign.5.1 Page 262 #3a, 4, 5, 6, 7, 10, 12

Topic 6 – (Day 2) 5.2 Properties of Functions (Domain, Range of Relations)

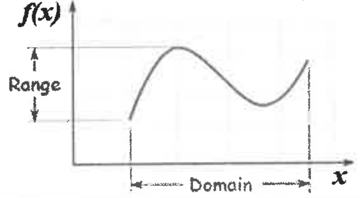
Concept #25: 5.2/5.5 Correctly determine the domain and range of linear & non-linear relations using interval notation, set notation or lists **(NC)**

[illegible]pendent Variable: the variable for which values are selected (Often the x –values)

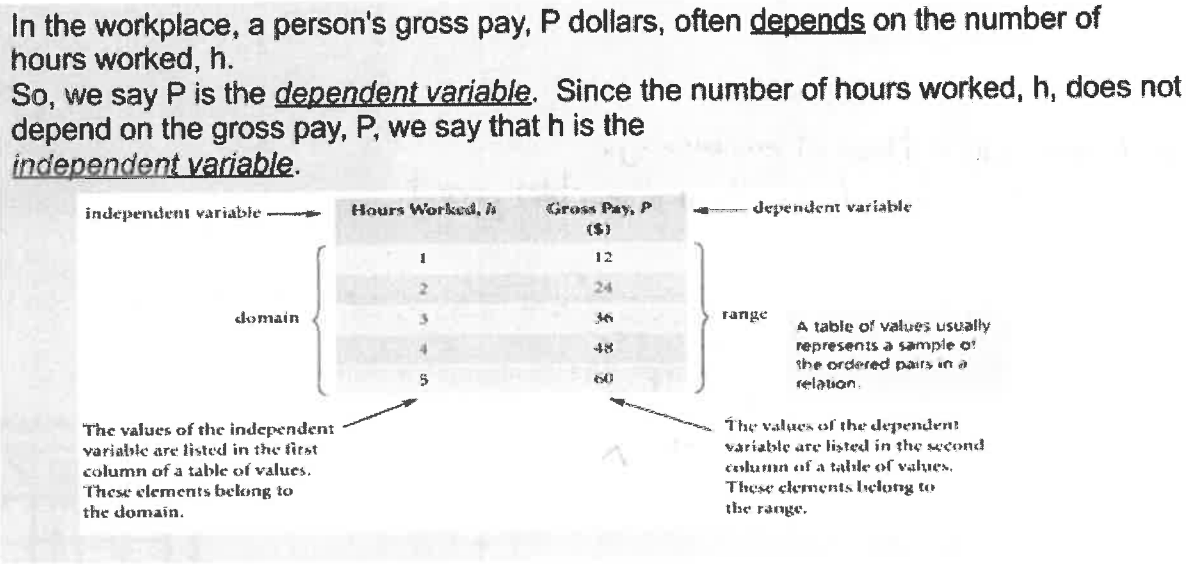
Dependent Variable: The variable whose values depend on those of the independent variable (Often the y- values)

Domain: the set of all possible values for the independent variable in a relation

Range: the set of all possible values for the dependent variable as the independent variable takes on all possible values of the domain



For example:



In the workplace, a person's gross pay, P dollars, often depends on the number of hours worked, h.
So, we say P is the *dependent variable*. Since the number of hours worked, h, does not depend on the gross pay, P, we say that h is the *independent variable*.

independent variable → domain

Hours Worked, *h*	Gross Pay, *P* ($)
1	12
2	24
3	36
4	48
5	60

← dependent variable; range

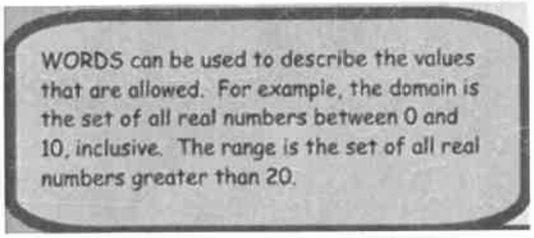
A table of values usually represents a sample of the ordered pairs in a relation.

The values of the independent variable are listed in the first column of a table of values. These elements belong to the domain.

