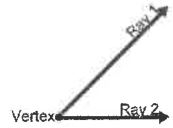
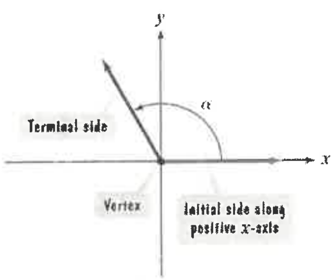
Fill in.

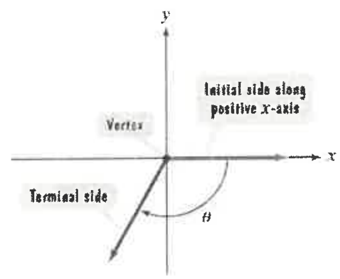
2.1 Angles in Standard Position and Special Triangles

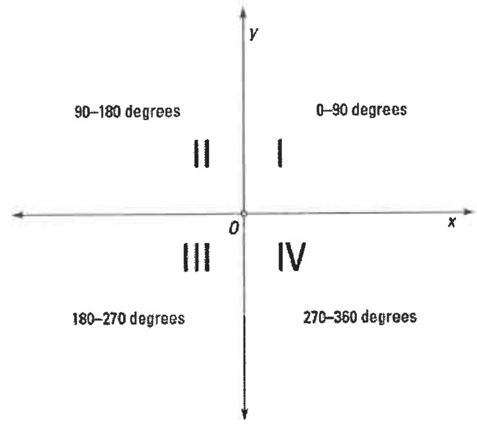
- In **GEOMETRY**, an angle is formed by two rays having a common endpoint



- In **TRIGONOMETRY** angles are often interpreted as rotations of a ray about a vertex.
  - These rotations are said to be in **STANDARD POSITION** when the vertex is located at the origin of the Cartesian plane.
  - In standard position the initial location of the ray is located on the positive x axis and is called the **INITIAL ARM**
  - Positive angles in standard position are created by rotating the initial arm counterclockwise about the vertex. The end location of the ray is called the **TERMINAL ARM.**
  - If the rotation of the terminal arm is clockwise, the angle has a negative measure





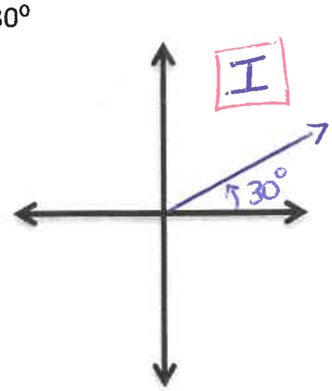


- We indicate the quadrant that an angle belongs in by the quadrant where the terminal arm is located
- If the terminal arm of an angle lies along an axis, then it is called a **QUADRANTAL ANGLE.**

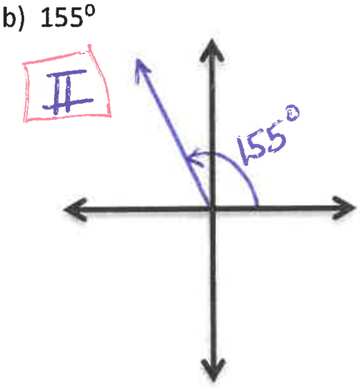
READ PART A #1 and 2 on pg 75 of textbook

EX #1/ Sketch each angle in standard position and state what Quadrant they lie in:

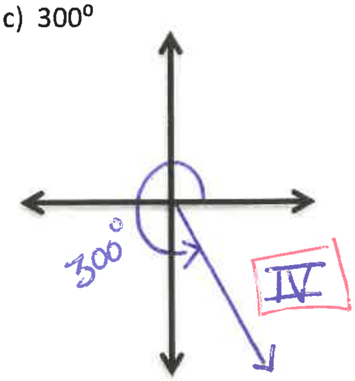
a) 30°



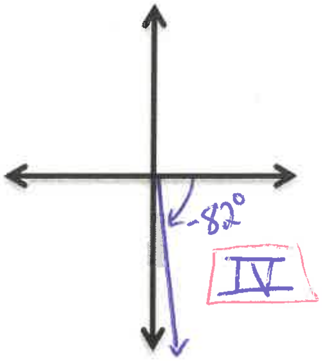
b) 155°



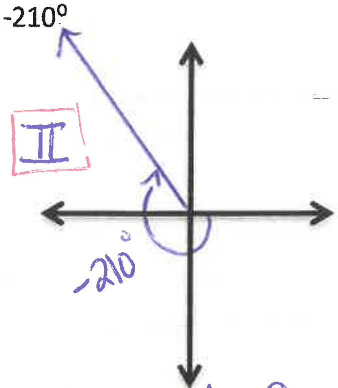
c) 300°



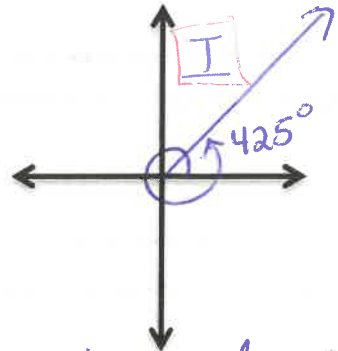
d) -82⁰



e) -210⁰



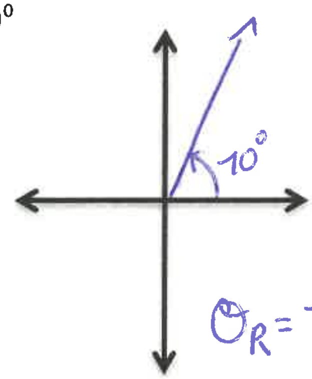
f) 425⁰



Reference Angle – the positive acute angle formed by the terminal arm and the nearest x-axis (θ_R = symbol)

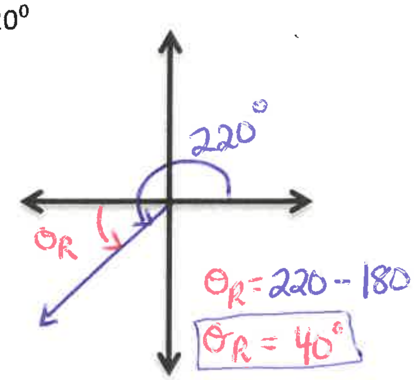
Ex.#2/ Find the reference angle for each of the following:

a) 70⁰



$\theta_R = 70°$

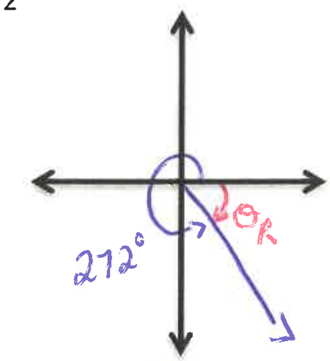
b) 220⁰



$\theta_R = 220 - 180$

$\boxed{\theta_R = 40°}$

c) 272⁰



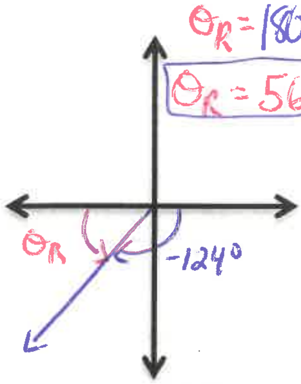
$\theta_R = 360 - 272$

$\boxed{\theta_R = 88°}$

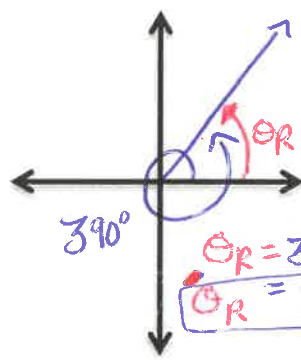
d) -124⁰

$\theta_R = 180 - 124$

$\boxed{\theta_R = 56°}$



e) 390⁰



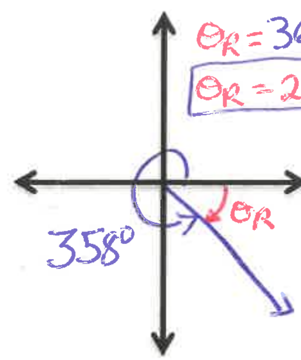
$\theta_R = 390 - 360$

$\boxed{\theta_R = 30°}$

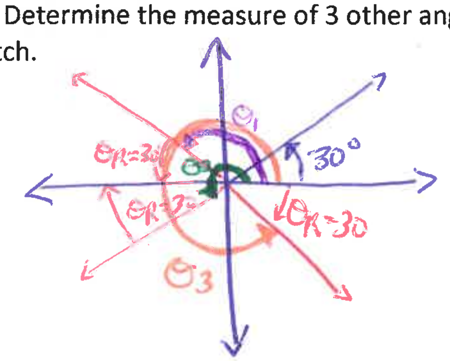
f) 358⁰

$\theta_R = 360 - 358$

$\boxed{\theta_R = 2°}$



EX #3: Determine the measure of 3 other angles in standard position 0°> Θ>360° that have a reference angle of 30°. Sketch.



$\theta_1 = 180 - 30 = 150°$

$\theta_2 = 180 + 30 = 210°$

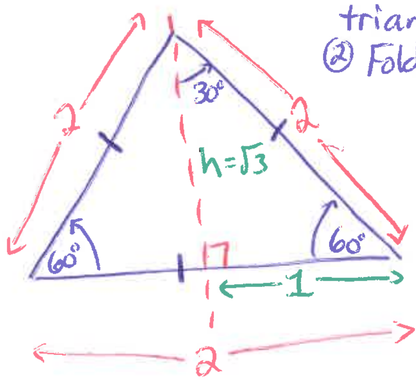
$\theta_3 = 360 - 30 = 330°$

2.1 DAY 2 Special Triangles:

Pg 76 Complete Part B: Create a 30^0- 60^0-90^0 triangle #6-8

Recall: SOH CAH TOA Pythagorean Theorem $a^2+b^2=c^2$

30^0- 60^0-90^0 triangle ① Start with an equilateral triangle with side lengths = 2 units
② Fold the triangle in half.



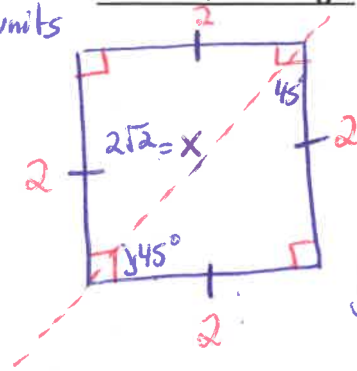
$h^2 = 2^2 - 1^2$

$h^2 = 4-1$

$\sqrt{h^2} = \sqrt{3}$

$h = \sqrt{3}$

45^0-45^0-90^0 triangle ① Start with a square with side lengths = 2units



$x^2 = 2^2 + 2^2$

$x^2 = 4+4$

$\sqrt{x^2} = \sqrt{8}$

$x = \sqrt{8}$

$x = \sqrt{4 \cdot 2}$

$x = 2\sqrt{2}$ units

- We can determine the ***exact values*** of the trig ratios of these angles

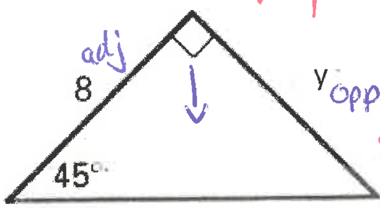
Complete the chart using **exact values**

θ	Sin θ	Cos θ	Tan θ
30^0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ ☆rationalize
60^0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{1} = \sqrt{3}$
45^0	$\frac{\cancel{2}^1}{\cancel{2}_1\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ☆rationalize	$\frac{\cancel{2}^1}{\cancel{2}\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ☆rationalize	$\frac{2}{2} = 1$

EX #4: Use special triangles to determine the missing lengths, (leave as exact values)

★replace Tan45 w/ exact value.

a)



$\tan 45 = \frac{y}{8}$

$8 \cdot 1 = \frac{y}{8} \cdot 8$

$\boxed{8 = y}$

$(x)\ \cos 45 = \frac{8}{x}(x)$

$\frac{x(\cos 45°)}{\cos 45} = \frac{8}{\cos 45}$

$x = \frac{8}{\frac{\sqrt{2}}{2}}$

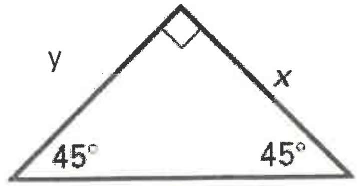
$x = 8 \div \frac{\sqrt{2}}{2} \rightarrow x = 8 \cdot \frac{2}{\sqrt{2}}$

