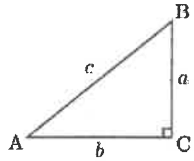
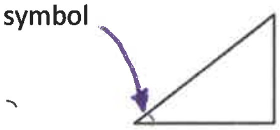
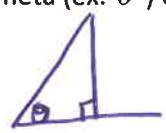
Chapter 2 (DAY 1): 2.1/4 Primary Trig Ratios and using them to find an Angle

Labeling Right Triangles



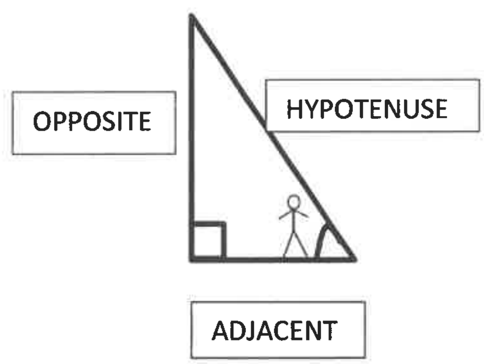
The right triangle will be labeled using letters such as the triangle above. Capital letters are used to label angles while lower case letters are used to label sides

- ***The Angle:*** The size of one angle we use as a reference point to use in our calculations. We will always be given a right angle (as the trig ratios only apply to a right triangle), but will never be the angle we use as a reference point. It will be one of the other two angles. The other angles may be indicated by an actual degree size (ex. 62^0) , a symbol  , the Greek symbol Theta (ex. θ) or capital letter. (as seen in the triangle above)

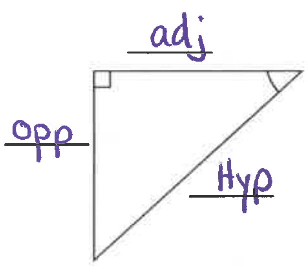


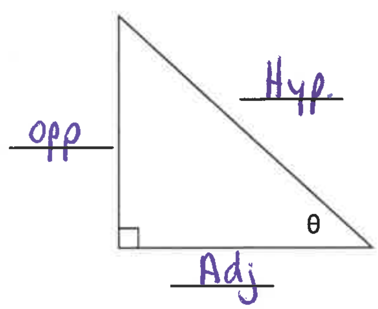
- ***Hypotenuse:*** This is the side of the right triangle directly across from the right angle. It is always the longest side.
- ***Opposite***: Imagine you are standing in the reference angle. The side we call "OPPOSITE" is the side that is directly across from you. The side you cannot touch.
- ***Adjacent***: The word adjacent means "beside". If you are standing in the reference angle , the adjacent side is the side" beside you". The side you could touch.

NOTE: Sometimes the Opposite and Adjacent sides are called the LEGS.



Example#1: Label the following triangles using the terms from above.





As you saw in our Right Triangle ratio activity, no matter the size of the triangle if the angles are the same, the ratio of $\frac{Opposite}{Hypotenuse}$, $\frac{Adajcent}{Hypotenuse}$ and $\frac{Opposite}{Adjacent}$ will always be the same value. So they were given specific names:

$$\text{The Cosine of Angle A} = \frac{\text{Length of the adjacent side}}{\text{Length of the hypotenuse}}$$

$$\text{The Sine of Angle A} = \frac{\text{Length of the opposite side}}{\text{Length of the hypotenuse}}$$

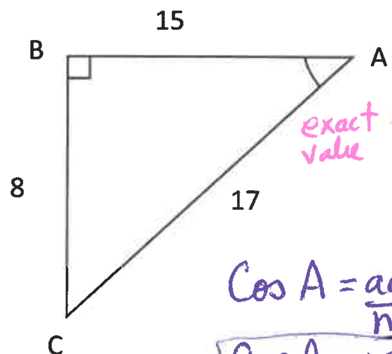
$$\text{The Tangent of Angle A} = \frac{\text{Length of the opposite side}}{\text{Length of the adjacent side}}$$