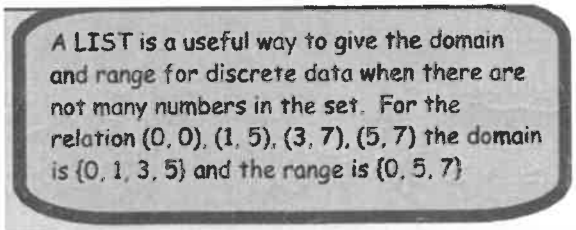
The values of the dependent variable are listed in the second column of a table of values. These elements belong to the range.

Different Ways to Describe the Domain and Range

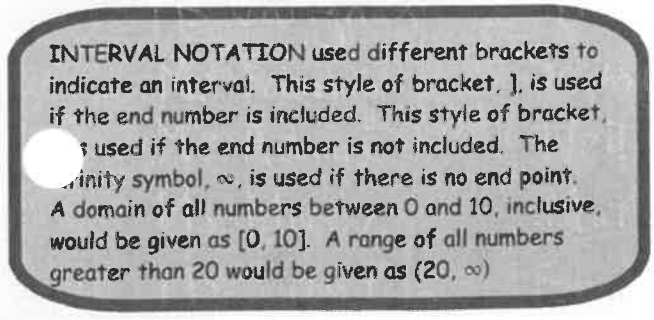
When a relation is continuous (a solid line) it makes more sense to use set notation or interval notation to describe the domain and range.
When a relation is not continuous (just points) it makes more sense to use a list to describe the domain and range



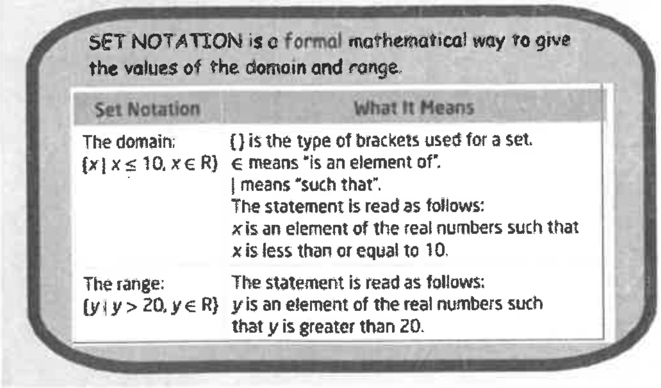
WORDS can be used to describe the values that are allowed. For example, the domain is the set of all real numbers between 0 and 10, inclusive. The range is the set of all real numbers greater than 20.



A LIST is a useful way to give the domain and range for discrete data when there are not many numbers in the set. For the relation (0, 0), (1, 5), (3, 7), (5, 7) the domain is {0, 1, 3, 5} and the range is {0, 5, 7}



INTERVAL NOTATION used different brackets to indicate an interval. This style of bracket,], is used if the end number is included. This style of bracket, [illegible]s used if the end number is not included. The [illegible]inity symbol, ∞, is used if there is no end point. A domain of all numbers between 0 and 10, inclusive, would be given as [0, 10]. A range of all numbers greater than 20 would be given as (20, ∞)



SET NOTATION is a formal mathematical way to give the values of the domain and range.

Set Notation	What It Means
The domain: $\{x \mid x \leq 10, x \in R\}$	{ } is the type of brackets used for a set. ∈ means "is an element of". | means "such that". The statement is read as follows: *x* is an element of the real numbers such that *x* is less than or equal to 10.
The range: $\{y \mid y > 20, y \in R\}$	The statement is read as follows: *y* is an element of the real numbers such that *y* is greater than 20.

"Domain"

Set Notation "x belongs to the real set of numbers" **Interval Notation**

$D=\{x \mid \text{left\#} \le \text{ or } < x \le \text{ or } < \text{right\#}, x \in \mathbb{R}\}$

variable of the independent

$R=\{y \mid \text{lowest\#} \le \text{ or } < y \le \text{ or } < \text{highest\#}, y \in \mathbb{R}\}$

"Range" variable of the dependent

$D=(\text{left\#}, \text{Right\#})$ $R=(\text{low\#}, \text{high\#})$

curved bracket indicated to NOT include the #. Use when there is an open circle

Square bracket use when the number is included. Closed circle.