$x = \frac{16 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$

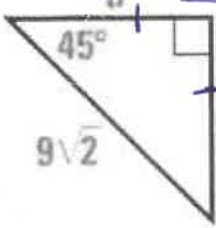
$x = \frac{16\sqrt{2}}{2}$

$\boxed{x = 8\sqrt{2}}$

b)



c)



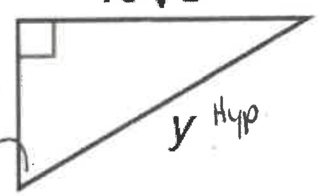
$\sin 45° = \frac{b}{9\sqrt{2}}$

$\frac{2}{2\sqrt{2}} = \frac{b}{9\sqrt{2}}$

$\frac{2 \cdot 9\sqrt{2}}{\sqrt{2}} = b$

$\boxed{b = 9 \quad a = 9}$

d)



$\sin 60° = \frac{10\sqrt{3}}{y}$

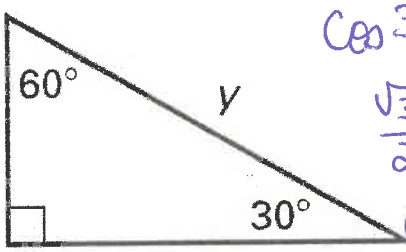
$\frac{\sqrt{3}}{2} = \frac{10\sqrt{3}}{y}$

$\left(\frac{2}{\sqrt{3}}\right) y \frac{\sqrt{3}}{2} = 10\sqrt{3}\left(\frac{2}{\sqrt{3}}\right)$

$y = 20$

$20^2 - (10\sqrt{3})^2 = x^2$

e)



$\cos 30° = \frac{x}{10}$

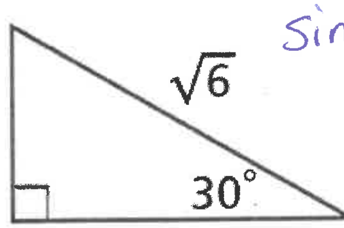
$\frac{\sqrt{3}}{2} = \frac{x}{10}$

$\boxed{5\sqrt{3} = x}$

$y^2 = (5\sqrt{3})^2 + 10^2$

$y = \sqrt{75 + 100}$ $\quad y = \sqrt{175}$ $\quad y = \sqrt{25 \cdot 7}$ $\quad \boxed{y = 5\sqrt{7}}$

f)



$\sin 30° = \frac{x}{\sqrt{6}}$

$\frac{1}{2} = \frac{x}{\sqrt{6}}$

$\boxed{\frac{\sqrt{6}}{2} = x}$

$(\sqrt{6})^2 - \left(\frac{\sqrt{6}}{2}\right)^2 = y^2$

$\frac{6 \cdot 4}{1 \cdot 4}\ \frac{6}{4} = y^2$

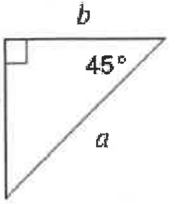
$y^2 = \frac{24}{4} - \frac{6}{4}$

$\sqrt{y^2} = \sqrt{\frac{18}{4}}$

$\boxed{y = \frac{3\sqrt{2}}{2}}$

2.1 Assignment: Page 83 #1 – 7, 9 and questions below:

1.



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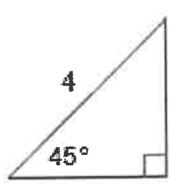
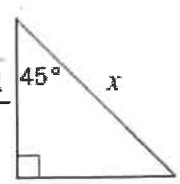
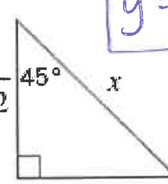


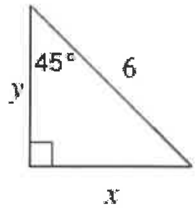
3.



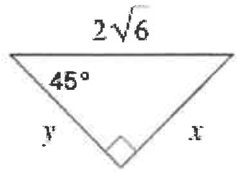
4.



5. ~~7.~~ ~~8.~~



6.



