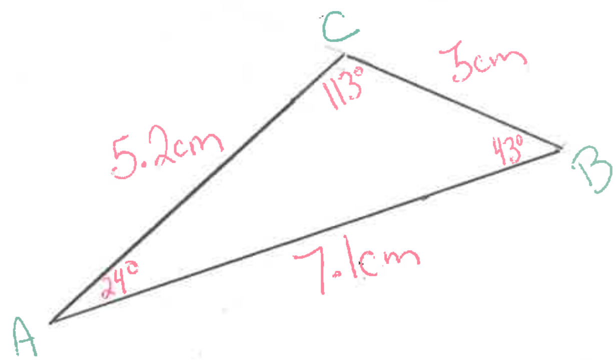
2.3 – The Sine Law

1. Read Page 100 in text (2 paragraphs in the middle of the page)

2. Define Oblique Triangle- Any triangle that does not contain a right angle.

3. Draw an oblique triangle. Label it ΔABC. Measure all sides and angles using a ruler and protractor. (Remember : side "a" is opposite $\angle$A, side "b" is opposite $\angle$B, side "c" is opposite $\angle$C)



4. Find the ratio of the sine of each angle with its corresponding side: (Round to the nearest Hundredth)

$\frac{\sin A}{a} = \frac{\sin 24°}{3cm} = 0.14$

$\frac{\sin B}{b} = \frac{\sin 43°}{5.2cm} = 0.13$

$\frac{\sin C}{c} = \frac{\sin 113°}{7.1cm} = 0.13$

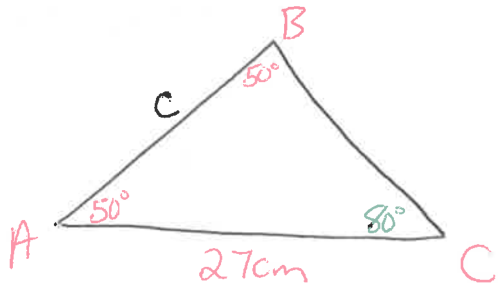
Students will have different answers but ratios should all equal the same value

Sine Law: For any triangle ΔABC where a, b, c are the sides opposite $\angle$A, $\angle$B, $\angle$C respectively:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Ex#1/ Sketch the triangle and determine the measure of the indicated side:

ΔABC $\angle A = 50^0$ $\angle B = 50^0$ and AC = 27 cm. Find AB. Round to the nearest tenth of a centimeter.



① Find $\angle C$

$180 - 50 - 50$

$\boxed{\angle C = 80^\circ}$

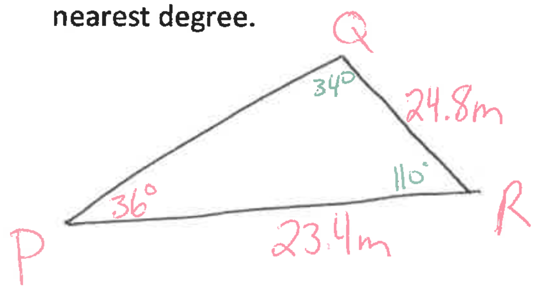
② Find AB or side c

$(\sin 80)\ \frac{c}{\sin 80} = \frac{27}{\sin 50}(\sin 80)$

$c = \frac{27(\sin 80^\circ)}{\sin 50^\circ}$

$\boxed{c = 34.7\text{cm}}$

Ex#2 / In ΔPQR $\angle P = 36^0$ p= 24.8m and q=23.4m. Determine the measure of $\angle R$, to the nearest degree.