Remember the acronym to help us remember the trig ratios: SOH CAH TOA

Concept #14: 2.1/2.4 Correctly set up the primary trigonometric ratios (sin, cos, tan) for acute angles in right triangles **(C)(Skill & Problem Solving)**

Example #2: Find the primary trigonometric ratios for the following Triangles. Leave answers in exact value and approx. to four decimal places.

a) Determine tan A, Sin A and Cos A



$\sin A = \frac{opp}{hyp}$ approx.

exact value

$\boxed{\sin A = \frac{8}{17} \approx 0.4706}$

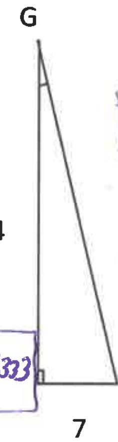
$\cos A = \frac{adj}{hyp}$

$\boxed{\cos A = \frac{15}{17} \approx 0.8824}$

$\tan A = \frac{opp}{adj}$

$\boxed{\tan A = \frac{8}{15} \approx 0.5333}$

b) Determine tan G, Sin G and Cos G



$\tan G = \frac{opp}{adj}$

$\boxed{\tan G = \frac{7}{24} \approx 0.2917}$

Need to calculate the hypotenuse to find the other trig ratios:

$c^2 = 24^2 + 7^2$

$c^2 = 576 + 49$

$\sqrt{c^2} = \sqrt{625}$

$c = 25$

$\sin G = \frac{opp}{hyp}$

$\boxed{\sin G = \frac{7}{25} \approx 0.2800}$

$\cos G = \frac{adj}{hyp}$

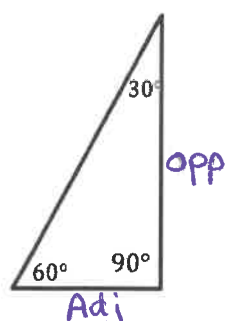
$\boxed{\cos G = \frac{24}{25} \approx 0.9600}$

All of these decimal values of the ratios have been stored in your scientific calculator. Let's check:

- First, make sure your calculator is in the correct "mode". It needs to say Deg, Degree or D at the top. If it says G, Grad, R or Rad, your calculator is in the incorrect mode and needs to fixed!
- Looking at this triangle if you were to approximate the value of $\frac{Opposite}{Adjacent}$ using the tangent ratio of 60^0. Do you think your decimal will be larger or smaller than 1? Larger

  large # (longer side)

  small # (shorter side)



- On your calculator press the following buttons tan and then 60: You should see 1.732050808. If you don't see that, you may need to press the buttons in the order 60 and then tan for your calculator.
- We always round off this decimal from our calculator to 4 decimal places. This will be 1.7321

This means that every triangle in the world with a 60˚angle in the corner will have an opposite side divided by an adjacent side that will always be rounded to 1.7321.

Example #3: Find the measure of each angle to the nearest degree.

Our calculator has the decimal value for every size angle and its tangent, sine and cosine ratios. This question is asking us to look at the question from the opposite direction – we are given the ratio (as a decimal or fractions) that came from dividing the side lengths associated with that ratio. "What size was the angle in the triangle?"

- We are going to have to use the SHIFT or the 2^{nd} button (Tan^{-1}, Sin^{-1}or Cos^{-1}) in our calculator along with tan, sin or cos to find this answer.

- How to do (a): You are either going to press the buttons 0.5418 then SHIFT/2nd then Tan or you are going to press SHIFT/2nd then Tan then 0.5418. Tan^{-1}(Tan inverse is the opposite operation of tan)

a) $\tan A = 0.5418$

$Tan^{-1}(Tan\,A) = Tan^{-1}(0.5418)$

what you put in calc.