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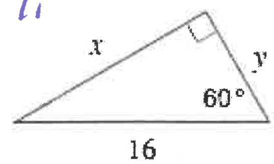

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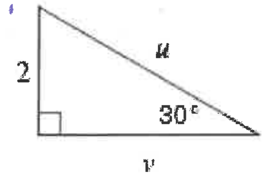

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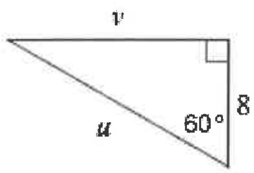

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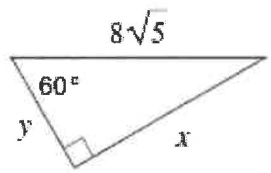

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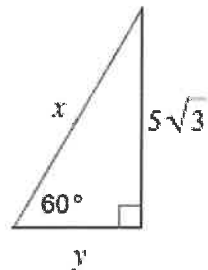

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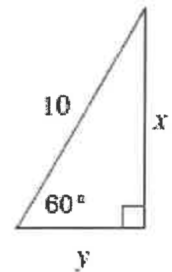
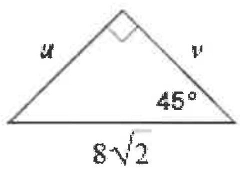


13.



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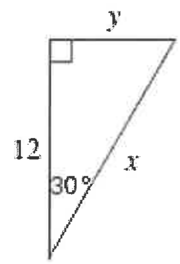

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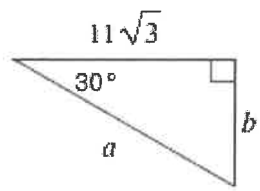
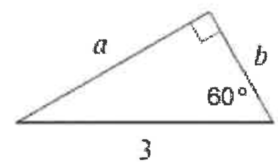


16.



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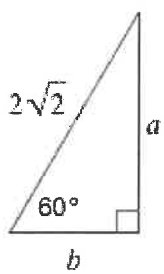
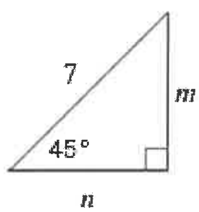




18.

SOLUTIONS TO EXTRA QUESTIONS

1. $a = 4; b = 2\sqrt{2}$
2. $x = 2\sqrt{2}; y = 2\sqrt{2}$
3. $x = 3; y = \frac{3\sqrt{2}}{2}$
4. $x = 6; y = 3\sqrt{2}$
5. $x = 3\sqrt{2}; y = 3\sqrt{2}$
6. $x = 2\sqrt{3}; y = 2\sqrt{3}$
7. $x = 8\sqrt{3}; y = 8$
8. $u = 4; v = 2\sqrt{3}$
9. $u = 16; v = 8\sqrt{3}$
10. $x = 4\sqrt{15}; y = 4\sqrt{5}$
11. $x = 10; y = 5$
12. $x = 5\sqrt{3}; y = 5$
13. $u = 8; v = 8$
14. $x = 8\sqrt{3}; y = 4\sqrt{3}$
15. $a = \frac{3\sqrt{3}}{2}; b = \frac{3}{2}$
16. $a = 22; b = 11$
17. $a = \sqrt{6}; b = \sqrt{2}$
18. $m = \frac{7\sqrt{2}}{2}; n = \frac{7\sqrt{2}}{2}$