① Find $\angle Q$

$(23.4)\ \frac{\sin Q}{23.4\text{m}} = \frac{\sin 36^\circ}{24.8}(23.4)$

$\sin^{-1}\left[\frac{\sin 36^\circ(23.4)}{24.8}\right]$

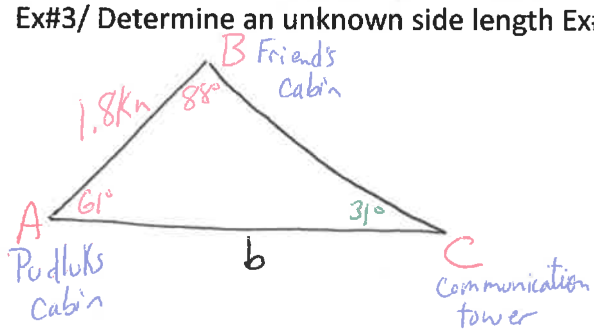
$\boxed{\theta \text{ or } \angle Q = 34^\circ}$

② $\angle R$

$180 - 34 - 36$

$\boxed{\angle R = 110^\circ}$

Ex#3/ Determine an unknown side length Ex#1 on Pg 102.



① $\angle C$

$180 - 88 - 61$

$\angle C = 31^\circ$

② $(\sin 88)\ \frac{b}{\sin 88} = \frac{1.8\text{Km}}{\sin 31^\circ}(\sin 88)$

$b = \frac{1.8(\sin 88^\circ)}{\sin 31^\circ}$

$\boxed{b = 3.5\text{Km}}$

<u>Assignment</u>: Page 108 #1ac, 2, 3b, 4bd, 5ab, 10, 12

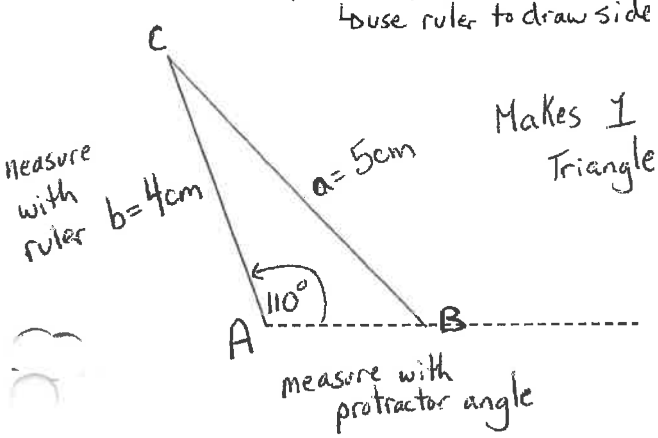
2.3 – Sine Law : The Ambiguous Case

The Sine Law gives the relationship between the sides and the angles of a triangle. If you are given two angles and one side, you can use the Law of Sines to find the lengths of the two unknown sides (ASA or AAS triangles).

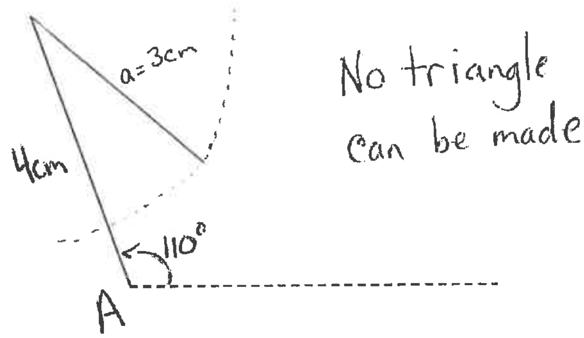
You can get into trouble when you are given two sides and one opposite angle (SSA triangles) in order to find the other opposite angle as the combination of side lengths and the angle measure does not always produce one unique triangle. Due to this uncertainty, this is called the Ambiguous Case.

Construct triangle ΔABC given the following information. *Use Compass and ruler*

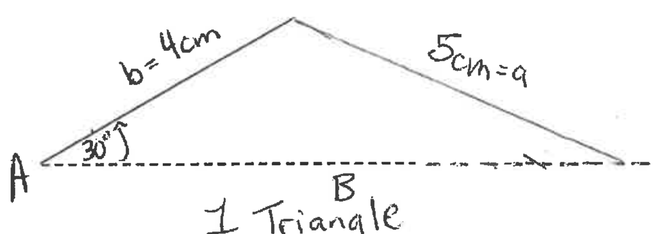
$\angle A = 110°$, $a = 5\text{cm}$, $b = 4\text{cm}$