$D=(\quad , \quad]$ $R=(\quad , \quad]$

$D=[\quad , \quad)$ $R=[\quad , \quad)$

$D=[\quad , \quad]$ $R=[\quad , \quad]$

Example #1 The table shows the masses, m, grams of different numbers of identical marbles, n.

a) Identify the independent variable and the dependent variable. Justify your choices

b) What would be an appropriate way to describe the domain and range of the relation?

c) Write the domain and range.

Number of Marbles, *n*	Mass of Marbles, *m* (g)
1	1.27
2	2.54
3	3.81
4	5.08
5	6.35
6	7.62

a) Independent variable = # of marbles

Dependent variable = Mass of marbles (g)

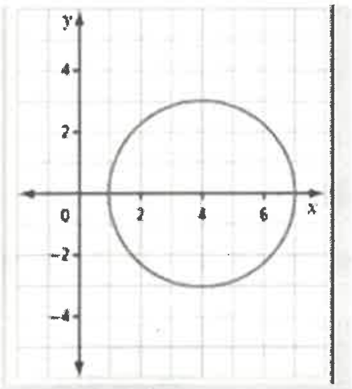
The mass of the marbles depends on the # of marbles present.

b) Use a List

c) $D=\{1, 2, 3, 4, 5, 6\}$ $R=\{1.27, 2.54, 3.81, 5.08, 6.35, 7.62\}$

Example #2- Describe the domain and range of each relation. (Use the appropriate method)

a)



Set Notation

$D=\{x \mid 1 \le x \le 7, x \in \mathbb{R}\}$

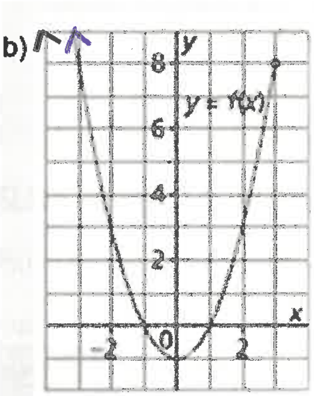
$R=\{y \mid -3 \le y \le 3, y \in \mathbb{R}\}$

Interval Notation

$D=[1, 7]$

$R=[-3, 3]$

B)



Set Notation

$D=\{x \mid x \le 3, x \in \mathbb{R}\}$

Note: the graph is continuous on the left side so do not include

$R=\{y \mid -1 \le y, y \in \mathbb{R}\}$

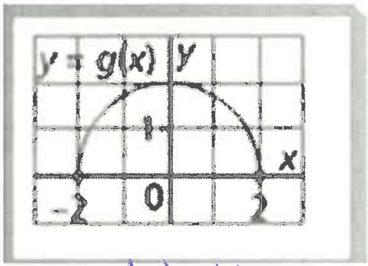
Note! the graph is continuous at the highest point so do not include.

Always use round bracket w infinity

Interval Notation

$D=(-\infty, 3]$ $R=[-1, \infty)$

c)



Set Notation

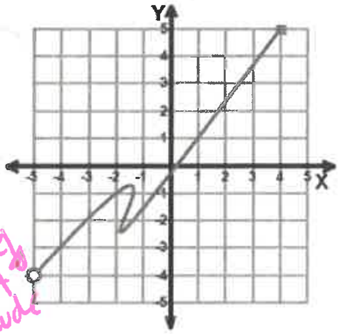
$D=\{x \mid -2 \le x \le 2, x \in \mathbb{R}\}$

$R=\{y \mid 0 \le y \le 1, y \in \mathbb{R}\}$

Interval Notation

$D=[-2, 2]$

$R=[0, 1]$



open circle Do not include

Set Notation

$D=\{x \mid -5 < x \leq 4, x \in \mathbb{R}\}$

$R=\{y \mid -4 < y \leq 5, y \in \mathbb{R}\}$

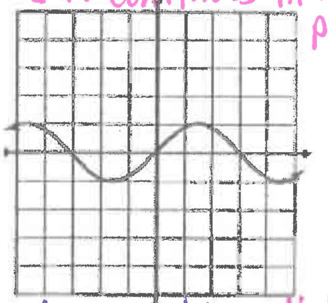
Interval Notation

$D=(-5,4]$

$R=(-4,5]$

Sinusoidal Graph
Assume it continues in this pattern

e)