$\angle A = 28°$

rounded to the nearest degree

b) $\sin B = \frac{13}{16}$

what you put in calculator

$\sin^{-1}(\sin B) = \sin^{-1}\left(\frac{13}{16}\right)$

$\angle B = 54°$

rounded to the nearest degree

c) $\cos T = \frac{6}{7}$

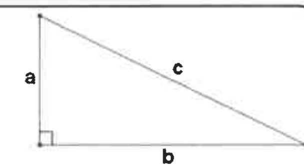
$\cos^{-1}(\cos T) = \cos^{-1}\left(\frac{6}{7}\right)$

$\angle T = 31°$

Things to Remember from Last Year:

All three Angles in a Triangle Add to 180˚.

The Pythagorean Theorem:



$a^2 + b^2 = c^2$

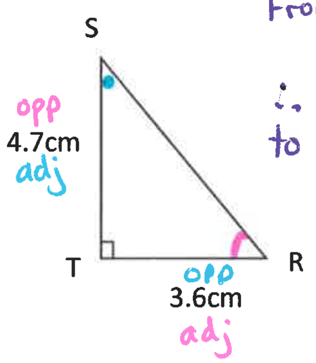
Concept #15: 2.1/2.4 Correctly solve for an acute angle measure in a right triangle using the primary trig ratios **(C) (Skill & Problem Solving)**

Example# 4: Determine the measures of each unknown angle to the nearest tenth of a degree.

a) Determine $\angle$**R and** $\angle$**S**

Step1

From $\angle R$ 4.7cm = opp
3.6cm = adj
∴ use the Tangent Ratio to solve for the angle.



$Tan\,R = \frac{opp}{adj}$

What you put into calc.

$Tan\,R = \frac{4.7}{3.6}$

$Tan^{-1}(Tan\,R) = Tan^{-1}\left(\frac{4.7}{3.6}\right)$

$\boxed{\angle R = 52.5°}$

rounded to the nearest tenth

Step2

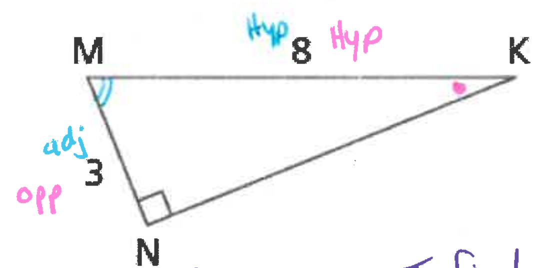
From $\angle S$ 4.7=adj 3.6=opp
∴ use Tangent ratio

$Tan\,S = \frac{3.6}{4.7}$

$Tan^{-1}(Tan\,S) = Tan^{-1}\left(\frac{3.6}{4.7}\right)$

$\boxed{\angle S = 37.5°}$

b) Determine $\angle$ **M and** $\angle$**K**



To find $\angle M$
3 = adj 8 = Hyp
∴ use Cosine ratio

$Cos\,M = \frac{3}{8}$

$Cos^{-1}(cos\,M) = Cos^{-1}\left(\frac{3}{8}\right)$

$\boxed{\angle M = 68.0°}$

To find $\angle K$
3 = opp 8 = Hyp
∴ use sine ratio

$Sin\,K = \frac{3}{8}$

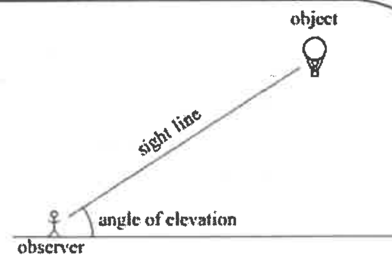
$Sin^{-1}(Sin\,K) = Sin^{-1}\left(\frac{3}{8}\right)$

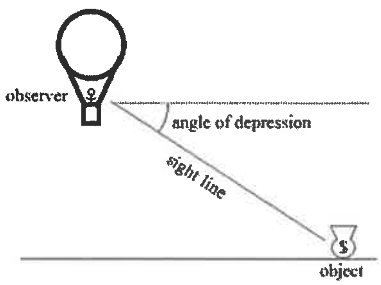
$\boxed{\angle K = 22.0°}$

Other Terms to Learn:

Acute Angle: An angle whose size is between 0˚ and 90˚

Angle of Inclination (Also known as Angle of Elevation):
The angle measured between a horizontal line and a line angling upwards.