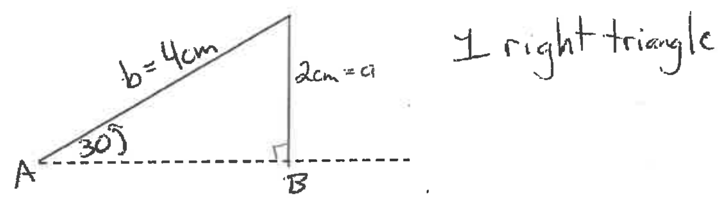
$\angle A = 110°$, $a = 3\text{cm}$, $b = 4\text{cm}$



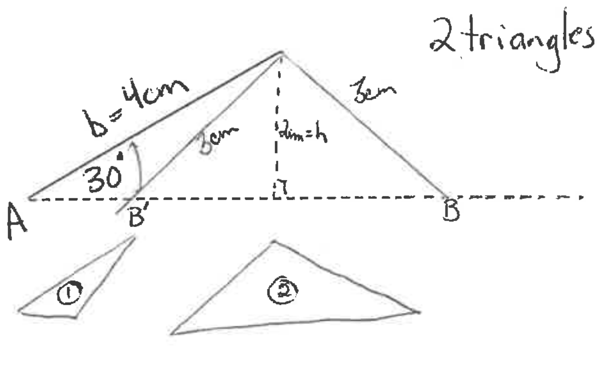
$\angle A = 30°$, $a = 5\text{cm}$, $b = 4\text{cm}$



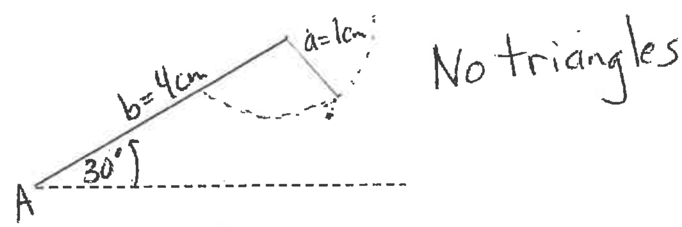
$\angle A = 30°$, $a = 2\text{cm}$, $b = 4\text{cm}$



$\angle A = 30°$, $a = 3\text{cm}$, $b = 4\text{cm}$



$\angle A = 30°$, $a = 1\text{cm}$, $b = 4\text{cm}$



Three different situations can arise when given ASS information: one triangle formed, two triangles formed or no possible triangle. In order to determine the number of triangles, you must first consider the type of angle then the lengths of the adjacent and opposite sides.

Type of Angle
↓
Relationship of the Given Sides
↓
Number of Triangles that can be formed
↓
Number of Solutions

Given an Obtuse Angle

Case 1

If the opp > adj

Note: opp = opposite side to the given angle
adj = adj side that was giving to the given angle

1 Triangle

Case 2

If opp ≤ adj

No triangles

Given an Acute Angle

Case 1

If opp ≥ adj

1 Triangle

Case 2

If opp < adj

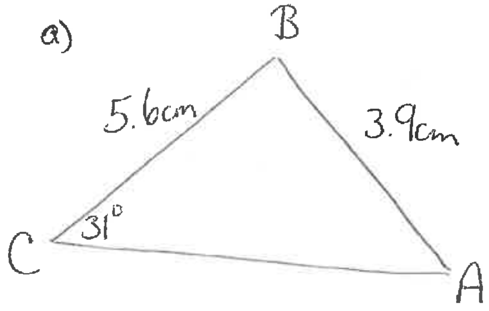
① Find "h" $h = (\sin A)\,\text{adj}$

- opp < height: NO triangles
- opp = height: 1 right triangle
- opp > height: 2 Triangles

Ex #1/ Given ΔABC $\angle C = 31°$, a = 5.6cm and c=3.9cm

a) Draw a diagram.
b) Decide how many solutions.
c) Determine the unknown sides and angles.

a)



b) side opposite the given angle ↓ opp < adj

Need to check "h"

$h = (\sin 31°)\,5.6$

$\boxed{h = 2.9\text{cm}}$

Since h < opp there are two triangles

Case 1 (acute ∠A)

(Use Sine Law)