Set Notation

No left # or right #

$D=\{x \mid x \in \mathbb{R}\}$

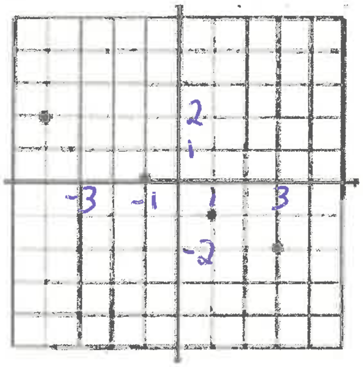
$R=\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$

Interval Notation

$D=(-\infty,\infty)$

$R=[-1,1]$

f)

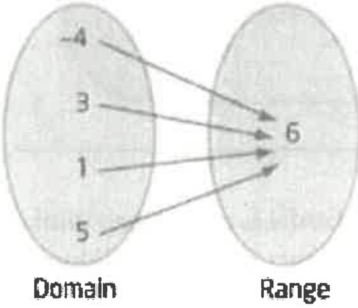


Note: You must use a list to describe domain and range because it is discrete data.

$D=\{-4,-1,1,3\}$

$R=\{-2,-1,0,2\}$

g)



Use a list

$D=\{-4,3,1,5\}$

$R=\{6\}$

h)

{(9, 9), (7, 9), (5, 9), (3, 9)}

Use a list

$D=\{9,7,5,3\}$

$R=\{9\}$

only need to write once

i)

x	y
2	−3
−1	0
5	5
3	2
2	1

only need to list once

Use a list

$D=\{2,-1,5,3\}$

$R=\{-3,0,5,2,1\}$

Topic 6 Day 2 Assignment: Domain and Range Handout

Topic 6 (Day 3) 5.2/5.5 Properties of Functions (Domain, Range and Functions)

Concept #25: 5.2/5.5 Correctly determine the domain and range of linear & non-linear relations using interval notation, set notation lists

Concept #24: 5.2 /5.5 Correctly identify whether a relationship is a function or not with justification

There is a special class of relations, called functions, where two quantities depend on each other in a particular way.

- The amount of tension on a guitar string determines the musical note played.
- The channel displayed on your television screen depends on the number you enter into the remote.

INVESTIGATION

Study the following relations. They are categorized as functions and non-functions.

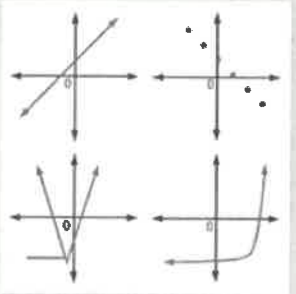
These 8 relations ARE functions

x	y
5	10
6	15
7	20

x	y
11	3
21	3
31	3

{(−2, −5), (0, 4), (2, 13), (4, 22)}

{(10, 10), (12, 10), (14, 12), (16, 12)}



What is similar about the functions?

What is similar about the non-functions?

How can you tell whether or not a relation is a function?

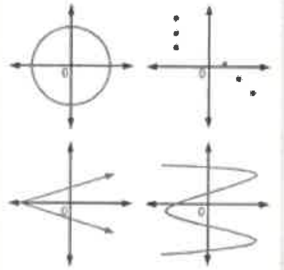
These 8 relations ARE NOT functions

x	y
6	10
6	15
7	20

x	y
3	11
3	21
3	31

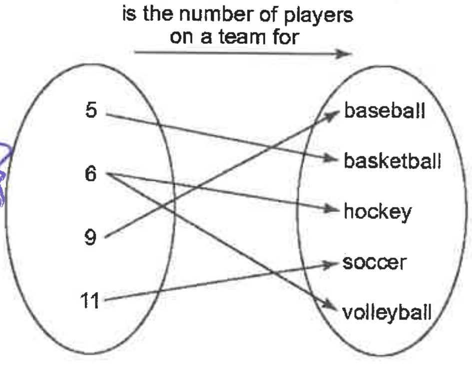
{(10, 10), (12, 10), (12, 14), (12, 16)}

{(7, 5), (7, 8), (9, 11), (11, 14)}



FUNCTION: A specific type of relation that occurs when each element in the DOMAIN is associated with exactly one element in the range. This means that you don't see any repeated elements in the first column of a table, in the first numbers of a set of ordered pairs, or more than one arrow coming from any element in the first oval of an arrow diagram.