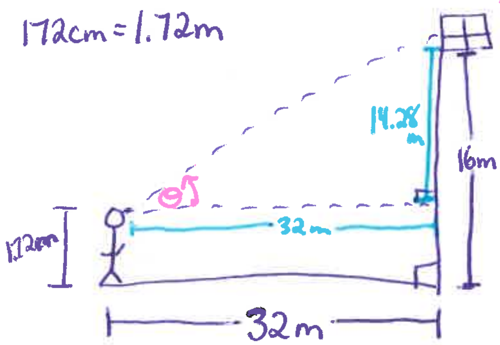




Angle of Depression (Also known as Angle of Elevation):
The angle measured between a horizontal line and a line angling upwards.

Example 5: **Mrs. Sundeen is standing in the teacher parking lot in front of her vehicle which is 32 m away from the school and directly across from her classroom. She looks up to the second floor and sees the windows of her classroom which are 16 m high (and is annoyed when she sees her students hanging out of the windows instead of doing their work). What is the angle of inclination that Mrs. Sundeen is looking up at? Leave your answer to the nearest hundredth. (Note: Mrs. Sundeen is 172cm tall)** Note: Make sure all units are the same before calculating

172cm = 1.72m



16m − 1.72m
= 14.28m

$$\tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{14.28}{32}\right)$$

$$\theta = 24.05^\circ$$

The angle of inclination that Mrs. Sundeen is looking up at is 24.05°

STEPS:

1. Draw a diagram. Be sure that your angle of elevation contains a horizontal line _____ and a line slanting upwards. Put all given numerical information on the diagram.
2. Label your angle, your hypotenuse, your opposite and your adjacent
3. Write down the formula

$$\tan A = \frac{Opp}{Adj}$$

4. Fill in the formula with information from your triangle. Divide your fraction.
5. Use the SHIFT/2nd procedure to find your angle.
6. Our answer is to hundredths which means round to two decimals!

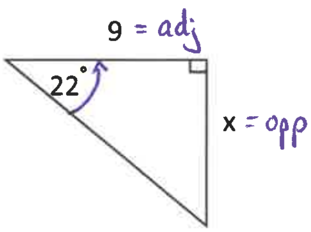
Chapter 2 DAY 1 - 2.1/4 Assignment: Page 75 #3ac, 4ad, 5ab, 8(no need to describe solution method), 10bd,14, 16, 17
Pg 95 #4, 6ad, 7bc, 8ad, 10,12,13

Chapter 2 DAY 2 : 2.2/2.4/2.5 Using the Primary Trig Ratios to Calculate Lengths

Concept #16: 2.2/2.5 Correctly solve for a side length in a right triangle (using primary trig ratios and/or the Pythagorean Theorem) & solving entire triangles **(C) (Skill & Problem Solving)**

STEPS:
1. Label all of your given information using: angle, hypotenuse, opposite, adjacent.
2. Decide which formula based on the info your given.
3. Fill in the formula with information from your triangle.
4. Use your "Tan, sin or cos" button to find the "Tan, sin or cos" of the angle. Do not use your shift or 2nd key when you know the number beside the word Tan!
5. Cross multiply to find the answer to your unknown side!