∠A =

$$\frac{(5.6\text{cm})\sin 31°}{3.9\text{cm}} = \frac{\sin\angle A\,\cancel{(5.6\text{cm})}}{\cancel{5.6\text{cm}}}$$

$$\sin^{-1}\left[\frac{(5.6\text{cm})(\sin 31°)}{3.9\text{cm}}\right] = \sin^{-1}\cdot\sin A$$

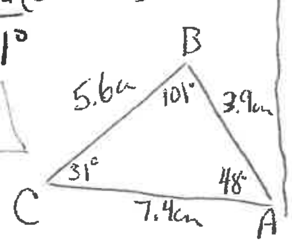
$\boxed{\angle A = 48°}$

$\boxed{\angle B = 180 - 31 - 48 = 101°}$

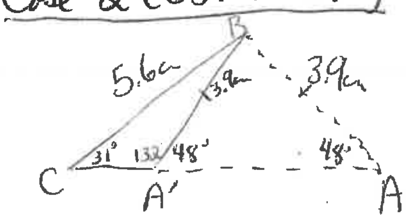
side b (use Sine Law)

$$\frac{(\sin 101°)\,b}{\sin 101°} = \frac{3.9\text{cm}(\sin 101°)}{\sin 31°}$$

$\boxed{b = 7.4\text{cm}}$



Case 2 (obtuse ∠A)



ΔAA'B is an isosceles triangle

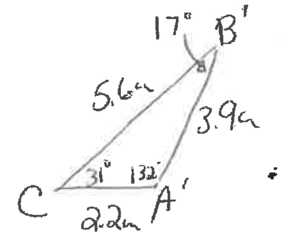
$\angle A' = 180 - 48 = \boxed{132°}$

$\angle B' = 180 - 31 - 132 = \boxed{17°}$

side b'

$$\frac{(\sin 17°)\,b}{\cancel{\sin 17°}} = \frac{3.9(\sin 17°)}{\sin 31°}$$

$\boxed{b = 2.2\text{cm}}$

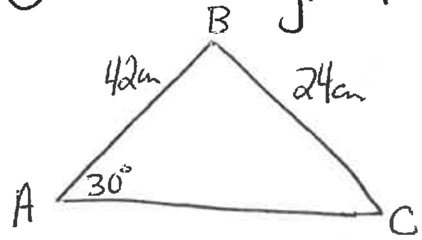


Note: to check answers you can compare ratios

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Ex #2/ Given ΔABC $\angle A = 30°$, a = 24cm and c=42cm. Determine the measures of the other sides and angles. Round your answers to the nearest unit.

① Draw a diagram and label angles and sides



② Decide how many solutions (# of triangles)

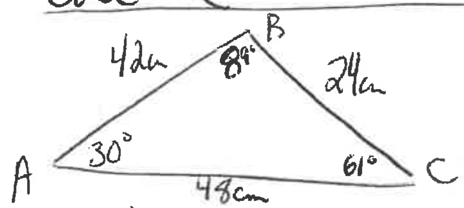
opp < adj so we need to find "h"

$h = 42(\sin 30°)$

$h = 21cm$

Since $h <$ opposite side there are 2 solutions

Case 1 ($\angle C$ is an acute angle)



① $\angle C = (42cm) \dfrac{\sin 30°}{24cm} = \dfrac{\sin C}{42cm}(42cm)$

$\sin^{-1} \dfrac{(42cm)(\sin 30°)}{24cm} = \sin^{-1} \sin C$

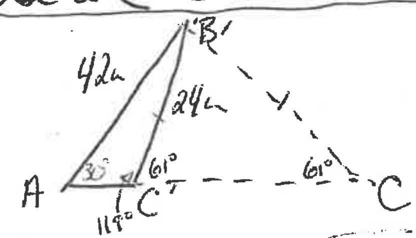
$\boxed{61° = \angle C}$

② $\boxed{\angle B = 180 - 61 - 30 = 89°}$

③ side b $(\sin 89)\dfrac{24cm}{\sin 30°} = \dfrac{b}{\sin 89°}(\sin 89)$

$\boxed{48cm = b}$

Case 2 ($\angle C$ is an obtuse angle)