Example #1: Answer the following questions using the following Arrow Diagram,



a) State the Domain

D={5, 6, 9, 11}

b) State the Range

R={baseball, Basketball, Hockey, Soccer, Volleyball}

c) Is the given relation is a function? Why or why not?

No, 6 in the domain relates to two different elements in the range

Example #2: The following list of ordered pairs describes the relationship between certain months and number of students in Ms. Sundeen's Math 10 class that have their birthdays in that month.

{(January, 4), (February, 7), (March, 3), (April, 4), (May, 6)}

a) State the Domain

$D=\{Jan, Feb, Mar, Apr, May\}$

b) State the Range

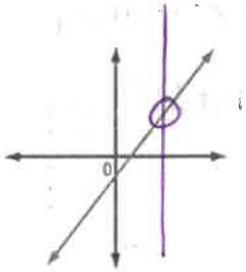
$R=\{4,7,3,4,6\}$

c) Is the given relation a function? Why or Why not?

Yes, each element in the domain is associated with only one element in the range. Nothing repeats in the domain.

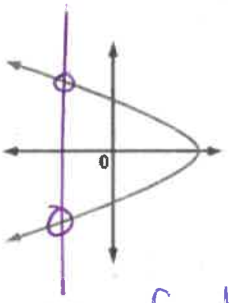
Example #3: For each pair of relations, decide which relation is a function and which relation is not a function. Explain your choice.

a) A



Function because it passes the VLT.

B



Not a function because the vertical line crosses the graph more than once So it fails the VLT

Vertical Line Test (VLT)

- * a test to see if a graph represents a function
- * if any vertical line intersects the graph at more than one point, the relation is not a function.

Hold pencil or ruler vertically at the left edge of the graph. Pass over the graph from left to right. "Does it cross the graph more than once at the same moment?"

Example #4: Determine whether each relation is a function or is not a function. Give a reason for your answer.

a) (−1, 2), (0, 1), (1, 2), (2, 5) Yes

b) (3, 12), (4, 12), (5, 14), (6, 14) Yes

c) (1, 2), (2, 3), (3, 4), (4, 5), (5, 6) Yes

d)

x	y
0	0
1	−1
1	1
4	−2
4	2

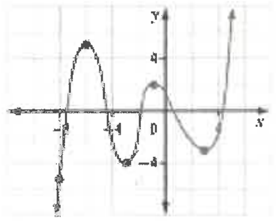
No

e)

Name	Age
Naomi	14
Wam	15
Brigid	14
Sharon	16
Arvind	15

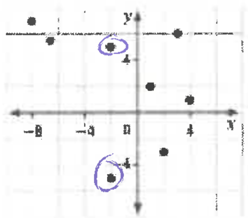
Yes

f)



Yes

g)



No

Topic 6 Day 3 Assignment: **5.2 Page 270 # 4, 5,8 10, 12**
5.5 Pg 294 #5,6,8

5.2 Properties of Functions (Functions Notation) (Day 3)

oncept #26: 5.2 Be able to change between function notation and equations in two variables and how to use function otation to find values

EQUATION IN TWO VARIABLES ⟷ FUNCTION NOTATION

Note: We use function notation to indicate the relation/equation is a function

- Any function that can be written as an equation in two variables can be written in function notation. For example, to write the equation d = 4t + 5(which relates distance and time) in function notation, we may write d(t) = 4t + 5. t represents an element of the domain and d(t) represents an element of the range.