Example#1: a) Find the length of x to the nearest tenth.



$$\tan 22^\circ = \frac{opp}{adj}$$

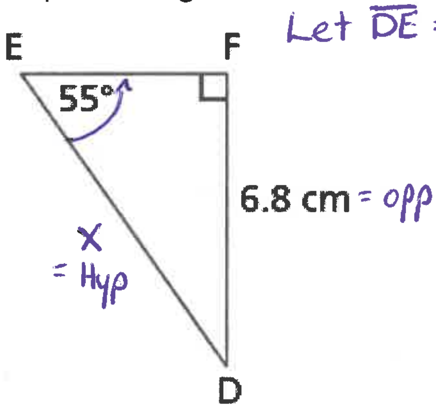
$$(9)(\tan 22^\circ) = \frac{x}{\cancel{9}}\ (9)$$

This is what you punch into the calc. → $(9)(\tan 22^\circ) = x$

$$\boxed{3.6 \text{ units} = x}$$

rounded to the nearest tenth

b) Find length DE to the nearest tenth of a cm.



Let $\overline{DE} = x$

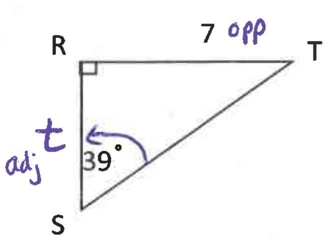
$$(x)(\sin 55^\circ) = \frac{6.8}{\cancel{x}}\ (x)$$

$$\frac{(x)(\sin 55^\circ)}{(\sin 55^\circ)} = \frac{6.8}{(\sin 55^\circ)}$$

$$x = \frac{6.8\text{ cm}}{\sin 55^\circ}$$

$$\boxed{x = 8.3\text{ cm}}$$

Example #3: Find the length of RS to the nearest hundredth.



Let $RS = t$

$7 = opp$ $t = adj$

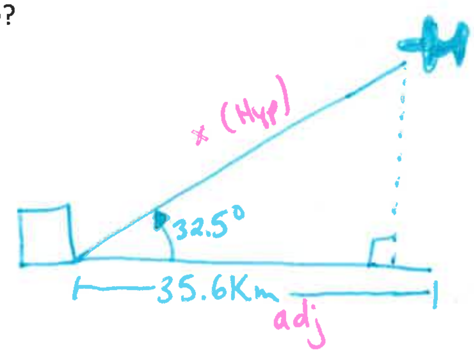
$$\frac{\tan 39}{1} = \frac{7}{t}$$ cross multiply

$$\frac{t(\tan 39^\circ)}{\cancel{\tan 39^\circ}} = \frac{7}{\tan 39^\circ}$$

$$t = \frac{7}{\tan 39^\circ}$$

$$\boxed{t = 8.64 \text{ units}}$$

Example #4: From a radar station, the angle of elevation of an approaching airplane is 32.5⁰. The horizontal distance between the plane and the radar station is 35.6km. How far is the plane form the radar station to the nearest tenth of a kilometre?



$(x)(\cos 32.5°) = \frac{35.6}{\cancel{x}}\cancel{(x)}$

$\frac{(x)\cancel{(\cos 32.5°)}}{\cancel{\cos 32.5°}} = \frac{35.6}{\cos 32.5°}$

$x = \frac{35.6}{\cos 32.5°}$

$x = 42.2\text{Km}$

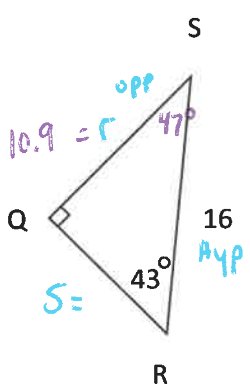
The Plane is 42.2Km from the radar station.