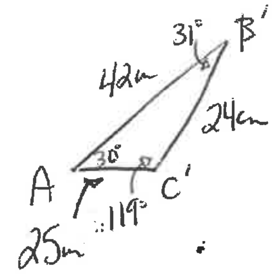


① $\angle C' = 180 - 61 = \boxed{119°}$

② $\angle B' = 180 - 30 - 119° = \boxed{31°}$

③ Side b' $(\sin 31°)\dfrac{24}{\sin 30} = \dfrac{c}{\sin 31°}(\sin 31°)$

$\boxed{25cm = c}$



Assignment: Handout & pg 108 #6, 8bc, 11

2.4 Cosine Law

Given $\triangle ABC$ is any oblique triangle:

$$c^2 = a^2 + b^2 - 2ab\,\text{Cos}\,C$$
$$b^2 = a^2 + c^2 - 2\overset{ac}{\cancel{ab}}\,\text{Cos}\,B$$
$$a^2 = b^2 + c^2 - 2\overset{bc}{\cancel{ab}}\,\text{Cos}\,A$$

If given SAS use the Cosine law to find the unknown side

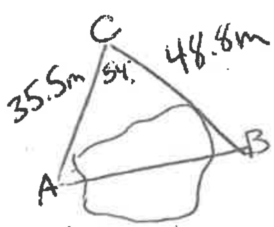
If given SSS use the Cosine law to find a particular angle

Use the sine law to solve the rest of the unknowns in the triangle

Hint: 1) Given all 3 sides (SSS), find the size of the largest angle first using Law of Cosines.
2) When using Law of Sines for finding angles, find smallest or second smallest angle first as you know they will be acute. The largest angle may be obtuse or acute.
3) The sum of the lengths of any 2 sides in a triangle must always be greater than the 3rd side.

Your Turn (page 117)

Nina wants to find the distance between two points, A and B , on opposite sides of a pond. She locates a point C that is 35.5m from A and 48.8m from B. If the angle at C is 54^0, determine the distance AB, to the nearest tenth of a metre.

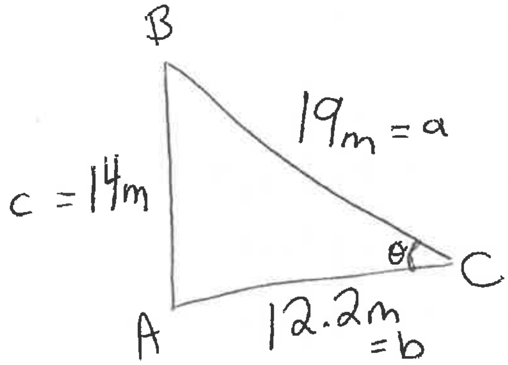


$$c^2 = a^2 + b^2 - 2ab\,\text{Cos}\,C$$
$$c^2 = 48.8^2 + 35.5^2 - 2(48.8)(35.5)\,\text{Cos}\,54$$
$$= 2381.44 + 1260.25 - 3464.8(\text{Cos}\,54)$$
$$\sqrt{c^2} = 3641.69 - 3464.8(\text{Cos}\,54^\circ)$$
$$\sqrt{c^2} = \sqrt{1605.131\ldots}$$
$$c = 40.1\text{m}$$

The distance is 40.1m

Example 2 (page 117)

The Lions Gate Bridge has been a Vancouver landmark since it opened in 1938. It is the longest suspension bridge in Western Canada. The bridge is strengthened by triangular braces. Suppose one brance has side lengths 14m, 19m and 12,2 m. Determine the measure of the anlge opposite the 14m side to the nearest degree.



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$14^2 = 19^2 + 12.2^2 - 2(19)(12.2)\cos C$$

$$196 = 361 + 148.84 - 463.6\cos C$$

$$-361-148.84 \quad -361 \quad -148.84$$

$$\frac{-313.84}{-463.6} = \frac{-463.6\cos C}{-463.6}$$

$$\angle C = \cos^{-1}\left(\frac{-313.84}{-463.6}\right)$$

$$\angle C = 47.3^\circ$$

$$= 47^\circ$$

Assignment: Page 119 #1ab, 2ab, 3ab, ~~7~~, 12, 15, ~~19~~