We can use any other letter such as g, h or k to name a function

$$d = 4t + 5 \longleftrightarrow d(t) = 4t + 5$$

- When we write an equation that is not related to a context, we use x as the independent variable and y as the dependent variable. Then an equation in two variables, such as y = 3x - 2 may be written as f(x) = 3x - 2.

$$y = 3x - 2 \longleftrightarrow f(x) = 3x - 2$$

- Conversely, we may write an equation in function notation as an equation in two variables. For example, the equation C(n) = 300 + 25n, we write C = 300 + 25n. And, for the equation g(x) = -2x + 5 we write y = -2x + 5
-

$$C(n) = 300 + 25n \longleftrightarrow C = 300 + 25n$$

Example #1:

a) Write the function f(x) = -5x + 11 as an equation in 2 variables. Note:

$y = -5x + 11$

b) Write the equation y = 2.54x in function notation.

$f(i) = 2.54$

Example #2 Given f(x) = -3x +2

a) Determine f(-6)

$x = -6$

Step ① This is the functions equation. → $f(x) = -3x + 2$

Step ② Substitute $x = -6$ into functions equation.

$f(-6) = -3(-6) + 2$

$f(-6) = 18 + 2$

$f(-6) = 20$

b) Determine x when f(x) = -10

" Substitute $y = -10$ into equation or replace $f(x) = -10$

$-10^{-2} = -3x + 2^{-2}$ ("solve for "x")

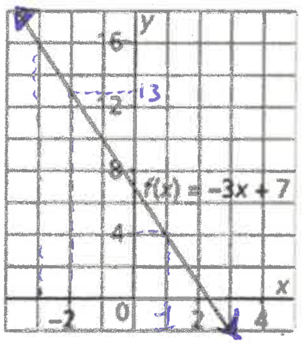
$-12 = -3x$

$\frac{-12}{-3} = \frac{-3x}{-3}$

$x = 4$

$\therefore f(4) = -10$

Example #3:



Here is a graph of the function f(x) = -3x + 7

a) Determine the range value when the domain value is -2

$f(-2)=13$ The range value is 13 when the domain value is -2.

$x=-2$

b) Determine the domain value when the range value is 4.

$f(1)=4$ The domain value is 1 when the range value is 4.

c) Determine the value f(-3)

$f(-3)=16$

"Graphically" or "Algebraically"

$f(-3)=-3(-3)+7$

$f(-3)=9+7$

$f(-3)=16$

Example #4:

The equation V = -0.08d + 50 represents the volume, V litres, of gas remaining in a vehicle's tank after travelling d kilometres. The gas tank is not refilled until it is empty.

a) Describe the function. Write the equation in function form.

The volume of gas remaining in a vehicle's tank is a function of the distance travelled.

In function Notation: $V(d)=-0.08d+50$

b) Determine the value of V(600). What does this number represent?

$x=600$

$V(600)=-0.08(600)+50$

$V(600)=-48+50$

$V(600)=2$

"This means that when the car has travelled 600Km the volume of gas remaining in the tank is 2Litres"

c) Determine the value of d when V(d) = 26. What does this number represent?

↳ This means V=26

It's asking, "If there are 26 Litres of gas left how far did the car travel?"

$V(d)=-0.08d+50$

$26 \overset{-50}{=} -0.08d+50-50$

$\frac{-24}{-0.08}=\frac{-0.08d}{-0.08}$

$d=300$

"This means the car has travelled 300Km when there is 26L left in the tank"

d) In example #4, what is the independent and dependent variable? What is the domain and range?

$\boxed{\text{Independent}}$ = "d" Distance (Km)

$\boxed{\text{Dependent}}$ = "V" Volume of gas in the vehicle's tank (Km)