Ch. 2 Day 2 Assignment Pg 82 #3ac,4ab, 5b,8, Pg 95 #5ac, Pg 101#5ac,7,

Chapter 2 Day 3 :2.2/2.4/2.5 Using the Primary Trig Ratios to Calculate Lengths

Concept #16: 2.2/2.5 Correctly solve for a side length in a right triangle (using primary trig ratios and/or the Pythagorean Theorem) & solving entire triangles **(C) (Skill & Problem Solving)**

Example #1: Solve ΔRQS . State the measures to the nearest tenth. (Note: When asked to solve a triangle you are to find all the angles and side lengths of that triangle)



Find Side $\overline{QS}$ or side r

$(16)(\sin 43°) = \frac{r}{\cancel{16}}\cancel{(16)}$

$(16)(\sin 43) = r$

$\boxed{10.9\text{units} = r}$

Find Angle S

$\angle S = 180 - 43° - 90°$

$\boxed{\angle S = 47.0°}$

Find Side $\overline{QR}$ or side s

To find the last side you could use : - pythagorean theorem

★ Remember to use exact values though★ - Cosine ratio

- Sine ratio

- Tangent ratio

$(16)\cos 43° = \frac{s}{\cancel{16}}\cancel{(16)}$

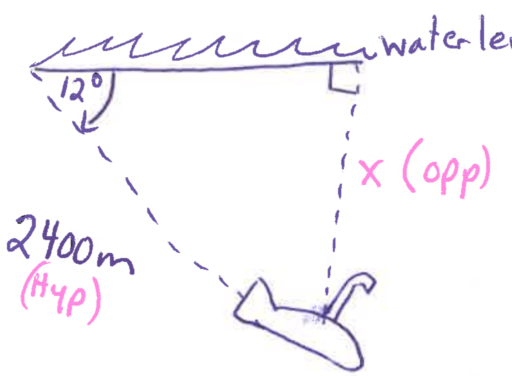
$(16)(\cos 43°) = s$

$\boxed{11.7u = s}$

u = units

Remember:
Lowercase letters can be used to label side lengths

Example #2: A submarine is diving below the surface of the ocean at an angle of depression of 12^0. If the submarine travelled 2400m along its dive path, how far will it be below the surface?



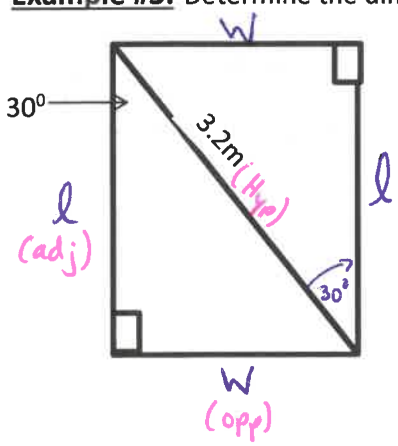
$$(2400m)\sin 12^\circ = \frac{x}{2400m}(2400m)$$

$$(2400m)(\sin 12^\circ) = x$$

$$498.988m = x$$

The submarine will be 498.988m below the surface of the water.

Example #3: Determine the dimensions of this rectangle to the nearest tenth of a centimetre.



To find the width:

$$(3.2)\sin 30^\circ = \frac{w}{\cancel{3.2}}\cancel{(3.2)}$$

$$(3.2)(\sin 30^\circ) = w$$

$$1.6m = w$$

To find the length:

$$(3.2)\cos 30^\circ = \frac{l}{\cancel{3.2}}\cancel{(3.2)}$$

$$(3.2)(\cos 30^\circ) = l$$

$$2.8m = l$$

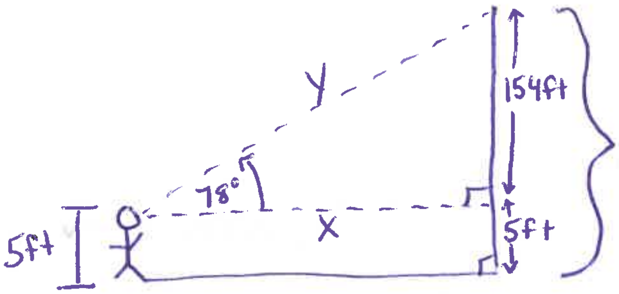
The dimensions of the rectangle are 1.6m by 2.8m.