2.2 Trigonometric Ratios of any Angle

1. Find the 3 trig values for θ

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

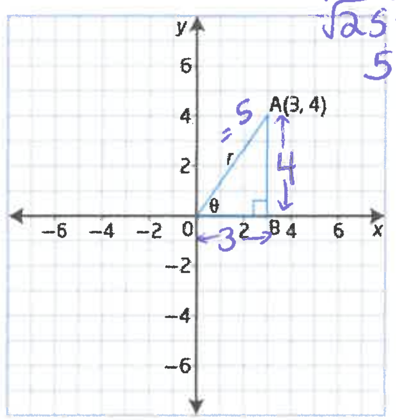
$\tan\theta = \frac{4}{3}$

$4^2+3^2=r^2$

$16+9=r^2$

$\sqrt{25}=\sqrt{r^2}$

$5=r$

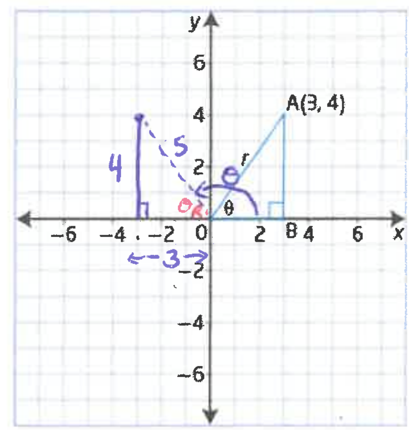


2. Reflect this triangle about the y-axis. Find the 3 trig values for this new angle

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{-3}{5}$

$\tan\theta = \frac{4}{-3}$

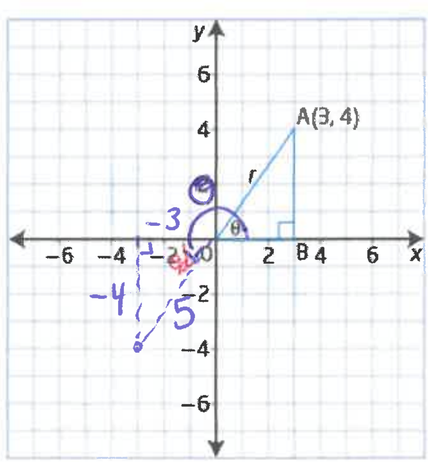


3. Reflect this triangle about the y-axis and then the x-axis. Find the 3 trig values for this new angle.

$\sin\theta = -\frac{4}{5}$

$\cos\theta = \frac{-3}{5}$

$\tan\theta = \frac{-4}{-3} = \frac{4}{3}$

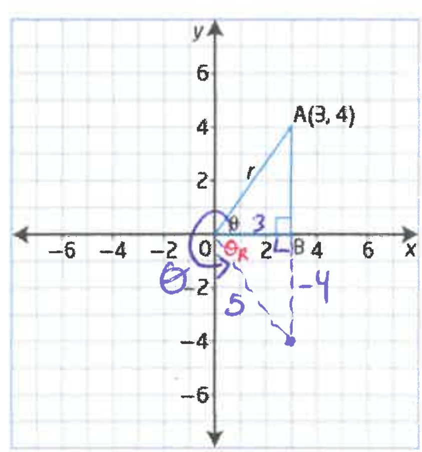


4. Reflect this triangle about the x-axis. Find the 3 trig values for this new angle

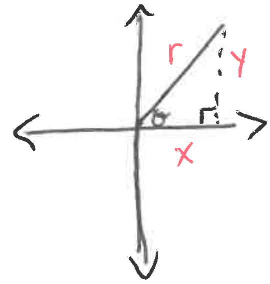
$\sin\theta = \frac{-4}{5}$

$\cos\theta = \frac{3}{5}$

$\tan\theta = \frac{-4}{3}$



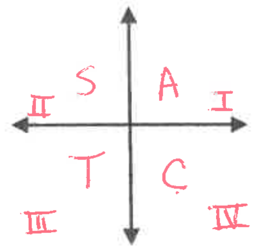
1. **The Trig ratios for θ can be written as follows:**



$\sin\theta = \frac{opp}{hyp}$ $\qquad \cos\theta = \frac{adj}{hyp}$ $\qquad \tan\theta = \frac{opp}{adj}$

$\sin\theta = \frac{y}{r}$ $\qquad \cos\theta = \frac{x}{r}$ $\qquad \tan\theta = \frac{y}{x}$

2. **The ASTC (All Soup Turns Cold) Rule**



- In the 1st quadrant, **ALL** trig functions have positive values.
- In the 2nd quadrant, the **SINE** function has positive values.
- In the 3rd quadrant, the **TANGENT** function has positive values.
- In the 4th quadrant, the **COSINE** function has positive values.