Domain = $[0,\infty)$

Range = $[0,50]$

5.2 (DAY 3 Function Notation) Assign. Pg 271 #6 ,7, 14, 15, 16, 17 , 18ab, 19

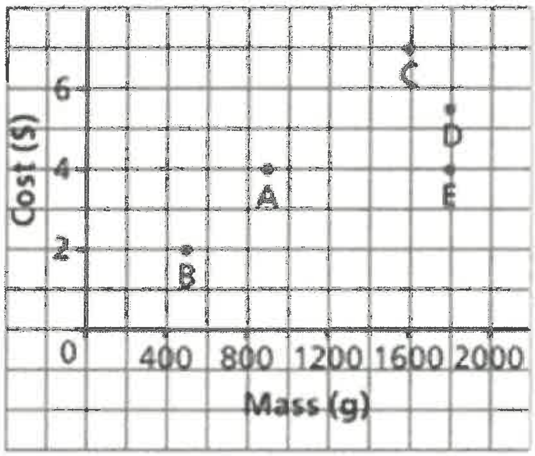
5.3 Interpreting and Sketching Graphs

ncept #27: 5.3 Sketch a graph to represent a situation, interpret a given situation, be able to identify the independent and dependent variables and determine if the data points should or should not be connected on the graph (discrete or continuous)**(NC)**

Interpreting Graphs

Example #1: Each point on this graph represents a bag of popping corn. Explain the answer to each question below.

Costs and Masses of Various Bags of Popcorn



a) Which bag is the most expensive? What does it cost?

Bag "C" $7.00

b) Which bag has the least mass? What is this mass?

Bag "B" 500g

c) Which bags have the same mass? What is this mass?

Bag D+E 1800g

d) Which bags cost the same? What is this cost?

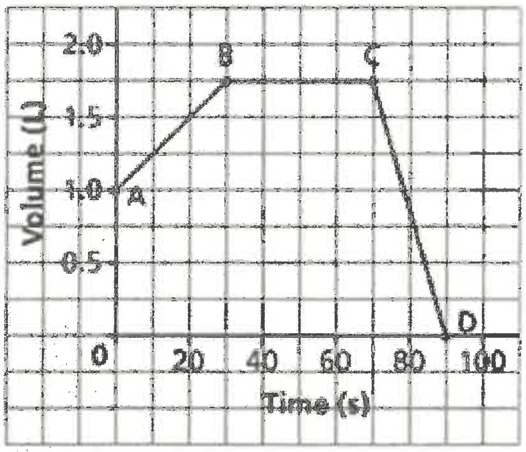
Bag A+E $4.00

e) Which of bags C or D has the better value for money?

Bag D, you get more grams for less.

Example #2: The graph shows how the volume of water in a watering can changes over time.

Volume of Water in a Watering Can



- What is the starting volume? Which point is this?

1.0 Litres at point A

- Describe segment AB and what that means.

The watering can began with 1.0L and is being filled with water till 1.75L

- Describe segment BC and what that means.

The watering cans' volume does not change therefore it may be stationary or being carried to what needs to be watered.

- Describe segment CD and what that means.

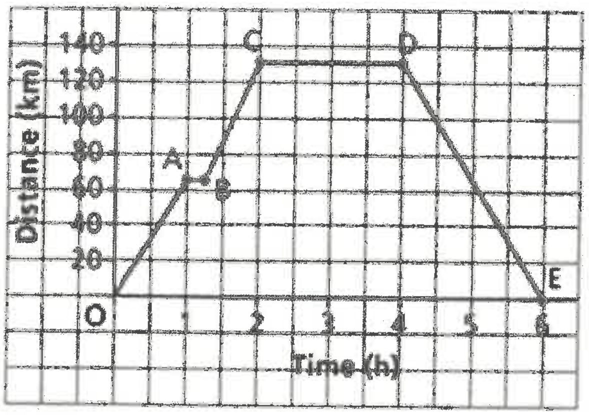
The water is being poured out of the watering can at a faster rate than it was poured in at.

- What does point D represent?

Represents when the watering can is empty.

Example #3: a) Describe the journey for each segment of the graph. Distance from Winnipeg(km) vs Time (hours).

Day Trip from Winnipeg to Winkler, Manitoba



The distance between Winnipeg and Winkler is 130 km.

OA -> In 1hour the car leaves Winnipeg and travels 65Km towards Winkler, MB

AB -> The car stops for approx. 15mins.

BC -> The car continues on it's journey to Winkler travelling 65Km more till it reaches Winkler, MB.

CD -> The car stops in Winkler MB for 2hours

DE -> The car returns to Winnipeg without Stopping. It travels 130Km in 2hours.

b) What was the total driving time? Explain.