Example #4: Mrs. Sundeen was walking towards the Colosseum in Rome , and as she had brought her handy-dandy math tools she whipped out and measured the angle of inclination to be 78° from her eyelevel to the top of the Colosseum.The Colosseum is 159ft tall and Mrs. Sundeen's eyeball is 5ft from the ground.

a) How far was she from the Colosseum at the time of her measuring?

b) How far was her eyeball from the top of the Colosseum?

c) How embarrassed was her husband at this show of math



a) $\tan 78° = \dfrac{154ft}{X}$

$(154)\dfrac{1}{\tan 78°} = \dfrac{X}{154ft}(154)$

$\dfrac{154}{\tan 78°} = X$

$32.734ft \approx X$

b) $(y)\sin 78° = \dfrac{154}{\cancel{y}}\cancel{(y)}$

$\dfrac{(y)(\sin 78°)}{\sin 78°} = \dfrac{154}{\sin 78}$

$y = \dfrac{154}{\sin 78}$

$y \approx 157.440ft$

c) Not at all, he's a math teacher too.

CH. 2 Day 3 Assignment Pg 101 #6, 12a Pg 127 #4 Pg 82#7, 11, 14

2.6/2.7 Applying Trigonometric ratios and Solving problems with more than one triangle

In this section, we will use Sin, Cos and Tan to Solve Right Triangles. Solving a right triangle means that at the end of each question you will know the length of all three sides and the size of all three angles. We can use the following to help us solve the triangles:

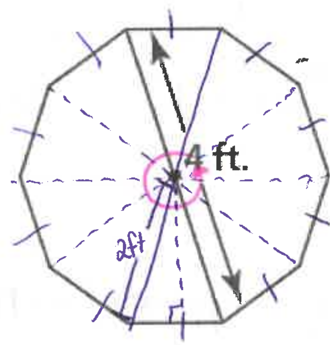
$$\text{Sin}A=\frac{\text{opp}}{\text{hyp}},\quad \text{Cos}\,A=\frac{\text{adj}}{\text{hyp}},\quad \text{Tan}\,A=\frac{\text{opp}}{\text{adj}},\quad a^2+b^2=c^2,\quad \measuredangle A+\measuredangle B+\measuredangle C=180°$$

Concept #17 2.6 & 2.7 Solve problems involving one or more than one right triangle **(C) (Skill & Problem Solving)**

Example #1: A window has the shape of a regular decagon. The distance from one vertex to the opposite vertex, measured through the centre of the window is approximately 4ft. Determine the length of the wood moulding material that forms the frame of the window, to the nearest foot.

Note: Decagon is a ten-sided polygon

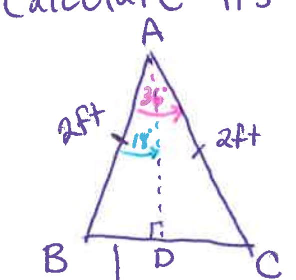
To find the length of the wood moulding material that frames the window we need to Calculate it's perimeter

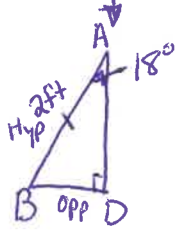


① To Find ∠A:

$360° \div 10$

$= 36°$





② To find $\overline{BD}$ use the sine ratio

$(2)\ \text{Sin}18° = \frac{\overline{BD}}{2}\ (2)$

$(2)(\text{Sin}18) = \overline{BD}$

$(2)(\text{Sin}18°) = \overline{BD}$

③ To Find $\overline{BC}$ double $\overline{BD}$

$\overline{BC} = 2(\overline{BD})$

$\overline{BC} = 2(2)(\text{Sin}18°)$

$\overline{BC} = 4\ (\text{Sin}18°)$

④ The perimeter of the Decagon will be $\overline{BC}$ times 10.