Ex#1/ Given the following description, in which quadrant does the terminal arm of the angle lie?

a) $\cos\theta > 0$ *and* $\tan\theta < 0$

Cos θ + $\qquad$ Tan θ −

IV

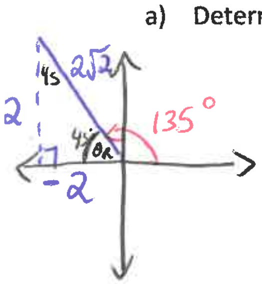
b) $\sin\theta > 0$*and* $\tan\theta > 0$

sin θ + $\qquad$ tan θ +

I

Ex#2/ **Determining the exact value of a trig ratio given its angle which has a reference angle of 30, 45 or 60**

a) Determine the exact value of $\cos 135^0$

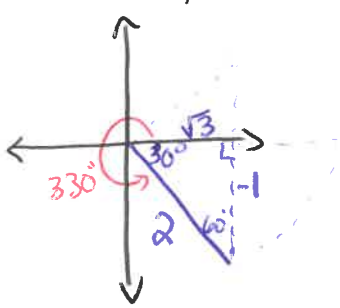


$\theta_R = 45°$

$\cos 135 = \frac{-2}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\boxed{\cos 135 = \frac{-\sqrt{2}}{2}}$

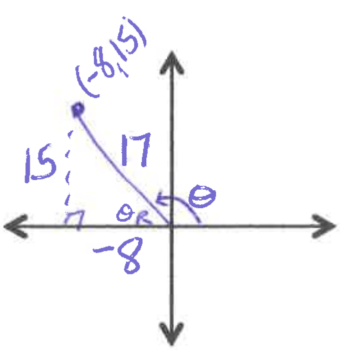
b) Determine the exact value of $\sin 330^0$



$\sin 330 = \frac{-1}{2}$

Ex #3/ **Determine the exact trig ratios of an angle given a point on the terminal arm**

The point (-8, 15) is on the terminal arm of an angle θ in standard position. Draw the angle. Create a right triangle. Determine the exact trig ratios for sin θ, cos θ, tan θ. Determine θ to the nearest thousandth of a degree.



$15^2 + (-8)^2 = r^2$

$225 + 64 = r^2$

$\sqrt{289} = \sqrt{r^2}$

$17 = r$

$\sin\theta = \frac{15}{17}$ $\qquad \sin^{-1}\left(\frac{15}{17}\right) = 61.928°$

$\cos\theta = \frac{-8}{17}$ $\qquad \cos^{-1}\left(\frac{-8}{17}\right) = 118.072°$

$\tan\theta = \frac{15}{-8}$ $\qquad \tan^{-1}\left(\frac{15}{-8}\right) = -61.928°$

$\boxed{\theta = 118.072°}$

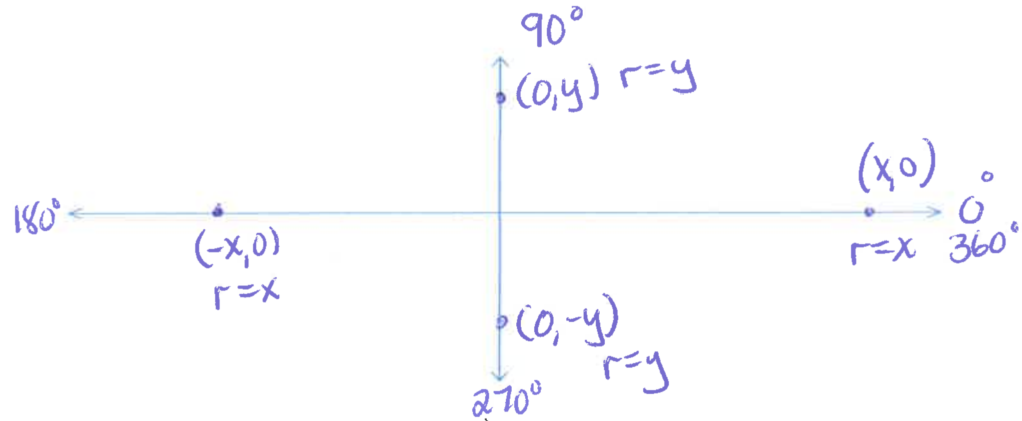
Determining Trig Ratios of Quadrantal Angles

	0°	90°	180°	270°
Sin Θ $= \frac{y}{r}$	0	1	0	−1
Cos Θ $= \frac{x}{r}$	1	0	−1	0
Tan Θ $= \frac{y}{x}$	0	Undefined	0	Undefined

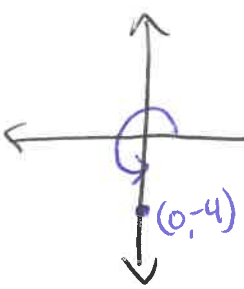
Recall: $\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$



Ex#4/ Suppose θ is an angle in standard position with point (0, -4) on the terminal arm . What are the exact values of cos θ , sin θ and tan θ? What is θ ?