Approx 4hours

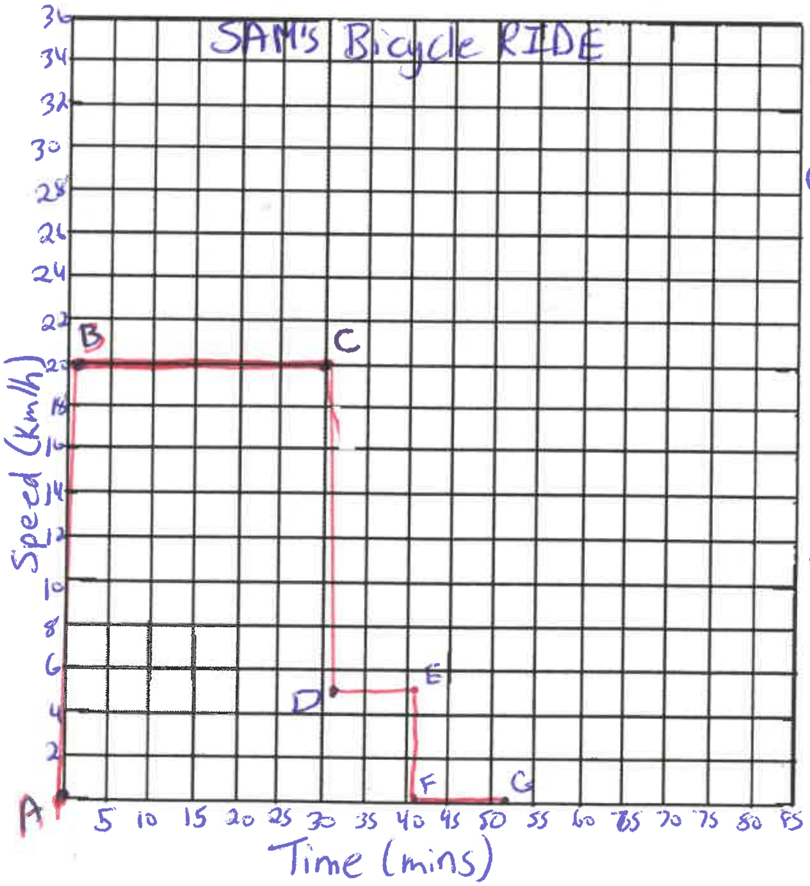
OA = 1hour DE = 2hours.
BC = 45mins

c) What are the dependent and the independent variables? What is the domain and range?

Dependent = Distance from Winnipeg, MB (Km)
Indepent = Time in hours

Sketching Graphs

Example #4: Samuel went on a bicycle ride. He accelerated until he reaches a speed of 20km/h, then he cycled for 30min at approx. 20km/h. Samuel arrived at the bottom of a hill, and his speed decreased to approx. 5km/h for 10 mins as he cycled up the hill. He stopped at the top of the hill for 10 min. Sketch a graph of the speed as a function of time. Label each section for the graph and explain what it represents.



AB -> Sam Accelerated to 20Km/h

BC -> Sam cycles at 20Km/h for 30mins. (His speed does not change, so it is a horizontal line)

CD -> Sam's speed decreases to 5Km/h, so the line segment is very steep.

DE -> Sam cycles uphill @ approx. 5Km/h for 10mins, so the line is horizontal.

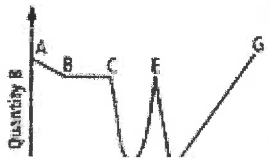
EF -> Sam slows down to 0Km/h, so his speed decreases quickly, so the line goes straight down very steep.

FG -> Sam is stopped at 0Km/h for 10mins, so the line is horizontal.

5.3 Assignment Page 281 #3, 5, 6, 10, 13

Extra Questions: As follows (on looseleaf)

1. The graph shows how quantity B is changing relative to quantity A. Describe each section of the graph as representing a constant increase, a constant decrease, an increase that is not constant, a decrease that is not constant, or no change. Explain your answers.



2. Formats for distributing recorded music have changed through the years. Study the multi-line graph. Predict which line represents each format: vinyl albums, cassette tapes, compact discs, and digital downloads. Explain your choices.