$P = 10\,\overline{BC}$

$P = 10(4)(\text{Sin}18°)$

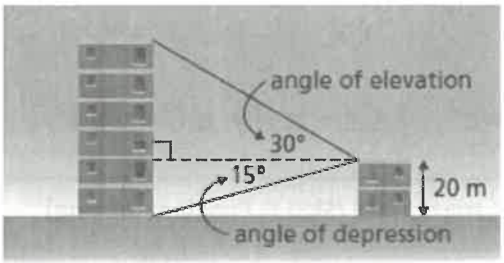
$P \approx 12.3606\ldots$ ft

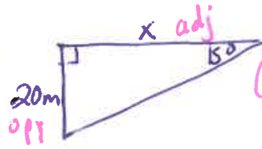
Rounded to the nearest ft is 12ft

The length of the wood moulding you need to frame the decagon window is approx. 12ft.

Example #2: From the top of a 20m high building, a surveyor measured the angle of elevation ~~of~~ to the top of another building and the angle of depression of the base of that building. The surveyor sketched this plan of her measurements. Determine the height of the taller building to the nearest tenth of a metre.

① Let x = distance between buildings





$$(x)\ \tan 15° = \frac{20}{x}\ \cancel{(x)}$$

$$\frac{(x)\ \tan 15°}{(\tan 15°)} = \frac{20}{(\tan 15°)}$$

$$x = \left(\frac{20}{\tan 15°}\right) \text{ meters.}$$

② Let y = Height of building minus smaller building

$$\left(\frac{30}{\tan 15°}\right) \tan 30 = \frac{y}{\left(\frac{30}{\tan 15°}\right)}\ \cancel{\left(\frac{30}{\tan 15°}\right)}$$

$$(\tan 30)\left(\frac{30}{\tan 15°}\right) = y$$

③ Height of Taller building

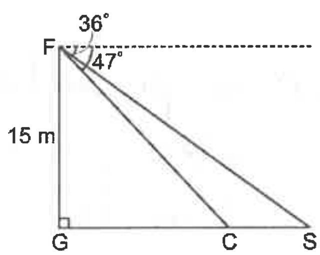
$$H = 20 + \left[\tan 30 \left(\frac{30}{\tan 15°}\right)\right]$$

$$H = 63.1\text{m}$$

$\boxed{\text{The taller building is approx. 63.1m high}}$

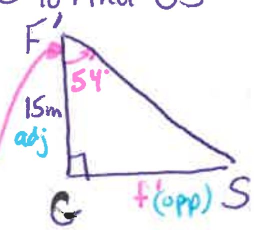
Example #3:

This diagram shows a falcon, F, on a tree, with a squirrel, S, and a chipmunk, C, on the ground. From the falcon, the angles of depression of the animals are 36° and 47°. How far apart are the animals on the ground to the nearest tenth of a metre?



① To find $\overline{CS}$, we need to find $\overline{GS}$ and $\overline{GC}$

② To find $\overline{GS}$

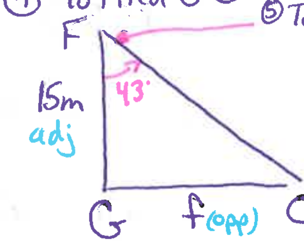


③ To find ∠F'
$90 - 36° = 54°$

$$(15)\ \tan 54° = \frac{f'}{15}\ \cancel{(15)}$$

$$(15)\tan 54° = f'$$

④ To find $\overline{GC}$



⑤ To find ∠F
$90° - 47° = 43°$

$$(15)\ \tan 43° = \frac{f}{15}\ \cancel{(15)}$$

$$(15)\ \tan 43° = f$$

⑥ $\overline{CS} = \overline{GS} - \overline{GC}$

distance between the animals → $\overline{CS} = [(15)(\tan 54°)] - [(15)(\tan 43)]$ $\overline{CS} = 6.7\text{m}$

Chapter 2 Day 4 Assignment 2.6 Pg 111 #6, 7, 8, 11, 12a (#14 (Extension question))

The animals are approx. 6.7m apart.

2.7 Assignment Pg 118 # 3a,4c,5a,9,11,14