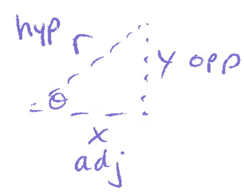


$\theta = 270°$

$\cos\theta = \frac{0}{-4} = 0$

$\sin\theta = -1$

$\tan\theta = \text{undefined.}$



Handout assignment: EXACT VALUES and Pg 96 #1-6

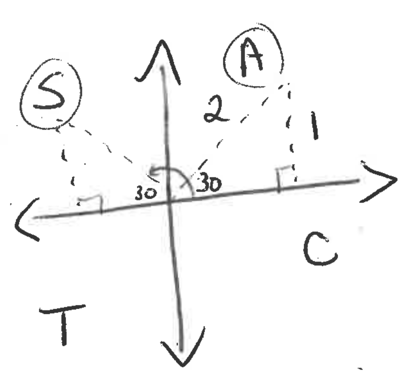
2.2 Day 2

Ex#5/ Solving for Angles Given Their Sine, Cosine, or Tangent ratio

Steps:
1. Determine which quadrant the solution(s) will be in by looking at the sign (+ or -) of the given ratio.
2. Solve for the reference angle (Θ_R)
3. Sketch the reference angle in the appropriate quadrant. For exact values: Use the diagram to determine the measure of the related angle in the standard position.

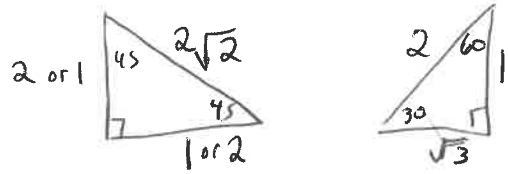
a) Given Exact Trig Value (Use special triangles)
Example: If sin Θ = ½ (0° ≤ Θ ≤ 360°), solve for Θ.

$\sin\theta = \frac{opp}{hyp} = \frac{Y}{r}$



$\theta = 30°$ in Quad I

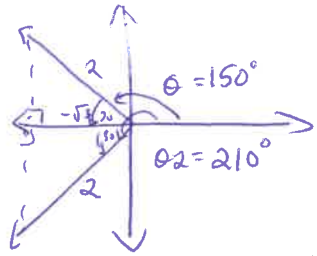
$\theta = 150°$ in Quad II



adj

b) If $\cos\theta = -\frac{\sqrt{3}}{2}$ (0° ≤ Θ ≤ 360°), solve for Θ.

hyp

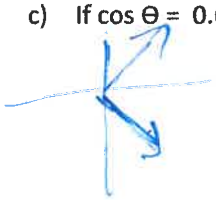


$\theta = 150°$

$\theta = 210°$

Given Approximate Trig Value (use calculator) Use $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ to find the angle.

c) If cos Θ = 0.6753 (0° ≤ Θ ≤ 360°), solve for Θ to the nearest tenth of a degree.

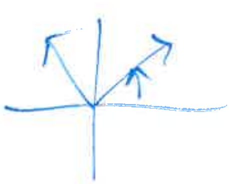


$\theta = \cos^{-1}(0.6753) = 47.5°$ and $360 - 47.5$

Q1 $312°$

Q4

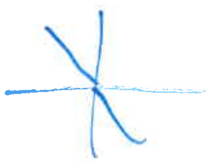
d) If sin Θ = 0.8090 (0° ≤ Θ ≤ 360°), solve for Θ to the nearest tenth of a degree.



$\theta = \sin^{-1}(0.8090) =$ $180 - 54°$

~~55~~ $54°$ & $125°$

e) If tan Θ = -0.3675 (0° ≤ Θ ≤ 360°), solve for Θ to the nearest tenth of a degree.



$\theta = \tan^{-1}(-0.3675)$

$\theta = -20° \rightarrow$ so $360 - 20°$ and $180 - 20°$

$340°$ $160°$

2.2 Day 2

Pg 96 #7,8,9,12,13,28,29